

# Student Worksheet in Mathematical Creative Thinking

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**Keywords:** Student worksheet, mathematical creative thinking.

**Abstract:** This study aimed to develop student worksheet on a calculus with APOS to improve mathematical creative thinking. The development model includes: identifying learning objectives, conducting learning analyzes, formulating specific objectives, developing assessment instruments, developing learning strategies, developing and selecting learning materials, designing and executing evaluation, revising learning materials, designing and carry out evaluation. The research is done by validating the student worksheet on calculus with APOS and the subject of the test was the mathematics education student. The results show that the trial of student worksheets on the calculus with APOS showed very good and very effective results and appropriate to be used in learning. This is because the student worksheet got positive responses from student and learning by using student worksheet is very interesting and students understand the calculus material more easily and student's ability of mathematical creative thinking improved with 83% achievement. The student worksheets on the calculus material with APOS has impact on the student's academic ability which has improved very well.

## 1 INTRODUCTION

One of the innovation efforts in order to improve students' creative thinking ability is by developing student worksheet. Student worksheet is developed by using APOS with stages that include action, process, object, and scheme. The developed student worksheet using APOS is on the calculus material. The development student worksheet with APOS is expected to improve students' creative thinking ability

APOS stands for action, process, object, and schema. APOS requires the ability of students to think creatively in solving mathematical problems. (Dubinsky and Mcdonald, 2002) argue that APOS is a mathematical learning approach at the college level, which integrates computer use, small group discussion, and considering the mental constructions by students in understanding a mathematical concept. These mental constructions are APOS which includes action, process, object, and scheme. (Weller, Clark and Dubinsky, 2003) states that APOS can be used to improve thinking ability that are particularly suitable for advanced mathematical materials, including: functions; abstract algebra topics such as binary, group, subgroup, normal group; discrete mathematical topics such as mathematical induction,

permutation, symmetry; statistical topics such as averages, standard deviations, and central calculus theorems; the topic of number such as place value in the n-base number, division, multiplication and the conversion of numbers from one base to another; topics of calculus such as calculus, chain rules, graphic derivations, and infinite sequences. In relation to the previous research on the of APOS and creative thinking (Arnawa, Kartasasmita and Baskoro, 2007) which showed that the proof ability in the APOS group is significantly better, thus it is strongly recommended to apply the APOS in abstract algebra. In addition, the results of the study (Cetin, 2015) were the first attempt to use the APOS framework in the context of educational programming with the results of the study indicating that the APOS with frameworks is useful for providing students with understanding of concepts. The results of the study (Marsitin, 2017b) indicated that the calculus learning module with developed APOS was very effective and feasible to be used for students. Students more easily understand the material calculus and the achievement of students' academic ability had improved very well.

In mathematics learning there are abilities that have to be achieved those are the understanding of mathematics, mathematical connections,

mathematical reasoning, mathematical problem solving, and mathematical communication mathematics (National Council Of Teachers Of Mathematics, 2000). Creative thinking has an important role in the process of mathematical thinking. Creative thinking has the meaning that a person has diversity in solving a problem. Mumford, Medeiros and Partlow (2012), Harris (2002) states of creative thinking development is the awareness of thinking, thinking observation, thinking strategy and thinking reflection. Improvements in developing the next level of mathematical creative thinking are based on students' creative thinking patterns that include: fluency, flexibility and novelty in problem solving and problem presenting. In the assessment of students' creative thinking, the student must have the ability to think creatively that includes fluency, flexibility and novelty and flexibility (Siswono, 2005; Marsitin, 2017a). Students use a method of planning solution, in solving mathematical problems so that it can implement through constructing the solution of the problem.

The process of creative thinking with indicators in the level of creative thinking that describes the creative process by understanding the problem, generating ideas / ideas and planning actions. Understanding the problems are finding goals, data (facts) and problems as target questions. Generating ideas / ideas is to find options to answer the problem. Planning action is finding a solution. At this stage, the individual analyzes, refines or develops the choice of appropriate ideas and then prepares a choice or alternative to increase the support and value (Isaksen and Aerts, 2011; Lithner, 2004; Maharani, 2014). In reality, students are accustomed to accept the material presented by the lecturers only, so that the mindset of the students tend to be passive and the lecturers need to make the learning innovation so that the students are active and have creative thinking ability in solving mathematical problems. The process of mathematical thinking in the students appears when students construct their knowledge in mathematical solution (Marsitin, 2016). The exposure and phenomenon above, is very interesting to be discussed related to mathematics learning, APOS and creative thinking ability, with the aim of developing a student worksheet with APOS on the calculus to improve mathematical creative thinking.

### 1.1 APOS

APOS is a in a mathematics learning approach at the college level by integrating the use of computers, learning in small groups, and paying attention to

mathematical connections and creative thinking to understand a mathematical concept, namely: action, process, object and scheme abbreviated as APOS developed (Dubinsky and Mcdonald, 2002; Parraguez and Oktaç, 2010). Learning with APOS includes: (a) Action is a transformation of mental objects that a person is said to experience an action if the person is focusing his mental processes on understanding a concept; (b) Process is a person who has experienced a process of a concept, if his thinking is calculus to the mathematical idea encountered, it is characterized by the emergence of the ability to reflect on the mathematical idea; (c) Object is a person who has the object conception of a mathematical concept, if he has been able to treat the idea or concept as a cognitive that includes the ability to act on the object, and provide a reason or explanation of its characteristics and be able to do the decomposition of an object into a process when the characteristics of the object in question will be used; (d) Scheme is when a person has the ability to construct examples of a mathematical concept in accordance with the characteristics of the concept.. The of APOS is a constructivist of how one learns a mathematical concept, thereby helping to develop the mathematical thinking process within itself (Parraguez and Oktaç, 2010; Arnawa, Kartasasmita and Baskoro, 2007).

### 1.2 Creative Thinking

Creative thinking has the meaning that a person has diversity in solving a problem. Mumford, Medeiros and Partlow (2012) states the definition of creative thinking development that is the awareness of thinking, thinking observation, thinking strategy and thinking reflection. Improvements in felling the ability of the next level of mathematical creative thinking are based on students' creative thinking patterns that include: fluency, flexibility and novelty in problem solving and problem presenting. In the assessment of students' creative thinking, the student must have creative thinking ability which includes fluency, flexibility and novelty and flexibility (Hu, Wu and Shieh, 2016). Students use a method of planning solution, in solving mathematical problems so that it can implement through constructing the solution of the problem. Creative thinking process with indicators in the level of creative thinking that describes the creative process by understanding problems, building ideas and planning actions. Understanding the problem includes the stages of finding goals, finding data or facts and finding problems as target questions. Generating ideas

includes decreasing options for answering open issues. Planning actions includes finding solutions and finding support. At this stage, the individual analyzes, refines or develops the choice of appropriate ideas and then prepares a choice or alternative to increase support and value (Harris, 2002; Runco and Acar, 2012) Any text or material outside the aforementioned margins will not be printed.

## 2 METHOD

This research is a development research. The research design of this module development uses development design (Richey, Klein and Tracey, 2011) with the model (Dick, Carey and Carey, 2009). The development model includes: identifying learning objectives, conducting learning analyzes, formulating specific objectives, developing assessment instruments, developing learning strategies, developing and selecting learning materials, designing and executing evaluation, revising learning materials, designing and carry out evaluation. The research subject is student of mathematics education of Kanjuruhan University of Malang who take calculus course in the third semester. The research subjects are form class A for small group trials is 9 students and class B for large group trials is 30 students. Student worksheets with APOS are validated by material validators, learning validators and student worksheet product design validators. After being validated, calculus field trials are conducted on small group and extensive field trials for large group to find out the lecturers' responses and students' responses to student worksheet. The effectiveness of student worksheet is analyzed qualitatively, while the ability of mathematical creative thinking is analyzed quantitatively. The instruments used for the collection of research data are: response questionnaire, test of mathematical creative thinking ability.

## 3 RESULTS AND DISCUSSION

The results of the research include several stages of analyzing, planning, developing, and evaluating. The first stage is the analyzing; analyzing the plan for lecturing implementation on calculus material, reviewing the learning objectives of calculus. Phase Two is planning, by developing instruments and designing product specifications and the structure of the student worksheet content. The third stage is

developing, analyzing and developing the components of the developed student worksheet, validating the student worksheet for calculus with APOS by material validation, learning validation and product design validation and then conducting trials on small groups, as well as large group tests. The fourth stage is evaluating, evaluating and analyzing the effectiveness of developed student worksheet by conducting creative thinking ability tests. The results include four stages of analyzing, planning, developing, and evaluating. Prior to the research trials, validation of research instruments and validation of student worksheet products are done by material validators, learning validators and design validators. The validation result shows that the validated instrument and student worksheet product has been declared valid. Furthermore, trials are conducted for small group trials with 9 student's form class A and large group trials with 30 student's form class B. The results are shown in table 1.

Table 1: Students' worksheet results in trials.

Description	Action	Process	Object	Scheme
Students' Worksheet-1	85	85	80	80
Students' Worksheet -2	85	80	80	80
Students' Worksheet -3	80	80	80	80
Students' Worksheet -4	85	85	85	85
Students' Worksheet -5	85	80	80	75
Total	420	410	405	400
Average	84	82	81	80

The results of the student worksheet in table 1, the obtained data is for the action the average score is 84, the average score of the process is 82, the average score of object is 81 and the average score of scheme is 80. It can be said that the students still have difficulties in the object and scheme stages, which means that students are still difficult to conclude from

the problem solving and it is difficult to conclude the formula used in the solution of the problem so that the students need the assistance in experiencing the difficulty, according to the opinion (Sutawidjaja and Afgani, 2015; Bikmaz *et al.*, 2010; Speer and Wagner, 2009) that the main role of lecturers in class discussions is as a facilitator/indirect intervention/scaffolding.

Student worksheet is very helpful for students to change old mindsets that just accept the material but can change themselves to be able to understand the mathematical concept itself. Through the student worksheet, the students learn to transfer the concept they have understood among friends and students to learn to improve if there is a misconception that he understands. The tasks in the student worksheet done in groups give concepts strengthen of constructed mathematics and apply the concepts or theorems of mathematics that have been studied and discussed to think about the concepts that have not been studied. This is in line with opinion (Speer and Wagner, 2009; Koçak, Bozan and Işık, 2009; Zakaria, Chin and Daud, 2010) which state that learning is an active and constructive process in which students try to solve problems by actively participating in training of mathematics during the learning process, so that creative thinking is a person's ability to generate new ideas that are effective and ethical.

Table 2: Recapitulation of research result.

Description	Achievement
Test of mathematical creative thinking ability	83%
Students' response toward the questionnaire	85%
Lecturers' response toward the questionnaire	90%

The results of the student worksheet in table 2, the achievement of mathematical creative thinking ability test is 83%, thus it can be concluded that the student worksheet with APOS can improve the creative thinking ability. Mathematical creative thinking ability test with indicators in the creative process by understanding problems, building ideas and planning actions with opinion (Lince, 2016; Williams, 2007). The result of students' response toward the questionnaire of the student worksheet

with APOS shows 85% students is agree, therefore it can be concluded that the student worksheet with APOS get good responses from the students, student worksheet calculus is in accordance with the needs of the students that is easy, interesting and useful for students. The result of lecturers' response toward the questionnaire of the student worksheet with APOS shows 90% is agree, thus it can be concluded that the student worksheet with APOS is in accordance with the needs of the student, it also can be said that the student worksheet provides practical, effective and feasible in learning. Mathematics learning not only provides concepts related to the definition of but must apply examples as solutions to mathematical problems with learning strategies that support learning achievement. Learning that involves the students then provide the opportunity for students to work independently by constructing their knowledge, thus can achieve maximum academic ability.

#### 4 CONCLUSIONS

The conclusions of this development research on student worksheet with APOS on the calculus material to improve creative thinking are: the achievement of creative thinking ability test in experiment is 83%, thus it can be concluded that the student worksheet with APOS can improve the creative thinking ability. The result of students' response toward the questionnaire of the student worksheet with APOS shows 85% students is agree and 90% lecturers' is agree, thus it can be concluded that the student worksheet with APOS is in accordance with the needs of the student is easy, interesting and useful for students. It also can be said that the student worksheet provides practical, effective and feasible in learning. Suggestions that can be given are lecturers who teach calculus courses should use teaching materials in the form of student worksheet to make the students master the mathematics concepts and academic quality of mathematics is better.

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