

An Investigation of Parameter Tuning in the Random Adaptive Grouping Algorithm for LSGO Problems

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Abstract: Large-scale global optimization (LSGO) is known as one of the most challenging problem for many search algorithms. Many well-known real-world LSGO problems are not separable and are complex for comprehensive analysis, thus they are viewed as the black-box optimization problems. The most advanced algorithms for LSGO are based on cooperative coevolution with problem decomposition using grouping methods. The random adaptive grouping algorithm (RAG) combines the ideas of random dynamic grouping and learning dynamic grouping. In our previous studies, we have demonstrated that cooperative coevolution (CC) of the Self-adaptive Differential Evolution (DE) with Neighborhood Search (SaNSDE) with RAG (DECC-RAG) outperforms some state-of-the-art LSGO algorithms on the LSGO benchmarks proposed within the IEEE CEC 2010 and 2013. Nevertheless, the performance of the RAG algorithm can be improved by tuning the number of subcomponents. Moreover, there is a hypothesis that the number of subcomponents should vary during the run. In this study, we have performed an experimental analysis of parameter tuning in the RAG. The results show that the algorithm performs better when using subcomponents of larger size. In addition, some improvement can be done by applying dynamic group sizing.

1 INTRODUCTION

Optimization problems with many hundreds or thousands of objective variables are called large-scale global optimization (LSGO) problems. LSGO is known as one of the most challenging problem for many search techniques from the field of mathematical programming and evolutionary optimization. Many well-known real-world LSGO problems are not separable and are complex for comprehensive analysis, thus they are viewed as the black-box optimization problems even the objective function has analytical representation (using mathematical formula).

Evolutionary algorithms (EAs) have proved their efficiency at solving many complex real-world black-box optimization problems. However, their performance usually decreases when the dimensionality of the search space increases. The most advanced algorithms for LSGO are based on cooperative coevolution (CC) with problem

decomposition using different grouping methods, which decomposes LSGO problems into multiple low-dimensional non-overlapping subcomponents. Unfortunately, the nonseparability of real-world LSGO problems excludes a straightforward variable-based decomposition. There exist at least three types of subcomponent grouping methods, including: static, random dynamic and learning dynamic grouping. The static grouping performs well only for well-studied LSGO problems with regular structures. The majority of state-of-the-art LSGO techniques are based on the random grouping and the learning-based grouping. The standard random grouping can be applied to the wide range of separable and non-separable LSGO problems, but it does not use any feedback from the search process for creating more efficient variables combinations. Many learning dynamic grouping techniques demonstrates too greedy adaptation, thus performs well only with separable LSGO problems.

In our previous study, we have proposed a novel grouping technique that combines the ideas of the random dynamic grouping and the learning dynamic grouping. The approach is called the random adaptive grouping (RAG). In our implementations, the RAG is combined with cooperative coevolution (CC) of the Self-adaptive Differential Evolution (DE) with Neighborhood Search (SaNSDE) (the whole search algorithm is called DECC-RAG). The RAG starts with random subcomponents of an equal predefined size. After some generations of the DECC (so-called adaptation period), we estimate the performance of each subcomponent. A portion of the best subcomponents is saved for the next adaptation period and for the rest of subcomponents we apply the random grouping again. Such a feedback forms different groups of variables and adaptively changes them during the search.

In this study, we have performed an experimental analysis of parameter tuning in the RAG. We have estimated how the performance of the DECC-RAG depends on the number of subcomponents. And, we have implemented and investigated a modification of the RAG with changing number of subcomponents. In this paper, we will present the experimental results for the LSGO CEC'10 benchmark only, because of great amount of time- and resource-costly fitness evaluations. Nevertheless, we will present and discuss the results for the LSGO CEC'13 benchmark in our further works and our presentation of the study.

The rest of the paper is organized as follows. Section 2 describes related work. Section 3 describes the proposed approach and experimental setups. In Section 4 the results of numerical experiments are discussed. In the Conclusion the results and further research are discussed.

2 RELATED WORK

There exist a great variety of different LSGO techniques that can be combined in two main groups: non-decomposition methods and cooperative coevolution (CC) algorithms. The first group of methods are mostly based on improving standard evolutionary and genetic operations. But the best results and the majority of approaches are presented by the second group. The CC methods decompose LSGO problems into low dimensional sub-problems by grouping the problem subcomponents. There are many subcomponent grouping methods, including: static grouping (Potter and Jong, 2000), random dynamic grouping (Yang et al., 2008c) and learning

dynamic grouping (Omidvar et al., 2014, Liu and Tang, 2013).

The first attempt to divide solution vectors into several subcomponents using the static grouping was proposed by (Potter and Jong, 1994). The approach proposed by Potter and Jong decomposes a n -dimensional optimization problem into n one-dimensional problems (one subcomponent for each variable). The CCGA employs CC framework and the standard genetic algorithm (GA). Potter and Jong had investigated two different modification of the CCGA: CCGA-1 and CCGA-2. The CCGA-1 evolves each variable of objective in a round-robin fashion using the current best values from the other variables of function. The CCGA-2 algorithm employs the method of random collaboration for calculating the fitness of an individual by integrating it with the randomly chosen members of other subcomponents. Potter and Jong had shown that CCGA-1 and CCGA-2 outperforms the standard GA. Unfortunately, search techniques based on the static grouping are inefficient for many non-separable LSGO problems.

One of the most popular and well-studied random grouping method had been proposed by Yang et al. (Yang et al., 2007, Yang et al., 2008c) and uses a DE-based CC method. The approach is called DECC-G and it is used as a core conception for many advanced techniques.

The learning dynamic grouping seems to be the most perspective approach as it collects and uses feedback information for improving the decomposition stage. There were proposed a CC algorithm based on the correlation matrix (Ray and Yao, 2009), a CC with Variable Interaction Learning (CCVIL) (Chen et al, 2010), an automated decomposition approach (DECC-DG) with differential grouping (Omidvar et al., 2014) and many others. In our previous studies, we have proposed the adaptive variable-size random grouping algorithm (AVS-RG CC) based on the Population-Level Dynamic Probabilities adaptation model (Sopov, E., 2018). A good survey on LSGO and methods is proposed in (Mahdavi et al., 2015).

The DECC-RAG algorithm (Vakhnin and Sopov, 2018), which is investigated in this study, combines the RAG approach with CC of the SaNSDE. The SaNSDE algorithm have been proposed by (Yang et al, 2008b). We have chosen this algorithm because of its self-adaptive tuning of parameters during optimization process. After each regrouping of variable in the CC stage of the DECC-RAG, we will deal with new optimization problems, thus we need to choose a new efficient search

algorithm for each subcomponent. The SaNSDE forms an efficient combination of DE's parameters (such as the type of mutation, the differential weight and the crossover probability) in an automated (self-adaptive) way.

The DECC-RAG operates with subcomponents of equal size. This limitation excludes such problems of the grouping methods as uneven distribution of computational resources between search algorithms and tuning the minimum and maximum number of variables in groups. The RAG divides the n -dimensional solution vector into m s -dimensional sub-components ($m \times s = n$), thus the number of subcomponents is a parameter of the algorithm. In our previous study we have estimated the performance of the DECC-RAG with $m = 10$ on the CEC'10 and CEC'13 LSGO benchmarks. The experiments have shown that the proposed approach outperforms on average some state-of-the-art algorithms such as DMS-L-PSO (dynamic multi-swarm and local search based on PSO algorithm) (Liang and Suganthan, 2005), DECC-G (cooperative coevolution with random dynamic grouping based on differential evolution) (Yang et al, 2008c), MLCC (Multilevel cooperative coevolution based on differential evolution) (Yang et al, 2008a) and DECC-DG (cooperative coevolution with differential grouping based on differential evolution) (Omidvar et al, 2014).

It should be noted that the comprehensive experimental analysis of LSGO algorithms is a challenging problem because computations are time- and resource-costly. According to the CEC LSGO competition rules, the performance of a LSGO algorithm is evaluated through 25 independent runs on each benchmark problem using 3×10^6 fitness evaluations in each independent run of the algorithm (Li et al., 2013). In a case of serial computations using 1 processor, the time for estimating the performance of one algorithm (or one configuration of an algorithm) is about 28 hours for LSGO CEC'10 benchmark and is about 255 hours for LSGO CEC'13 benchmark. We have implemented all our numerical experiments using the C++ OpenMP framework for parallel computing. The time of computations using 16 processors is about 5 hours for LSGO CEC'10 benchmark and is about 22 hours for LSGO CEC'13 benchmark.

3 EXPERIMENTAL SETUPS

The general scheme of the DECC-RAG can be described by the following pseudo code:

Pseudocode of the DECC-RAG algorithm.

```

1: Set  $FEV\_max$ ,  $FEV\_global$ ,  $T$ ,
 $FEV\_local = 0$ ;
2: An  $n$ -dimensional object vector is
randomly divided into  $m$ 
 $s$ -dimensional subcomponents;
3:  $i = 1$ ;
4: Evolve the  $i$ -th subcomponent with
SaNSDE algorithm;
5:  $FEV\_local++$ ,  $FEV\_global++$ ;
6: If  $i < m$ , then  $i++$ , and go to
Step 4 else go to Step 7;
7: Choose the  $best\_solution_i$  for each
subcomponents;
8: If ( $FEV\_local < T$ ) then go to
Step 3 else go to Step 9;
9: Choose  $m/2$  subcomponents with the
worse performance and randomly mix
indices of their variables, restart
parameters of SaNSDE for the chosen
 $m/2$  subcomponents,  $FEV\_local = 0$ ;
10: If ( $FEV\_global < FEV\_max$ ) go to
Step 3, else go to Step 11;
11: Return the best found solution.

```

Here FEV_max is maximum number of fitness evaluations, T is an adaptation period, FEV_local and FEV_global are counters for fitness evaluation inside the adaptation period and for the whole algorithm, respectively.

We use the following general settings for the experiments:

- 20 CEC'10 LSGO benchmark problems;
- Dimensionality of all problems are $D = 1000$;
- FEV_max is 3×10^6 in each independent run ;
- 25 independent runs for each benchmark problem;
- The performance of algorithms is estimated using the median value of the best found solutions;
- Population size for each subcomponent is 50;
- Adaptation period T is 3×10^5 ;
- Number of subcomponents m is $\{4, 8, 10, 20, 40, 50, 100\}$. We use the following notation: DECC-RAG(m);
- The DECC-RAG(4-8) algorithm starts with $m=4$, and after 4 periods of adaptation ($T=4$, $FEV=1.2 \times 10^6$) m is changed to 8;
- All parameters of the SaNSDE algorithm are self-adaptive.

Table 1: The experimental results on the CEC'10 LSGO benchmark problems.

Problem	DECC-RAG(4)	DECC-RAG(8)	DECC-RAG(10)	DECC-RAG(20)	DECC-RAG(40)	DECC-RAG(50)	DECC-RAG(100)	DECC-RAG(4-8)
1	7.96E-10	1.02E-17	1.97E-18	3.30E-10	1.61E+02	1.58E+04	1.91E+07	6.55E-14
2	3.00E+03	1.23E+03	7.87E+02	1.82E+03	4.62E+03	5.16E+03	5.85E+03	2.99E+03
3	1.09E+01	2.75E+00	1.49E+00	1.21E-08	1.09E-02	1.45E-01	3.53E+00	1.06E+01
4	6.85E+11	9.67E+11	1.02E+12	2.24E+12	3.45E+12	4.58E+12	8.99E+12	9.35E+11
5	7.96E+07	1.43E+08	1.63E+08	2.27E+08	3.94E+08	4.86E+08	7.00E+08	7.67E+07
6	2.03E+01	2.03E+01	2.04E+01	2.07E+01	1.98E+07	2.00E+07	2.00E+07	2.03E+01
7	5.93E+00	8.76E+00	1.70E+02	5.06E+05	1.08E+08	3.39E+08	3.09E+09	2.00E+00
8	2.09E+04	1.03E+07	2.36E+07	4.07E+07	1.17E+08	1.47E+08	3.01E+08	2.76E+06
9	4.22E+07	5.46E+07	6.60E+07	1.15E+08	1.97E+08	2.31E+08	3.44E+08	4.40E+07
10	4.98E+03	3.70E+03	3.24E+03	2.11E+03	5.27E+03	8.85E+03	1.17E+04	4.89E+03
11	1.11E+02	2.15E+02	2.16E+02	2.35E+02	2.35E+02	2.35E+02	2.35E+02	1.15E+02
12	2.73E+04	9.05E+03	8.68E+03	2.51E+04	9.51E+04	1.32E+05	3.34E+05	1.38E+04
13	1.33E+03	1.69E+03	1.30E+03	2.92E+03	1.06E+04	3.32E+04	2.20E+05	2.09E+03
14	1.70E+08	1.70E+08	2.00E+08	4.07E+08	1.09E+09	1.40E+09	3.09E+09	1.61E+08
15	5.78E+03	5.27E+03	5.00E+03	4.32E+03	1.29E+04	1.36E+04	1.53E+04	5.73E+03
16	2.79E+02	4.11E+02	4.28E+02	4.29E+02	4.29E+02	4.29E+02	4.28E+02	2.63E+02
17	2.22E+05	1.46E+05	1.67E+05	3.91E+05	9.90E+05	1.27E+06	1.74E+06	1.80E+05
18	3.81E+03	4.92E+03	5.95E+03	9.74E+03	1.18E+05	1.07E+05	1.01E+05	5.80E+03
19	1.86E+06	1.98E+06	2.20E+06	4.41E+06	1.29E+07	1.56E+07	1.57E+07	1.68E+06
20	2.29E+03	1.91E+03	1.82E+03	1.13E+03	1.04E+03	1.06E+03	2.97E+03	2.16E+03

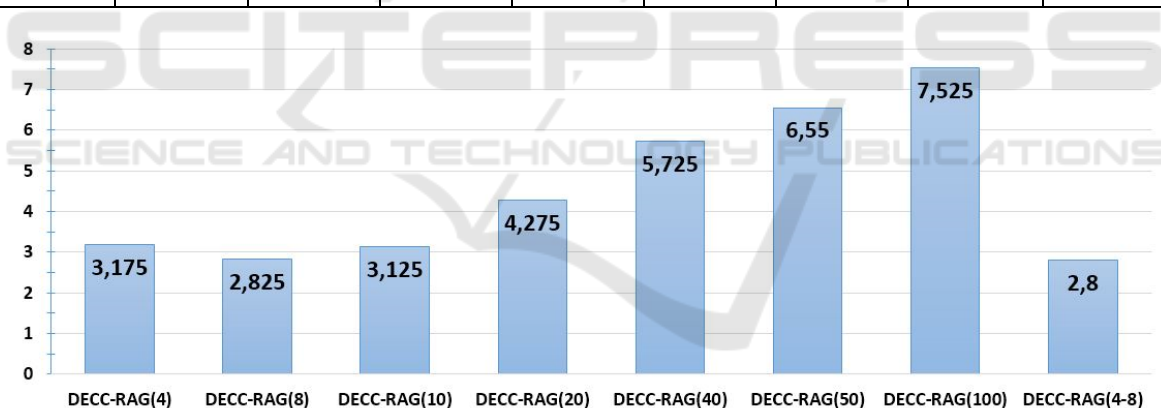


Figure 1: The DECC-RAG(m) ranking.

The experimental results for the DECC-RAG(m) are presented in Table 1. Figure 1 shows the results of ranking the investigated algorithms (ranks are averaged over the benchmark problems). Figures 3-12 demonstrate the convergence of the best-found solutions of each algorithm averaged over 25 independent runs.

As we can see from the Table 1, the DECC-RAG algorithm performs better with small number of subcomponents, which contains large number of variables. This means that the approach is able to provide an efficient decomposition of a problem and

to form efficient combinations of variables in subcomponents. The best results are achieved by the DECC-RAG(8), DECC-RAG(4) and DECC-RAG(10). At the same time, we can see in the Figures 3-12 that different configurations of the algorithm have different speeds of the convergence. Some configurations have good convergence at the early stages of the search process, but have worse the best-found value. Some configurations demonstrate low convergence speed, but are able to improve the best-found value at the finally stage of the search process. Thus, we can conclude that the

dynamic sizing of subcomponents may improve the performance of the algorithm.

The DECC-RAG(4-8) is a straightforward approach of changing the number of subcomponents during the run of the algorithm. As we can see in the Figure 1, this configuration has taken the best average rank over the benchmark.

In real-world LSGO problems, the best settings for the algorithms are unknown beforehand and the m value can be chosen at random. Our hypothesis is that the dynamic sizing is more preferable than the random choice of m . The performance of the random choice of m can be estimated by evaluating the average performance of all configurations. We have compared the DECC-RAG(4-8) with the following average values:

- The average of 4 and 8 for establishing the difference in the performance against the random choice of $m=4$ and $m=8$;
- The average of 4, 8, 10 and 20, because we have found that lower values of m perform better;
- The average of 4, 8, 10, 20, 40, 50, 100 for investigating the random choice of m .

Figure 2 shows the results of ranking the DECC-RAG(4-8) and the estimated performance of random choice of m .

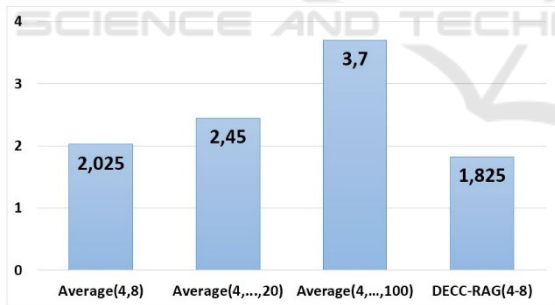


Figure 2: Comparison of ranks for the DECC-RAG(4-8) and the average of DECC-RAG(m).

Table 2 contains the results of Mann–Whitney U test with p -value equal to 0.05. We use the following notations in Tables 2: the sign “<” means that the first algorithm outperforms the second one, otherwise the sign “>” is used, and the sign “ \approx ” is used when there is no statistically significant difference in the results.

As we can see from the results, the dynamic sizing is more preferable than the random choice when the optimal value of m is unknown. Also, we can see from Table 1 that the dynamic sizing

(DECC-RAG(4-8)) is better than the best configuration of the DECC-RAG(m) with $m=8$.

It is obvious that the dynamic sizing can be implemented with other combinations of parameter m . Moreover, we can use not only deterministic schemes, but we can change sizes adaptively using a feedback. We will design and investigate these approaches in our further works.

Table 2: Results of Mann–Whitney U test for the DECC-RAG(4-8) vs Average(m).

DECC-RAG (4-8) vs Average (4, 8)									
F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀
<	>	>	\approx	<	\approx	<	<	<	>
F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅	F ₁₆	F ₁₇	F ₁₈	F ₁₉	F ₂₀
<	<	\approx	<	>	<	\approx	>	<	\approx
DECC-RAG (4-8) vs Average(4, 8, 10, 20)									
F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀
<	>	>	<	<	<	<	<	<	>
F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅	F ₁₆	F ₁₇	F ₁₈	F ₁₉	F ₂₀
<	<	\approx	<	>	<	<	\approx	<	>
DECC-RAG (4-8) vs Average(4, 8, 10, 20, 40, 50, 100)									
F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀
<	<	>	<	<	<	<	<	<	<
F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅	F ₁₆	F ₁₇	F ₁₈	F ₁₉	F ₂₀
<	<	<	<	<	<	<	<	<	>

4 CONCLUSIONS

In this study, we have presented the investigation of tuning the number of subcomponents in the random adaptive grouping algorithm that combines the ideas of random dynamic grouping and learning dynamic grouping for solving LSGO problems. The experimental results have shown that the algorithm is able to provide an efficient decomposition of a problem and to form efficient combinations of variables in subcomponents, thus performs better when using subcomponents of larger size. We have also demonstrated that the dynamic sizing is more preferable than choosing the number of subcomponents at random, and the straightforward scheme DECC-RAG(4-8) outperforms the best configuration of the DECC-RAG. In our further works, we will investigate other settings of the proposed approach and other schemes of the dynamic sizing of subcomponents.

ACKNOWLEDGEMENTS

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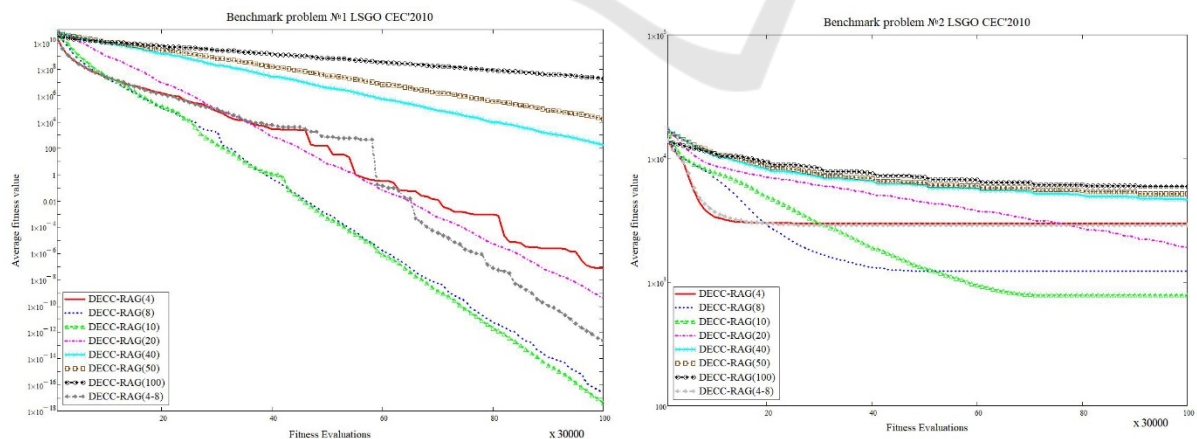


Figure 3: Convergence of the best found for benchmark problems 1 and 2.

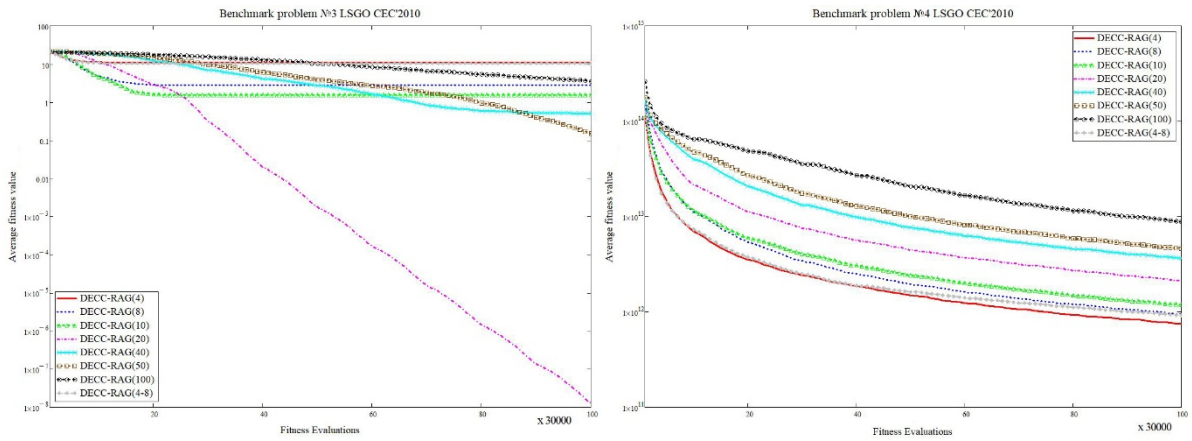


Figure 4: Convergence of the best found for benchmark problems 3 and 4.

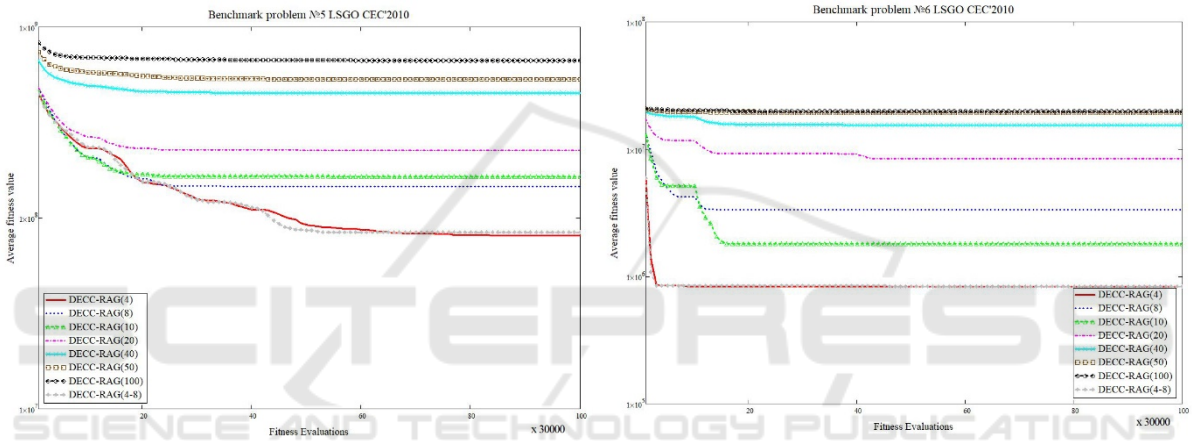


Figure 5: Convergence of the best found for benchmark problems 5 and 6.

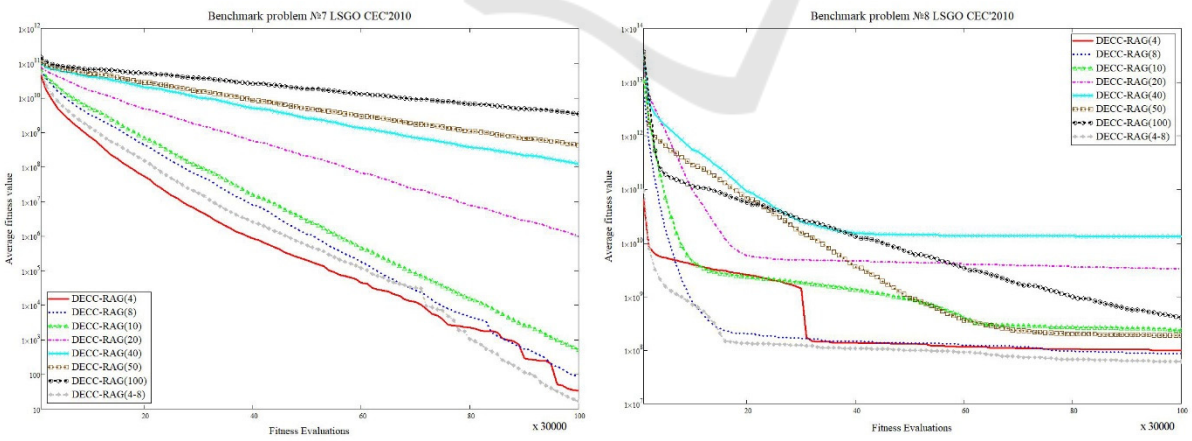


Figure 6: Convergence of the best found for benchmark problems 7 and 8.

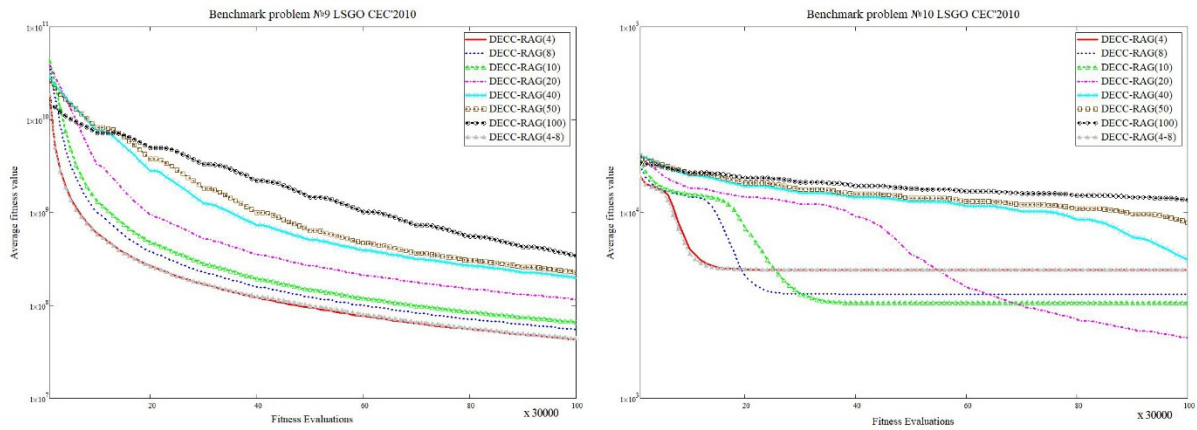


Figure 7: Convergence of the best found for benchmark problems 9 and 10.

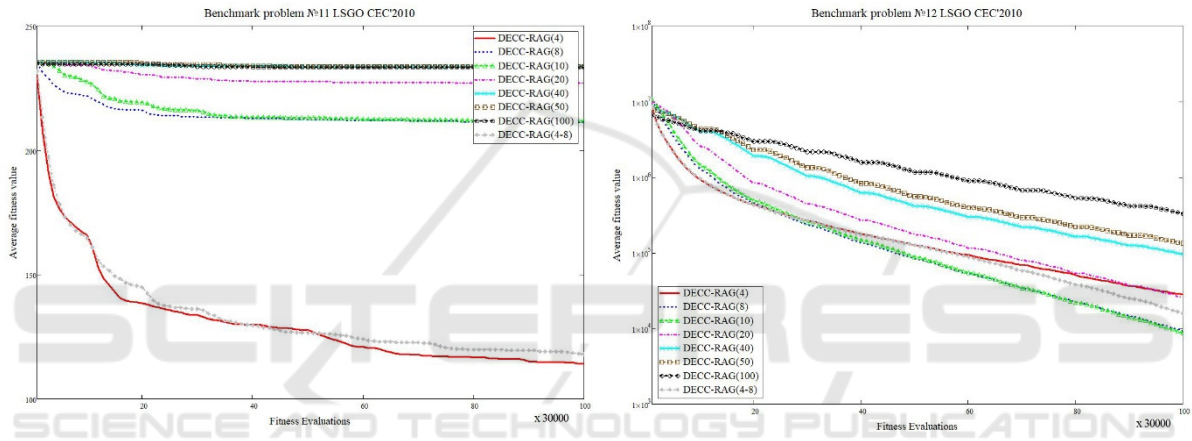


Figure 8: Convergence of the best found for benchmark problems 11 and 12.

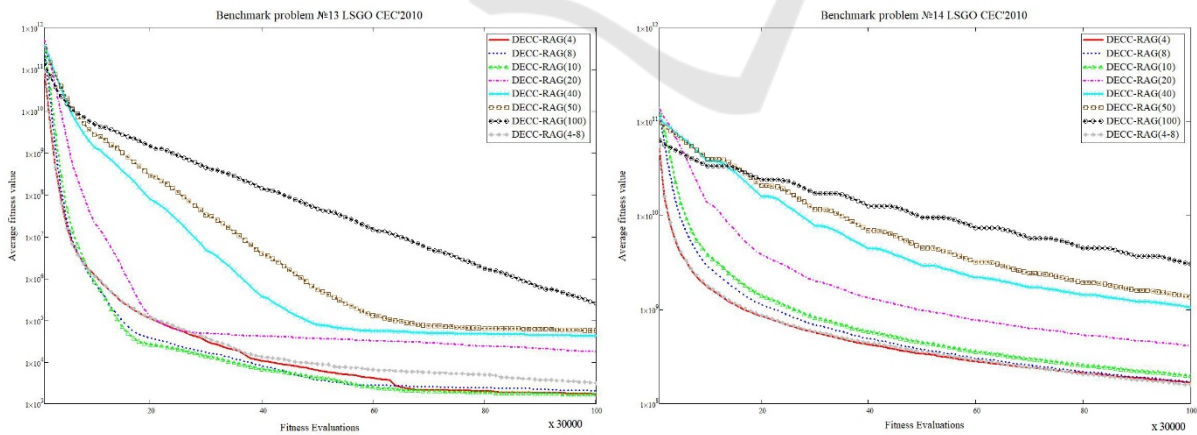


Figure 9: Convergence of the best found for benchmark problems 13 and 14.

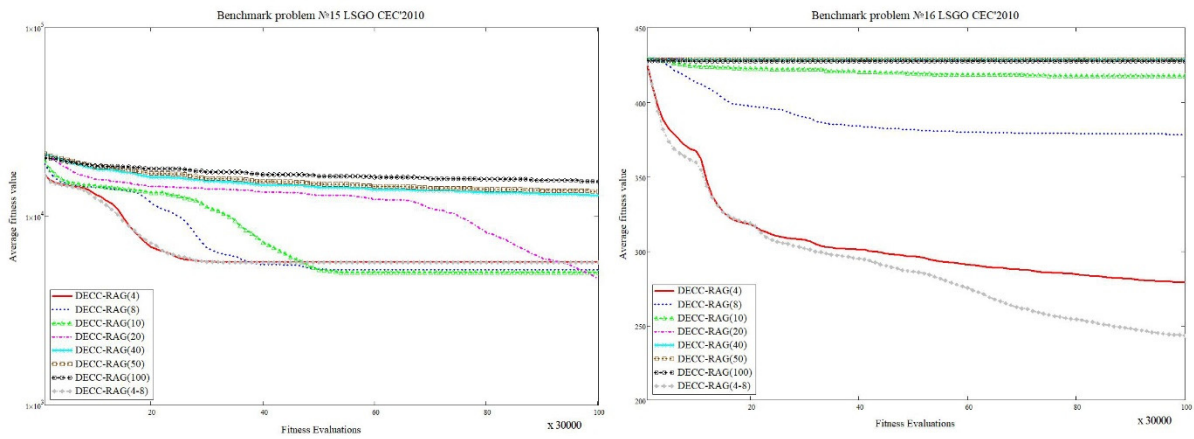


Figure 10: Convergence of the best found for benchmark problems 15 and 16.

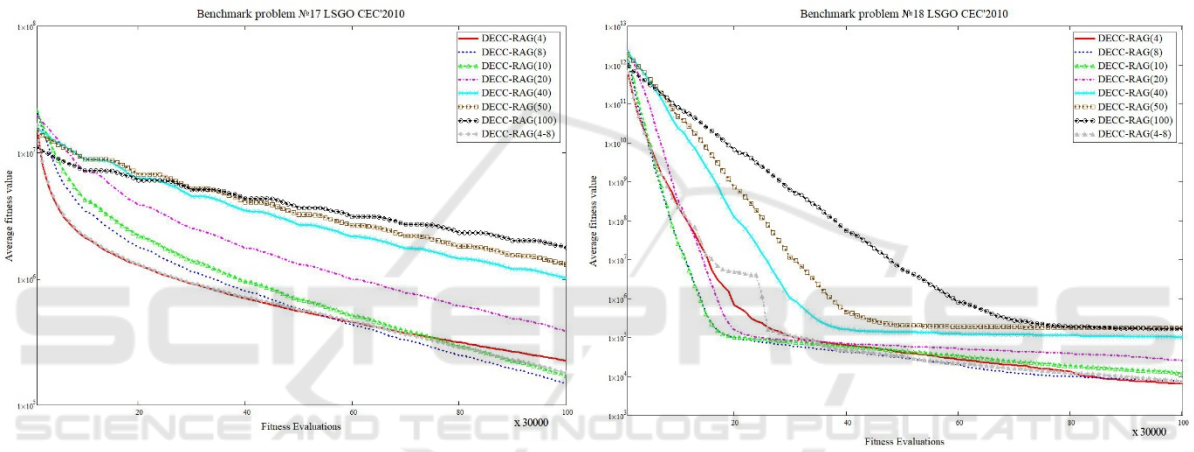


Figure 11: Convergence of the best found for benchmark problems 17 and 18.

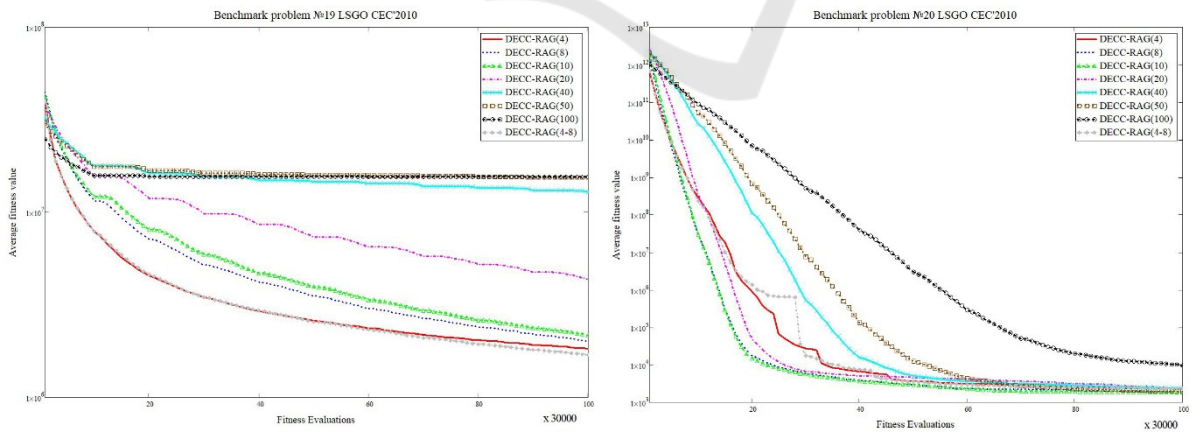


Figure 12: Convergence of the best found for benchmark problems 19 and 20.