Students' Learning Difficulty in Infinite Sequence and Series

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Abstract: This research aims to describe a difficulty encountered by students of Mathematics education department of State Institute for Islamic Studies of Lhokseumawe in solving problems of infinite sequence and series, particularly problems related to concepts and principles. Students' performance is low in a test about infinite sequence and series. The students' answer sheets of the test are subsequently used as data sources for a difficulties analysis by identifying the errors they made related to concepts and principles. Qualitative descriptive analysis is applied as the analysis methods. The result shows that there are three types of difficulties related to understanding concept and two types of difficulties related to understanding principles which could be categorized as high-level difficulty. The difficulties related to understanding concepts included: (1) identifying example and non-example of concepts; (2) using models, figures, and symbols to represent concepts; and (3) translating from one presentation model into another presentation models. The difficulties related to understanding principles included: (1) recognizing when a principle is required; and (2) appreciating the roles of principles in mathematics. Based on the results, possible future research and suggestions on effective teaching and learning for infinite sequence and series will be discussed.

1 INTRODUCTION

Calculus is a branch of mathematics which plays an important role in the development of technology and other knowledge fields. It is a means in the knowledge world to elucidate changes (Purwanto, Indriani, &Dayanti, 2005), which has been used in various life aspects. Almost all applied science such as engineering, agriculture, medical, pharmacy etc. make use of calculus concepts in their development. Besides, the social science such as economy, psychology, etc. also require calculus concepts.

Despite the strategic role of the calculus in the development of technology, many studies showed students' low performance and learning difficulties in the course. More particularly, students' learning difficulties in calculus have been among the research focuses in the area of mathematics education research since around 1990s (e.g., Anderson &Loftsgaarden, 1987; Tall, 1993). Some topics which have been found as common difficulties faced by students in calculus include the concepts of limit (Maharaj, 2010) and infinity (Kim, Sfard, &Ferrini-Mundy, 2005).

Several factors, included affective and cognitive ones, have also been identified with relate to the difficulties of learning calculus among university students in Indonesia. The affective factors included low of interest (Mutakin, 2015), metacognition, motivation, and attitude (Hidayat, 2013) in learning the course. On the other hand, the cognitive factors included low of prior knowledge in algebra and limit (Wahyuni, 2017), and in trigonometric concepts and principles (Abidin, 2012).

The findings from the latter two studies showed that the cognitive factors affecting the difficulties in learning calculus were the low of prior knowledge in basic concepts of mathematics. This might be explained by the fact that the topics in mathematics are interrelated to each other. The lack of comprehension in one concept might affect the learning in the more advanced concepts. Therefore, students need to acquire complete knowledge in basic concepts of mathematics, which are known as competency standards and basic competency of mathematics in the curriculum, in order to be able to learn more advanced mathematics concepts. In fact, many calculus students encountered difficulties in comprehending learning materials in calculus(e.g. Wahyuni, 2017)and in achieving the specified competency standards and basic competency of mathematics. In the calculus classes in Mathematics Department of State Institute for Islamic Studies

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(IAIN)Lhokseumawe, for instance, there have been always more than a half of students who could not achieve the minimum mastery learning score of 65. If such phenomena are ignored, students would encounter more obstacles in other advanced courses which require calculus as a prerequisite knowledge, such as differential equation, real analysis, complex analysis, etc.

Based on the above arguments, the goal of this study was to analyze students' learning difficulties in the topic of infinite sequence and series, particularly in solving problems related to understanding concepts and principles. The topic of infinite sequence and series was the beginning topic in the Advanced Calculus Course for students of Mathematics Education Department of IAIN Lhokseumawe. By knowing the difficulties students encountered in the topic, it may shed light on the appropriate steps to be taken in improving the teaching and learning of the topics.

2 METHODS

The qualitative descriptive approach was used to describe students' difficulties in learning the topic of infinite sequence and series in this study. The participants involved, and the instruments used in this study are described in the followings.

2.1 Participants

The 28 third year students of Mathematics Education Department of IAIN Lhokseumawe were involved as the participants of this study. These students were selected from the total of 38 students who were taking the Advanced Calculus Course based on their low performance (i.e., whose grades were C+ or below) on a test in the topic of infinite sequence and series. The Advanced Calculus course is a compulsory course for these students, which consists of several topics including Infinite Sequence and Series, Positive Series of Integral Test and Other Tests, Alternating Series, Absolute Convergence, Power Series, Operations for Power Series, Taylor and McLaurin Series, and Multiple Integral. The students were separated in two classes, no difference in their ability. One of the researchers was the lecturer of the course. The research was conducted in the first semester of academic year of 2017/2018.

2.2 Instruments

A test on the topic of infinite sequence and series was used as the instrument in this study, which was administered to the participants after they learned the topic. The test consisted of six items designed to examine students' difficulties in understanding concepts and principles related to the topic.

Six indicators of concept understanding and three indicators of principle understanding proposed by Cooney, Davis, and Henderson (1975) were referred to analyze students' answers to the test items. The six indicators of concept understanding include:

- assigning, describing by using words and defining a concept;
- identifying example and non-example of a concept;
- using models, figures, and symbols to represent a concept;
- translating from one representation model into another representation model;
- identifying the characteristics of a given concept;
- comparing and emphasizing concepts.

The three indicators of principle understanding include:

- recognizing when a principle is required;
- using a principle properly;
- appreciating the roles of principles in mathematics.

3 RESULTS AND DISCUSSIONS

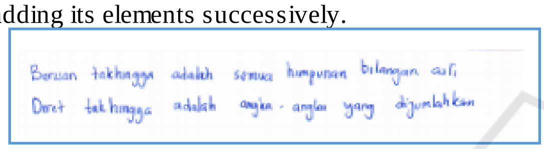
The results of the analyses on students' answers on the test showed that students encountered different level of difficulties in solving the given problems. The percentages of students who performed errors in solving the problems were used to categorize the levels of difficulties for each indicator of understanding concept and principle. We elaborated these difficulties for the six indicators of understanding concepts and those for the three indicators of understanding principles in the followings. The test item related to each indicator and students' errors in solving the item would be used to discuss the difficulties.

3.1 Difficulties in understanding concepts

There were different levels and types of difficulty encountered by students related to the six indicators of understanding concepts.

3.1.1 The indicator of assigning, describing by using words and defining concept

The test item used related to this indicator was: "*What are the definitions of infinite sequence and infinite series?*" The answer should be that infinite sequence is a function whose domain is the set of natural numbers. Infinite series is a sequence of numbers from which a new sequence can be produced by adding its elements successively.



Translation: Infinite sequence is all sets of natural numbers. Infinite series is the numbers which are added together.

Figure 1: One of student's error in assigning, describing by using words and defining a concept.

One of the students answered to this question as illustrated in Figure 1. From her answer, it could be inferred that the student understood the idea of infinite sequence and series. However, she had difficulty to describe the definitions by using proper mathematical words.

There were about half of students (50.98%) who encountered such difficulty, which means that less than a half of students who could describe the definitions correctly. Thus, the indicator of assigning, describing by using words and defining concept was categorized into the medium level of difficulty.

3.1.2 The indicator of identifying examples and non-examples of a concept

The test item for this indicator was: "*Identify the convergence of the sequence* $a_n = \frac{n}{(2n-1)!}$"

A sample of student's answer is given in Figure 2. The error in the student's answer in Figure 2 was that directly replacing *n* with ∞, which might indicate that the student did not have full understanding on how to solve limit problems.

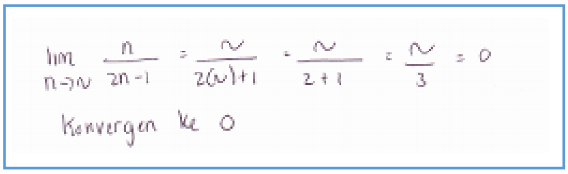


Figure 2: One of student's error in identifying examples and non-examples of a concept

There were many students (83.33%) in this study encountered difficulty related to solving limit problems. Therefore, the indicator of identifying examples and non-examples of a concept was categorized as very high level of difficulty.

3.1.3 The indicator of using models, figures, and symbols to represent a concept

The test item related to this indicator was: "*Determine the explicit formula for* a_n *from* $-1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, -\frac{5}{9}, \ldots$".

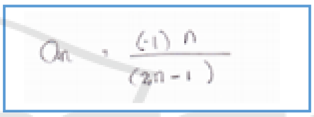


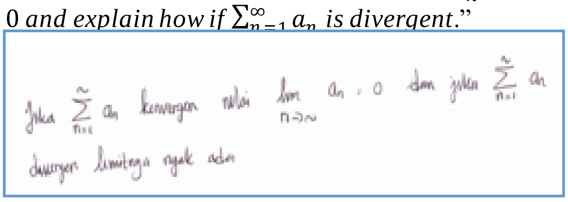
Figure 3: One of student's error in using models, figures, and symbols to represent a concept

A sample of student's answer is shown in Figure 3. The error in the above answer was that the formula would result in the negative values. The student might refer to the first number in the sequence in assigning (-1) for the formula and ignore the negative or positive value in the subsequent numbers in the sequence.

Such kind of errors was performed by many students in this study (80.39%). Hence, the indicator of using models, figures, and symbols to represent a concept was categorized as very high level.

3.1.4 The indicator of translating from one representation model into another representation model

The test item for this indicator was: "*Change into the mathematics words:* $\sum_{n=1}^{\infty} a_n$ is convergent $\lim_{n\to\infty} a_n = 0$ *and explain how if* $\sum_{n=1}^{\infty} a_n$ *is divergent*."



Translation: If $\sum_{n=1}^{\infty} a_n$ is convergent, the value of $\lim\limits_{n\to\infty} a_n = 0$ and if $\sum_{n=1}^{\infty} a_n$ is divergent, the limit is not available.

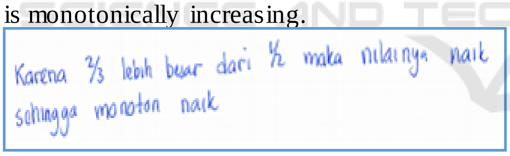
Figure 4: One of student's error in translating a model.

Figure 4 shows a sample of student's answer. The answer was improper, since there is no further explanation what it means that the limit is not available. There were many students (86.27%) encountered difficulty in solving this problem, which indicated that the indicator of translating from one representation model into another representation model was in the very high level of difficulty.

3.1.5 The indicator of identifying the characteristics of a given concept and recognizing the conditions specified by a concept

The test item used related to this indicator was: "*Specify whether or not the sequence* $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots, \frac{n}{(n+1)}$ *is monotonic*."

Figure 5 presents a sample of student's answer, which was actually true that the sequence is monotonically increasing. However, to specify that the sequence is monotonic, the definition of monotonic sequence should be used. Based on the answer, the student seems misunderstood about the characteristics of monotonic sequence. It is not always true that when a sequence is increasing then it is monotonically increasing.



Translation: Since 2/3 is larger than 1/2 then the value is monotonically increasing.

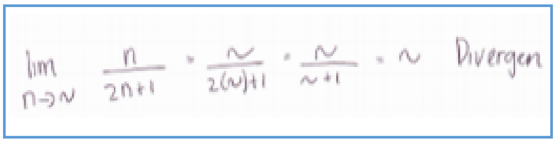
Figure 5: One of student's error in identifying the characteristics of a given concept and recognizing the condition specifies by a concept.

There were more than a half of students (56.86%) incorrectly answer this problem. Thus, the indicator of identifying the characteristics of a given concept and recognizing the conditions specified by a concept was categorized as the medium level of difficulty.

3.1.6 The indicator of comparing and emphasizing concepts

The test item related to this indicator was: "*Prove that* $2n + 1$ *is convergent to* ½."

Figure 6: One of student's error in comparing and



emphasizing concepts.

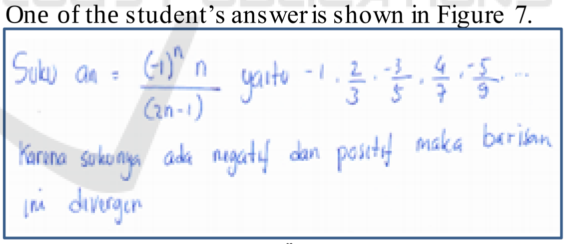
Based on the procedure used in the above answer, it could be inferred that the student had lack of basic knowledge related to solving limit problems. We found that there were some students (54.36%) encountered such difficulty with this item. Hence, the indicator of comparing and emphasizing concepts was categorized in the medium level of difficulty.

3.2 Difficulties in understanding principles

There were different errors and different difficulty levels encountered by students related to understanding principles in each indicator. These different errors are elaborated for the three indicators below.

3.2.1 The indicator of recognizing when a principle is required

The test item related to this indicator was: "*State the definition of the convergence of infinite sequence*." One of the student's answer is shown in Figure 7.



Translation: The term $a_n = \frac{(-1)^n n}{(2n-1)}$ are $-1, \frac{2}{3}, \frac{-3}{5}, \frac{4}{7}, \frac{-5}{9}, \cdots$ Since the terms have negative and positive values, then the sequence is divergent.

Figure 7: One of student's error in recognizing when a principle is required.

We could see that the above answer did not correspond to the definition of convergent and divergent sequence. The student showed an example of infinite sequence to explain the meaning of divergent sequence, which might indicate that she did not understand how to state the definition

There were almost seventy percent of students (68.62%) showed errors in solving this problem.

Hence, the indicator of recognizing when a principle is required was categorized in high level of difficulty.

3.2.2 The indicator of using principles properly

The test item related to this indicator was: "*Specify whether the sequence* $a_n = \frac{(-1)^n n}{(2n-1)}$ *is convergent or divergent*."

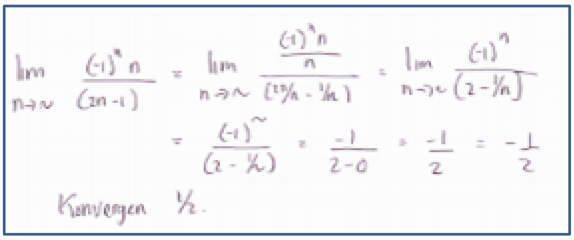


Figure 8: One of student's errors in using principle properly.

Figure 8 presents one of the student's answer to the item. The error shown in the answer is related to the incorrect defining of $(-1)^{\infty}$. It should be $(-1)^{\infty} = \infty$, not -1.

There were some students (59.80%) found difficult in solving infinite sequence and series with regard to this item. Thus, the indicator of using principles properly was in the medium level of difficulty.

3.2.3 The indicator of appreciating the roles of principles in mathematics

The item related to this indicator was the same item used for the previous indicator. Figure 9 showed one of student's answer. From the above answer, the student solved the limit correctly. However, when deciding whether the sequence is convergent or divergent, the student did not apply the definition of convergent sequence, which resulted in the incorrect of drawing conclusion about the convergence of the sequence.

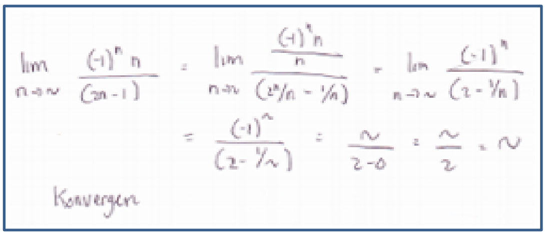


Figure 9: One of student's error in appreciating the roles of principles in mathematics.

There were many students (70.58%) performed error related to this indicator. In this case, they found difficult in solving infinite sequence and series by using definitions and theorems which have been learned. Therefore, the indicator of appreciating the roles of principles in mathematics was categorized in the high level of difficulty.

3.3 Discussions

The analysis results presented in the previous subsections showed different types and levels of difficulty encountered by students in solving problems in the topic of infinite sequence and series. The error performed by most of students related to understanding concepts was when they need to determine an explicit formula of a_n. When the formula is incorrectly determined, specifying its convergence or divergence will be also incorrect.

In specifying convergence and divergence of a_n in test item 2, many students encountered difficulty with finding limit value. It seems that they did not have adequate knowledge related to solving limit problems, which should has been learned in Calculus I. Student's difficulty related to limit problems has been acknowledged in several studies (e.g.,Wahyuni, 2017). Therefore, in order to overcome this difficulty, more attention in the learning of topic of limit in Calculus I is required. For instance, conceptual understanding of limit can be developed by using algorithmic contexts (Pettersson & Scheja, 2008).

The use of principles in each step of solving a mathematics problem is mostly interrelated. Thus, the incorrect use of the principle in the previous step would continue to the following steps. Despite the concepts and principles of infinite sequence and series given in the test items in this study have been taught to students, many students failed to recall and use them in solving the test items. This might be the result of the students not having appropriate mental structures (Maharaj, 2010) or that they had the instrumental understanding (Skemp, 1976) of the concepts and principles related to infinite sequence and series. Since the desired goal of calculus should be relational understanding (Skemp, 1976), providing students with a range of experiences in the Advanced Calculus class that develop the ideas of the topic so that they both knows and understands might be an alternative way of teaching and learning the course.

Based on the findings, the effective ways to facilitate and improve students' conceptual understanding of limit might be one of the topics of interest for future research in the area of mathematics education. Although such study has been conducted by Pettersson & Scheja (2008), more recent studies involving diverse participants from different cultures could enrich and contribute on a more fruitful discussions. Other directions for future research might be about how to improve students' memory related to definitions and theorems and their skills in using definitions and theorems to solve problems in calculus.

4 CONCLUSIONS

The results of this study have shown the different types and levels of difficulties encountered by the second-year students of Mathematics Education Department of IAIN Lhokseumawe in solving problems in the topic of infinite sequence and series. The difficulties included three types of difficulties in understanding a concept and two types of difficulties in understanding a principle which can be categorized into high level of difficulty. The difficulties in understanding a concept included the difficulty in identifying example and non-example of concepts (83.33%), the difficulty in using models, figures, and symbols to represent concepts (80.39%), and the difficulty in translating from one presentation model into another presentation model (86.27%). On the other hand, the difficulties in understanding a principle included difficulty in recognizing when a principle is required (68.62%) and the difficulty in appreciating the roles of principles in mathematics (70.58%).

The errors performed by most of the students related to understanding a concept was due to their low of prior knowledge related to solving limit problems and in determining the convergence or divergence of an infinite sequence or series. Besides, they could not recall the definitions and theorems related to infinite sequence and series. With relate to understanding principles, many students encountered difficulties in using definitions and theorems which have been learned in solving problems.

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