Detection of Heat Conduction Disturbance in Cylindrical-Shaped Metal Chip using Kalman Filter and Ensemble Kalman Filter

Nina Fitriyati, Gina Isma Kusuma and Irma Fauziah
Mathematics Study Program, Faculty of Sciences and Technology
UIN Syarif Hidayatullah Jakarta, Jl. Ir. H. Juanda No. 95 Ciputat, Banten, Indonesia

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Abstract: The heat transfer process will be disrupted when a leak occurs. Therefore, we need a method that can be used to detect the leak. In this paper, the leak detection in cylindrical-shaped metal chip simulated by give the heat disturbances in some positions. We discuss the estimation of heat disturbance position using the Kalman filter (KF) and the Ensemble Kalman Filter (EnKF) method where the state-space equation is constructed by discretization of the diffusion equation using Forward-Time Central Space Method. We divide the radius of this metal chip into 17 grids and simulate the detection of 1–4 disturbances in different positions. The simulation result shows that the KF and EnKF method succeed to detect the disturbances. However, the EnKF is more accurate than KF. The heat disturbances can be detected more clearly if the temperature of disturbance is large enough, especially for detection in the edge of chip (close to inner radius and outer radius). In addition, the detection of disturbances location is also affected by the number of grids. The more number of grids, the more accurate the position of detection.

1 INTRODUCTION

Heat transfer is the process of transferring heat from objects that have high temperatures to the objects with lower temperatures. The flow of heat is all-pervasive. There are three modes of heat transfer i.e. conduction, convection, and radiation. Conduction is one process of heat transfer from one solid to another one that has a different temperature. Heat convection is transfer of heat in fluid or gases, and thermal radiation occurs in a range of electromagnetic of energy emission (Lienhard, 1930).

One obstacle that can cause resistance to heat conduction is the leakage of the conductor media. Mathematically, several methods have been developed to detect leaks in metals including the Kalman filter and its development methods: adaptive particle filter (Liu et al., 2005), Extended Kalman filter (Emara-Shabaik et al., 2002), and EnKF (Apriliani, 2011).

Inspired by Apriliani (2011), in this study, we will detect the heat disturbances and its location in the cylindrical-shaped metal chip using the Kalman filter method and EnKF. The state-space equation will be formed by the result of discretization of the diffusion equation using Forward-Time Central Space (FTCS) Method.

2 METHODOLOGY

According to Carslaw and Jeager (1959), the three dimensional of heat equation in cylindrical coordinates can be expressed by:

$$\frac{\partial v}{\partial t} = k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2}\right), \quad (1)$$

where v is temperature, t is time, r is radius and k is conductivity. If we heat the cylindrical with the axis coincides with the z axis, the initial and boundary conditions are independent of the coordinates of θ and z.

The steady-state is a condition when several process variables such as pressure, temperature, location or position do not change with time. With this steady-state, a process will be more easily

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managed and planned. One-dimensional heat conduction in a steady-state condition based on Equation (1) is

$$\frac{dv}{dr}\left(r\frac{dv}{dr}\right) = 0, \quad a < r < b. \qquad (2)$$

The solution for equation (2) with the initial conditions and boundaries: $v(r,0) = 0,\ v(a,t) = v_1$ and $v(b,t) = v_2,\ a < r < b$ is

$$v = v_1 + v_2 \ln r, \qquad (3)$$

where v is temperature. This heat transfer can be illustrated in figure 1 and this object called cylindrical-shaped metal chip.

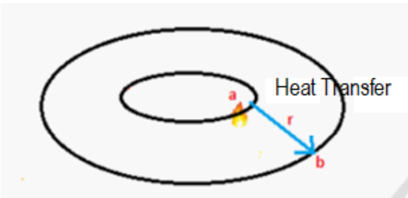

Figure 1: The heat transfer in the cylindrical metal chip.

The FTCS discretization for Equation (2) is

$$v_i^{k+1} = pv_{i+1}^k + (1-2)v_i^k + pv_{i-1}^k, \qquad (4)$$

where $p = -r\frac{\Delta r}{\Delta r^2}$. The general form of equation (4) is

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ \vdots \\ v_n \end{bmatrix}^{k+1} = \begin{bmatrix} 1-2p & p & 0 & \dots & \dots & 0 \\ 0 & 1-2p & p & 0 & \dots & 0 \\ 0 & p & 1-2p & \ddots & \ddots & \vdots \\ \vdots & 0 & p & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & p \\ 0 & 0 & 0 & 0 & p & 1-2p \end{bmatrix}^{k} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ \vdots \\ v_n \end{bmatrix}^{k} + \begin{bmatrix} pv_0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

Let

$$x_{k+1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ \vdots \\ v_n \end{bmatrix}^{k+1}, \quad A = \begin{bmatrix} 1-2p & p & 0 & \dots & \dots & 0 \\ 0 & 1-2p & p & 0 & \dots & 0 \\ 0 & p & 1-2p & \ddots & \ddots & \vdots \\ \vdots & 0 & p & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & p \\ 0 & 0 & 0 & 0 & p & 1-2p \end{bmatrix}^{k},$$

$$B = pv_0,\ u_k = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}.$$

Therefore, we can write:

$$x_{k+1} = Ax_k + Bu_k. \qquad (5)$$

In equation (5), it is assumed that the system is completely isolated but in fact there is a disturbance, called noise, in the transfer of heat between metal pieces and air. Let us denote this noise by w_k. Then

$$x_{k+1} = Ax_k + Bu_k + Ed_k + w_k, \qquad (6)$$

where w_k is assumed to be $N(0,Q)$ distributed. Equation (6) is called the equation of state (Kalman, 1960). The measurement equation is formed from

$$z_{k+1} = Hx_k + \eta_k, \qquad (7)$$

where η_k is a matrix represents the disturbance in the measurement equation which is assumed to be $N(0,R)$ distributed. From equation (6) and (7) we can form the state-space representation.

A state-space representation is a basic equation in Kalman filter. The Kalman filter is an algorithm for updating linear projections of this system sequentially (Hamilton, 1994). Kalman filters can estimate the state of a process by minimizing the mean square error. This filter is very resilient in several aspects: it can estimate the past state, current state, and future state, and can be used on systems that contain unknown observations (Tan, 2011). There are 2 steps in the Kalman filter algorithm: the prediction and the correction step with the initial state generated from the normal distribution.

The Kalman filter algorithm is (Kalman, 1960):

Initialisation step: $x_0^u \sim N(\mu_0, P_0)$

Prediction Step:

State: $x_k^f = Ax_{k-1}^u \qquad (8)$

Covariance matrix: $P_k^f = AP_{k-1}^u A^T + Q \qquad (9)$

Correction Step:

State: $x_k^u = x_k^f + K\left(z_k - Hx_k^f\right) \qquad (10)$

where Kalman gain

$$K_k = P_k^f H^T \left(HP_k^f H^T + R\right)^{-1} \qquad (11)$$

Covariance matrix: $P_k^u = (I - K_k H)P_k^f \qquad (12)$

The generalization of Kalman filter for the non-linear dynamical system is EnKF which is introduced by Evensen (Evensen, 2003). This method has been widely used as a sequential data assimilation technique. The EnKF algorithm is based on state-space representations formulated in Equations (6) and (7).

For the EnKF linear convergent linear system to Kalman Filter (Butala et al., 2008, Gland et al., 2009, Mandel et al., 2009, and Tan, 2011). The basic idea in the EnKF algorithm is to obtain a filter that is used for large scale on non linear systems. EnKF is an implementation of Monte Carlo from Kalman Filter for non-linear dynamic systems where the initial state is generated using a sample, called an ensemble, and the covariance matrix is approximated by sample covariance. The EnKF simulation is carried out repeatedly and then assimilates new data and updates the model simultaneously.

Basically, the equations used in the EnKF method are the same with those in the Kalman fiter, equation (8) – (12), but, in the EnKF method, the initial state is generated by the number of ensembles, N_ε. The EnKF algorithm is (Evensen, 2003):

Initialisation step:

$$x^u_{0,i} \sim N(\mu_0, P_0), i = 1,2,\dots,N_\varepsilon$$

Prediction Step:

State: $x^f_{k,i} = f\left(x^u_{k-1,i}\right),$ (13)

Covariance matrix:

$$P^f_k = \frac{1}{N_\varepsilon - 1}\sum_1^{N_\varepsilon}\left(x^f_{k,i} - \bar{x}^f_k\right)\left(x^f_{k,i} - \bar{x}^f_k\right)^T, \quad (14)$$

where $\bar{x}^f_k = \sum_{i=1}^{N} \bar{x}^f_{k,i}$.

Correction Step:

State: $x^u_{k,i} = x^f_{k,i} + K_k\left(z_k - Hx^f_{k,i}\right)$ (15)

where Kalman gain

$$K_k = P^f_k H^T\left(HP^f_k H^T + R_k\right)^{-1}, \quad (16)$$

Covariance matrix: $P^u_k = (I - K_k H)P^f_k.$ (17)

Performance of detection of heat disturbance using KF and EnKF will be analyzed using the average of norm of error covariance matrix.

3 SIMULATION RESULT AND DISCUSSIONS

In the simulation, we divide the radius of cylindrical-shaped metal chip into 17 grids (the t^{th} grid is equal to $r = a + t\frac{b-a}{17}$ for $t = 0,1,\dots,17$) with initial and boundary conditions for equation (2) are $v(r,0) = 0$, $v(a,t) = 100$ and $v(b,t) = 25$. Figure 1 shows the heat transfer in every grid. If we give a heat disturbance in that metal chip, then the heat transfer will be different from figure 1.

To evaluate the performance of KF in detecting heat disturbances, we will try several heat disturbances, i.e. 1 – 4 disturbances with different positions. Heat disturbance detection uses KF with initial state, x^u_0, generated from $N(50, 0.1)$ and assume that the error variance of data is $R = 0.1$.

The detection of one heat disturbance is shown in Figure 2. Heat disturbance in the top figure is given at 30^0 and the bottom figure is given at 56^0 on the same position i.e. 11^{th} grid. From Figure 2, it can be seen that, in every grid, estimation of correction state in KF close to the data (star symbol). Therefore, KF is able to detect the disturbance on the 11^{th} grid. The heat detection can be identified more clearly if the disturbance is large enough (the bottom figure) so that the temperature at that location will be higher than its surrounding.

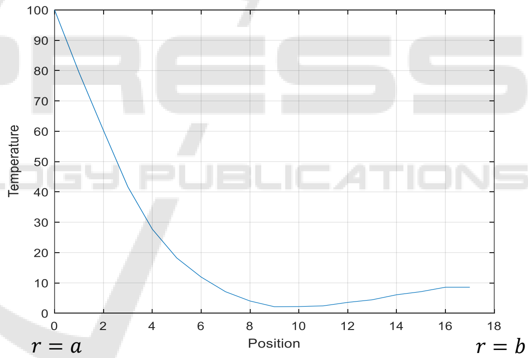

Figure 1: The heat transfer in the cylindrical-shaped metal chip using 16 grids with initial and boundary conditions for equation (2) are $v(r,0) = 0$, $v(a,t) = 100$ and $v(b,t) = 25$.

The detection using KF for two heat disturbances can be seen on figure 3. On figure 3 (above), we give disturbance at 60^0 on the 10^{th} grid and at 70^0 on the 11^{th} grid. On figure 3 (middle), we give disturbance at 60^0 on the 10^{th} grid and at 30^0 on the 11^{th} grid. On figure 3 (bottom), we give disturbance at 30^0 on the 10^{th} grid and at 60^0 on the 11^{th} grid. Figure 3 shows that KF is able to detect these disturbances if these disturbances are in high temperature (figure above), but it's rather difficult to detect one of these disturbances if one of them is in lower temperature (figure middle and bottom).

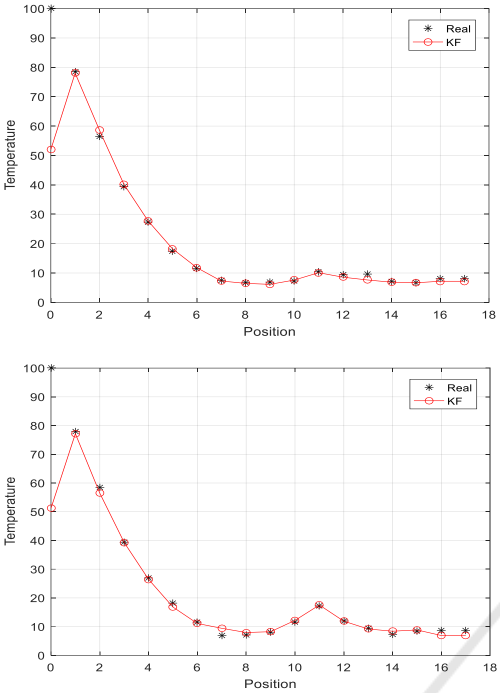

Figure 2: Detection of one heat disturbance using KF. The heat disturbance is given on the 11^{th} grid at 30^0 (top) and at 56^0 (bottom). The greater heat disturbance given, the disturbance will be easier to detect since the temperature in this area is higher than the others.

The heat disturbances on the edges of metal chip will be easier to detect if the disturbances are on higher temperature than the boundaries conditions (figure 4 above), but it's rather difficult to detect if one or two of them are in lower temperature (figure 4 bottom). It can be seen that figure 4 (bottom) is almost same as figure 1. So it's rather difficult to detect disturbances in this condition.

The conditions described for Figure 2-4 also apply to the other number of disturbances i.e. 3, 4, and 5. Figure 5 show the detection of three, four and five disturbances with temperature given are listed on table 1. The disturbance detection also depends on the number of grids. The more number of grids, the more accurate the position of detection but these results are not shown in this paper.

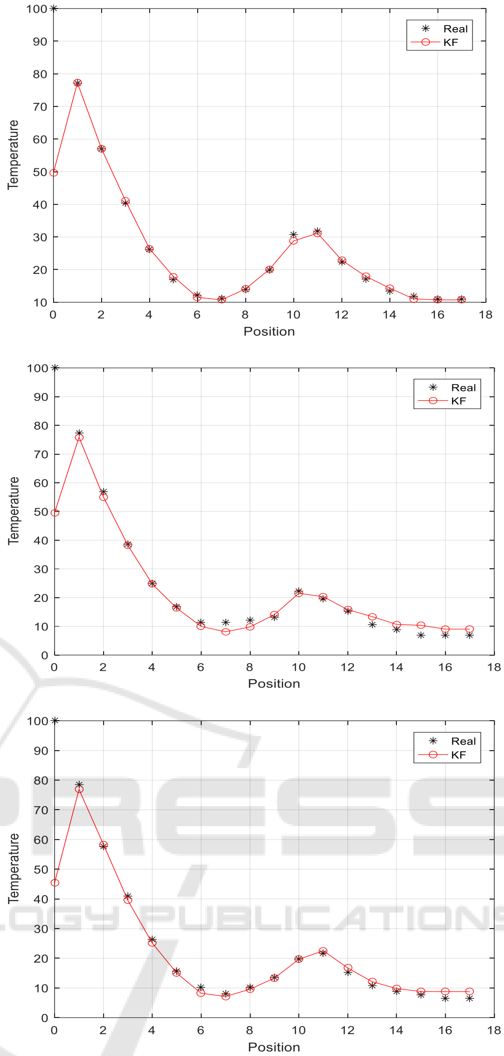

Figure 3: Detection of two disturbances using KF. The heat disturbances are given on the 10^{th} grid at 60^0 and on the 11^{th} grid at 70^0 (above); on the 10^{th} grid at 60^0 and on the 11^{th} grid at 30^0 (middle); and on the 10^{th} grid at 30^0 and on the 11^{th} grid at 60^0 (bottom).

Besides using KF, we also use EnKF to detect the heat disturbances on the metal chips. We use some difference numbers of ensembles (i.e. $N_\varepsilon = 50$, 75, 100, 150) with initial state are generated from $N(50, 0.1)$. The disturbance detection using EnKF result has same conditions as in KF and the figure is also almost same as Figure 2 – 4. Table 2 shows that the more ensemble number, the more accurate the estimation of correction state to the real state.

Table 1: Temperatures are given on the disturbances on figure 5.

Number of disturbances	Grid	Temperature
Three	6th	60^0
	8th	50^0
	12th	50^0
Four	6th	60^0
	8th	50^0
	11th	50^0
	12th	70^0
Five	2th	90^0
	6th	60^0
	8th	50^0
	11th	50^0
	12th	70^0

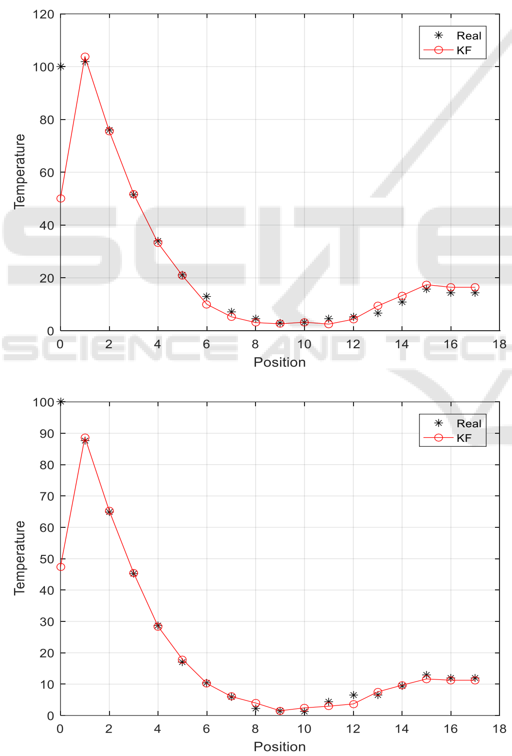

Figure 4: Detection of two disturbances on the edges of the metal chip using KF. The heat disturbances are given on the 1st grid at 170^0 and on the 15th grid at 30^0 (above) and on the 1st grid at 70^0 and on the 15th grid at 20^0 (bottom).

We will use the average of norm of error covariance matrix to compare disturbances detection using EnKF and KF and focus on one and two disturbances. The comparison result can be seen in Table 3. It can be seen that the average of norm of error covariance matrix in EnKF are smaller than the average of norm of error covariance matrix on KF. Therefore, for this case the EnKF is more accurate in the disturbance detection than KF.

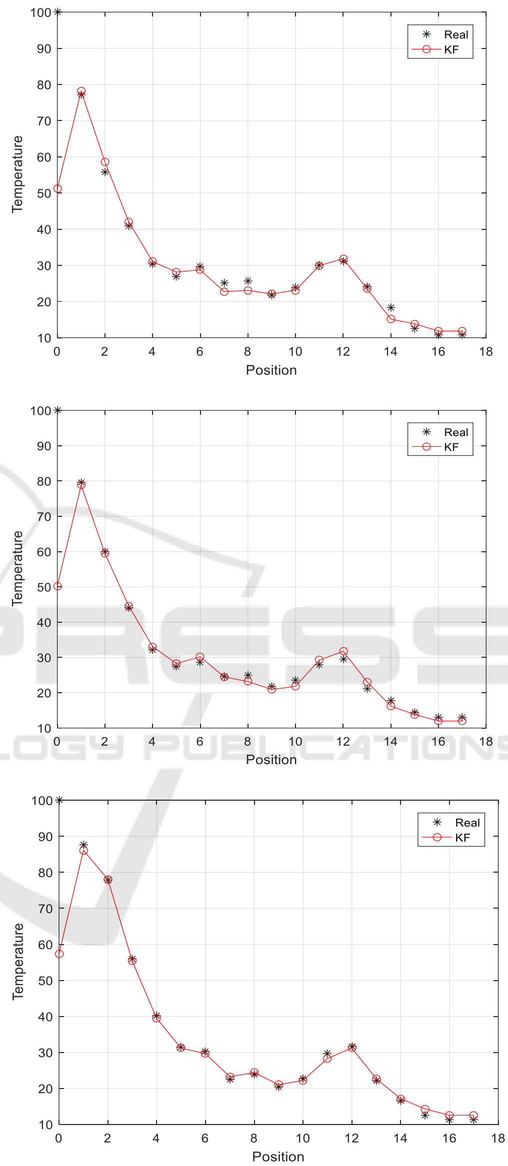

Figure 5: Detection of three disturbances (above), four disturbances (middle) and five disturbances (bottom) using KF.

Table 2: The average of norm of error covariance matrices for different numbers of ensembles in the disturbance detection using EnKF.

	$N_\varepsilon = 50$	$N_\varepsilon = 75$	$N_\varepsilon = 100$	$N_\varepsilon = 150$
Detection one disturbance				
Average of Norm of Error Covariance matrices	0.00635	0.006	0.00582	0.00574
Detection two disturbances				
Average of Norm of Error Covariance matrices	0.00637	0.006	0.00584	0.00574

Table 3. Comparison of the average of norm of error covariance matrices between EnKF ($N_\varepsilon = 150$) and KF in detection of one and two disturbances.

Detection one disturbance		
	EnKF with $N_\varepsilon = 150$	KF
Average of Norm of Error Covariance matrices	0.00567	0.00944
Detection two disturbances		
	EnKF with $N_\varepsilon = 150$	KF
Average of Norm of Error Covariance matrices	0.00566	0.00944

4 CONCLUSIONS

In this study, the KF and EnKF method succeed to detect the heat disturbances in cylindrical-shaped metal chip. Detection of heat disturbances has been carried out for 1–4 disturbances in different positions. Based on the average of norm of error covariance matrices, the EnKF is more accurate detect the disturbance than KF. The heat disturbances can be detected more clearly if the temperature of disturbance is large enough, especially for detection in the edge of chip (close to inner radius and outer radius). In addition, the more number of grids, the more accurate the position of detection.

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