# On Trimmed Data Effect in Parameter Estimation of Some Population Growth Models

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Keywords: Growth Model, Parameter Estimation, Chicken Weight, Trimmed Data.

Abstract: Logistic model, Gompertz model, Richard model, Weibull model and Morgan-Mercer-Flodin model are commonly used to describe growth model of a population. In this paper, we study the effect of trimmed data on parameter estimation results of those models. We use chicken weight data cited from literature. Parameter values of the models from the complete data and the trimmed data are compared. Then, the sensitivity index of all parameters is evaluated. We found that that sensitivity order of the models from the highest sensitivity was the Morgan-Mercer-Flodin, Weibull, Richards, logistic and Gompertz growth model. For practical applications, Gompertz model and Richards are recommended in order to modelling growth of a population.

## 1 INTRODUCTION

Mathematical growth models have been widely applied to explain body weight and age relationship in veterinary sciences. From the mathematical growth model, one can evaluate some important and practical parameters, e.g. the mature weight, the maturing rate and the growth rate of an animal. The parameters are beneficial tool to give estimations of the daily feed needs or to evaluate the effect of environmental condition on the weight growth of an animal. In addition, the mathematical growth models could be applied to forecast the optimum slaughter age. Therefore, mathematical growth models could be considered as an optimization instrument for the animal production (López et al., 2000; Vázquez et al., 2012; Teleken et al., 2017).

The mathematical growth model could be classified into two groups, namely empirical growth models and the empirical growth model and dynamical growth models (the growth model derived from ordinary differential equations). The empirical growth models include Weibull growth model and MMF (Morgan-Mercer-Flodin) growth model. The Weibull and the MMF growth model have been applied to describe chicken growth dynamic (Topal and Bolukbasi, 2008). The dynamical growth model includes logistic growth model, Gompertz growth model, and Richards growth model. These dynamical growth models have been used to describe the growth kinetics of many animals, including chicken (Aggrey, 2002), mammal (Franco et al., 2011), fish (Santos et al., 2013), reptile (Bardsley et al., 1995) and amphibian (Mansano et al., 2013).

Topal and Bolukbasi reported that the MMF, Weibull and Gompertz the MMF, Weibull and Gompertz growth model can be useful for describing chicken growth performance, since these models were the best fitted models (Topal and Bolukbasi, 2008). Aggrey found that the Richards and Gompertz growth model have the best fitted model in explaining rooster and hen growth dynamics (Aggrey, 2002). Zadeh and Golshani also reported that the Richards growth model provided the best fit to the growth curve of Iranian Gulian sheep (Zadeh and Golshani, 2016).

A mathematical growth model could be said as a good model if the model give accurately predicted result and it is robust with trimmed data. In this context, we compare robustness of some mathematical growth model due to trimmed data effect. We use sensitivity index to measure robustness performance of the models. We use chicken weight data cited from literature.

This paper is organized as follows. Section 2 briefly presents some mathematical growth models. Section 3 presents effect of trimmed data on robustness performance of the selected models. Finally, conclusions are presented in Section 4.



Windarto, ., Eridani, . and Purwati, U.
On Trimmed Data Effect in Parameter Estimation of Some Population Growth Models.
DOI: 10.5220/0008516300050008
In *Proceedings of the International Conference on Mathematics and Islam (ICMIs 2018)*, pages 5-8
ISBN: 978-989-758-407-7

## 2 SOME MATHEMATICAL GROWTH MODELS

In this section, we briefly present some mathematical growth models including empirical growth models and dynamical growth models. Let $y(t)$ represents chicken body weight at time t. The Weibull and MMF growth model are given by

$$y(t) = K - (K - A)\exp(-Bt^D), \quad (1)$$

and

$$y(t) = \frac{AB + Ct^D}{B + t^D}, \quad (2)$$

respectively. Here, $K$ is chicken mature weight, while $A, B, C, D$are empirical parameters (Topal and Bolukbasi, 2008).

Logistic growth model is derived from the following differential equation

$$\frac{dy}{dt} = ry\left(1 - \frac{y}{K}\right), y(0) = y_0 > 0. \quad (3)$$

Here $r$ is per capita growth rate. The logistic growth model is analytical solution of Eq. (3), which is given by (Aggrey, 2002; Windarto et al., 2014)

$$y(t) = \frac{K}{1 + exp\left(-r\left(t - t_{inf}\right)\right)} \quad (4)$$

where $t_{inf} = \frac{1}{r}\ln\left(\frac{K}{y_0}\right)$. Here $t_{inf}$ is the inflection time, where at chicken growth is maximum at the inflection time.

The Gompertz growth model is derived from the following Gompertz differential equation

$$\frac{dy}{dt} = ry\ln\left(\frac{K}{y}\right), y(0) = y_0 > 0. \quad (5)$$

The exact solution of Eq. (5) represents the Gompertz growth model. The Gompertz growth model is given by

$$y(t) = \frac{K}{exp\left(\exp\left(-r\left(t - t_{inf}\right)\right)\right)} \quad (6)$$

where $t_{inf} = \frac{1}{r}\ln\left(\ln\left(\frac{K}{y_0}\right)\right)$.

The Richards growth model is derived from the Richards differential equation

$$\frac{dy}{dt} = ry\left(1 - \left(\frac{y}{K}\right)^\beta\right), y(0) = y_0 > 0. \quad (7)$$

Here $\beta$ is the shape parameter in the Richards differential equation. For $\beta$=1, then the Richards differential equation could be simplified into logistic differential equation. Hence, Richards differential equation could be considered as an extension of the logistic differential equation. The exact solution of the Richards differential equation in Eq. (7) is given by

$$y(t) = \frac{K}{\left[1 + \beta\exp\left(-r^*\left(t - t_{inf}\right)\right)\right]^{1/\beta}} \quad (8)$$

where $r^* = r\beta,\ t_{inf} = \frac{1}{r\beta}\ln\left(\frac{\left(\frac{K}{y_0}\right)^\beta - 1}{\beta}\right)$.

## 3 EFFECT OF TRIMMED DATA ON THE ROBUSTNESS PERFORMANCE

In this section, we study effect of trimmed data on robustness performance of the growth models presented in the previous section. We used rooster weight data cited from literature (Aggrey, 2002; Windarto et al., 2014). The rooster weight data $(y)$ at the day $(t)$is presented in the Table 1.

Table : Means of the rooster weight data (y)

| t (days) | y (grams) | t (days) | y (grams) |
|---|---|---|---|
| 0 | 37 | 42 | 519.72 |
| 3 | 41.74 | 45 | 577.27 |
| 6 | 59.19 | 48 | 633.59 |
| 9 | 79.94 | 51 | 667.18 |
| 12 | 102.96 | 54 | 717.17 |
| 15 | 132.13 | 57 | 786.35 |
| 18 | 170.18 | 71 | 1069.28 |
| 21 | 206.56 | 85 | 1326.49 |
| 24 | 250.71 | 99 | 1589.71 |
| 27 | 285.27 | 113 | 1859.26 |
| 30 | 324.92 | 127 | 2015.44 |
| 33 | 372.83 | 141 | 2142.31 |
| 36 | 417.41 | 155 | 2220.54 |
| 39 | 469.13 | 170 | 2262.63 |

At the first step, we estimate parameters in the growth model before trimmed data. We estimate the parameters such that the mean square error (MSE) which is given by

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2, \quad (9)$$

is minimum. Here, $y_i$ and $\hat{y_i}$ are rooster weight data and predicted rooster weight at the $i$-th day, while $n$ is number of observation data.

We used Lavenberg-Marquardt algorithm to find the optimal parameters for the optimization problem given in Eq. (9). Estimation results of the Weibull, MMF, logistic, Gompertz and the Richards growth model for the rooster weight and the mean squared error of the models are presented in the Table 2. From the Table 2, we found that the Weibull was the best models, while the logistic growth model was the worst model. We also obtained that accuracy of the Weibull model and the Richards model did not considerably differ. We also found that mean squared error of the Richards model and the Gompertz model did not significantly differ. This was apparently caused by the shape parameter β in the Richards model was almost zero.

Table 2: Estimated parameters value for the whole data

| Growth Model | Parameters | Estimated value | MSE |
|---|---|---|---|
| Weibull | K | 2426.1709 | 347.743 |
| | A | 58.2211 | |
| | B | 0.000197 | |
| | D | 1.8699 | |
| MMF | A | 67.7095 | 793.779 |
| | B | 14411.3917 | |
| | C | 2996.0317 | |
| | D | 2.1030 | |
| Logistic | K | 2279.9041 | 1887.461 |
| | r | 0.0403 | |
| | $t_{inf}$ | 74.6775 | |
| Gompertz | K | 2539.6505 | 384.666 |
| | r | 0.0220 | |
| | $t_{inf}$ | 63.4975 | |
| Richards | K | 2512.9724 | 376.277 |
| | r* | 0.0230 | |
| | $t_{inf}$ | 64.3072 | |
| | β | 0.0541 | |

In order to study the effect of trimmed data, we also estimated parameters of the models for trimmed data at the end of the original data (the data from t = 0 until 127 days). We estimated parameters in the models for the trimmed data. We presented estimation results for the trimmed data in the Table 3. From the Table 3, we found that the Weibull model and the MMG model were the best models, while the logistic growth model was the worst model. It indicates that the empirical models are more suit when they are applied in a short data. We also obtained that accuracy of the Gompertz model and the Richards model did not considerably differ.

Table 3: Estimated parameters value for the trimmed data

| Growth Model | Parameters | Estimated value | MSE |
|---|---|---|---|
| Weibull | K | 2992.8983 | 111.903 |
| | A | 41.9675 | |
| | B | 0.000334 | |
| | D | 1.6772 | |
| MMF | A | 43.5080 | 139.359 |
| | B | 5355.9663 | |
| | C | 4540.7213 | |
| | D | 1.7266 | |
| Logistic | K | 2132.0511 | 1708.691 |
| | r | 0.0433 | |
| | $t_{inf}$ | 70.3077 | |
| Gompertz | K | 2694.6160 | 230.084 |
| | r | 0.0206 | |
| | $t_{inf}$ | 66.8981 | |
| Richards | K | 2694.3571 | 230.053 |
| | r* | 0.0206 | |
| | $t_{inf}$ | 66.8987 | |
| | β | 0.0002 | |

In order to measure effect of trimmed data on robustness performance of the models, we defined a sensitivity index of all parameters in the model. For any parameter α, we defined the sensitivity index as

$$SI_\alpha = \left|\frac{\alpha - \alpha_{trim}}{\alpha}\right|, \alpha \neq 0. \quad (10)$$

Here $\alpha_{trim}$ is the parameter value after trimmed data process. Sensitivity index of all parameters was presented in the Table 4.

From the Table 2 and Table 3, we found that the mean squared error of the Weibull model and the MMF model drastically increased due to adding a few data. From the Table 4, we found that average value of the sensitivity index varied from 5.94% until 42.01%. In addition, we found that the shape parameter β in the Richards model was very sensitive, while the remaining parameters in the Richards model were robust. Furthermore, we found that the Gompertz growth model was a robust model with respect to trimmed data. We also obtained that sensitivity index of the empirical model were more sensitive than the dynamical model studied in this paper. Hence, we found that the empirical growth model was more sensitive than the dynamical growth models. For practical applications, Gompertz model and Richards are recommended in order to describing a population growth.

Table 4: Sensitivity index of all parameters.

| Growth Model | Parameters | Sensitivity index | Average value |
|---|---|---|---|
| Weibull | K | 0.2336 | 0.3278 |
| | A | 0.2792 | |
| | B | 0.6954 | |
| | D | 0.1031 | |
| MMF | A | 0.3574 | 0.4201 |
| | B | 0.6284 | |
| | C | 0.5156 | |
| | D | 0.1790 | |
| Logistic | K | 0.0649 | 0.0659 |
| | R | 0.0744 | |
| | $t_{inf}$ | 0.0585 | |
| Gompertz | K | 0.0610 | 0.0594 |
| | R | 0.0636 | |
| | $t_{inf}$ | 0.0536 | |
| Richards | K | 0.0722 | 0.3034 |
| | r* | 0.1049 | |
| | $t_{inf}$ | 0.0403 | |
| | β | 0.9961 | |

# 4 CONCLUSIONS

We have studied effect of trimmed data on parameter estimation results of some empirical models (Weibull and Morgan-Mercer-Flodin) and some dynamical models (logistic, Gompertz and Richards growth model). We found that the empirical models were more sensitive than the dynamical models. We also found that the dynamical models were more robust with respect to trimmed data. For practical applications, Gompertz model and Richards are recommended in order to modeling growth of a population.

# ACKNOWLEDGEMENTS

Part of this research was supported by Ministry of Research, Technology and Higher Education, Republic of Indonesia through "Penelitian Unggulan Perguruan Tinggi" research project.

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