Linear Subset Size Scheduling for Many-objective Optimization using NSGA-II based on Pareto Partial Dominance

Makoto Ohki

Field of Technology, Tottori University,
4, 101 Koyama-Minami, Tottori, Tottori 680-8552, Japan

Keywords: Many-Objective Evolutionary Algorithm, Pareto Partial Dominance, Subset Size Scheduling, NSGA-II, 0/1 Knapsack Problem.

Abstract: This paper describes techniques for improving the solution search performance of a multi-objective evolutionary algorithm (MOEA) in many-objective optimization problems (MaOP). As an MOEA for MaOP, we focus on NSGA-II based on Pareto partial dominance. NSGA-II based on Pareto partial dominance requires beforehand a combination list of the number of objective functions to be used for Pareto partial dominance. Moreover, the contents of the combination list greatly influence the optimization result. We propose to schedule a parameter r meaning the subset size of objective functions for Pareto partial dominance. This improvement not only releases users from the schedule of the parameter r but also improves the convergence to Pareto optimal solutions ($\mathcal{POS}$) and the diversity of the individual set obtained by the optimization. Moreover, we propose to kill individuals of the archive set, where the individuals have the same contents as the individual created by the mating. This improvement excludes individuals with the same contents which obtained relatively good evaluations. The improved technique and other conventional techniques are applied to a many-objective 0/1 knapsack problem for verification of the effectiveness.

1 INTRODUCTION

In the real world, there are many problems with more than four objectives. Such the multi-objective optimization problems (MOP) with objective number of four or more are called many-objective optimization problem (MaOP). MaOP is difficult to solve and is tackled by many researchers (Zitzler and Thiele, 1998; Zitzler, 1999; Zitzler et al., 2001; Deb et al., 2000; Deb, 2001; Coello et al., 2007). Although SPEA2 (Zitzler and Thiele, 1998; Zitzler, 1999; Zitzler et al., 2001) and NSGA-II (Deb et al., 2000; Deb, 2001) are well known as powerful algorithm for MOPs, they do not work so effectively for MaOPs (Purshouse and Fleming, 2003; Hughes, 2005; Aguirre and Tanaka, 2007). In this paper, we handle the case of solving an MaOP by NSGA-II based algorithm.

When applying NSGA-II or SPEA2 to MaOP, as the objective number increases, most of the solutions in the solution set, or population, become a relation that is not superior or inferior to each other. This relation is called non-dominated (ND) relationship. As a result, the convergence of the obtained set of Pareto Optimal Solutions ($\mathcal{POS}$) to the optimum Pareto front remarkably decreases. Sato et al. have proposed a Pareto partial dominance that makes it easier to determine the superiority/inferiority relationship between solutions by using several objective functions instead of all objective functions as an algorithm for such MaOP (Sato et al., 2010). Since NSGA-II based on Pareto partial dominance focuses on a relatively small number of objectives, solutions are easy to decide superiority/inferiority even on MaOP, and an effective selection pressure can be expected.

NSGA-II based on Pareto partial dominance has the following three problems. The first problem is that a combination list of the number of objects to be used for Pareto partial dominance must be specified before the optimization. The second one is that an appropriate number of selected objectives according to the complexity of the problem in undecided. Moreover, the contents of the combination list greatly influence the optimization result. NSGA-II based on Pareto partial dominance performs ND sorting using all objective functions at a specific generation cycle, and preserves parents as an archive set for the next generation. This process generates child individuals

Ohki, M.
Linear Subset Size Scheduling for Many-objective Optimization using NSGA-II based on Pareto Partial Dominance.
DOI: 10.5220/0006905402770283
In *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2018) - Volume 1*, pages 277-283
ISBN: 978-989-758-321-6

having the same contents as the already existing individual in the archive set in some cases. As a result, the same individuals increases in the first front set, which disturbs effective ranking in the front selection. This is the third problem. By consideration of these problems, this paper proposes a simple scheduling technique of partial objective set used for Pareto partial dominance and a technique of killing individuals having the same contents in preserving the archive set. In order to verify the effectiveness of the proposed techniques, we examine a many-objective 0/1 knapsack problem(Zitzler and Thiele, 1998).

2 MANY-OBJECTIVE OPTIMIZATION PROBLEM

MOP is a problem that optimizes, or maximizes in this paper, multiple objective functions under several constraints. Since the objective functions are in a trade-off relationship with each other, it is not possible, in general, to obtain the only one solution that completely satisfies all the objective functions. Therefore, we require to obtain $\mathcal{POS}$ of compromised solutions without superiority or inferiority to each other. For the objective function vector $\mathbf{f}$ consisting m objective functions, f_i, the problem of finding the variable vector $\mathbf{x}$ that maximizes the value of f_i in the feasible region S in the solution space is defined as follows.

$$\begin{cases} \text{max.} & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x})]^T \\ \text{s.t.} & \mathbf{x} \in \mathbf{S} \end{cases} \tag{1}$$

When the values of the objective function, f_i, of two solutions x and y satisfy the following relation, we say that the solution x dominates the solution y.

$$\mathbf{f}(\mathbf{x}) \succ \mathbf{f}(\mathbf{y}) \triangleq \\ \forall i \in \mathbf{M} : f_i(\mathbf{x}) \geqq f_i(\mathbf{y}) \wedge \exists i \in \mathbf{M} : f_i(\mathbf{y}) > f_i(\mathbf{y}) \tag{2}$$

where $\mathbf{M}$ denotes a set of the indexes for the objective function, $\{1,2,\ldots,m\}$. When there is no solution dominates a solution $\mathbf{x}$, the solution $\mathbf{x}$ is called non-inferior solution. A set of such the non-inferior solutions is defined as the following $\mathcal{POS}$.

$$\mathcal{POS} = \{\mathbf{x} \in \mathbf{S} | \neg \exists \mathbf{y} \in \mathbf{S}. \mathbf{f}(\mathbf{y}) \succ \mathbf{f}(\mathbf{x})\} \tag{3}$$

A Pareto front showing the the trade-off relation between the objective functions is defined as follows.

$$\mathcal{F}ront = \{\mathbf{f}(\mathbf{x}) | \mathbf{x} \in \mathcal{POS}\} \tag{4}$$

Several effective studies (Zitzler and Thiele, 1998; Zitzler, 1999; Zitzler et al., 2001; Deb et al., 2000; Deb, 2001; Coello et al., 2007) have been made on MOP as defined by Eq.(1). NSGA-II shown in Fig.1 is a powerful multi-objective optimization scheme as a method proposed on one of these studies. NSGA-II applies non-dominated sorting (ND sorting) to the population $\mathbf{Q}$, and the individuals are classified to several ranked subsets, $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \cdots$. While not exceeding the size of the parent set $\mathbf{P}$, the individuals of each subset are moved to the parent set in order. Individuals of the subset that exceeds the size of the parent set is sorted using crowding distance (CD sorting) and moved to the parent set. The individuals not selected are culled. The mating operators generates the child set $\mathbf{C}$ from the parent set $\mathbf{P}$ by using the crossover and mutation operators.

Although NSGA-II effectively solves MOP with less than four objective functions, as the objective number m increases, an appropriate $\mathcal{POS}$ could not be obtained even by those methods containing the conventional NSGA-II. When ND is performed based on the conventional Pareto dominance using all m objective functions, as the number of objective function increases, a subset of solutions satisfying Eq.(2) is difficult to obtain (Tsuchida et al., 2009). Then most solutions of the population become non-inferior solutions. As a result, the superiority/inferiority relationship between solutions is difficult to determined, and the selection pressure in the optimization is significantly reduced. This paper focuses to NSGA-II with Pareto partial dominance shown in Fig.2 for solving MaOP. Pareto partial dominance is based on a concept of partially applying Pareto domination to r objective functions extracted from all m objective functions. The Pareto partial dominance is defined by the following formula.

$$\mathbf{f}(\mathbf{x}) \sqsupset \mathbf{f}(\mathbf{y}) \triangleq \\ \forall i \in \mathbf{R} \subset \mathbf{M} : f_i(\mathbf{x}) \geqq f_i(\mathbf{y}) \\ \wedge \exists i \in \mathbf{R} \subset \mathbf{M} : f_i(\mathbf{y}) > f_i(\mathbf{y}) \tag{5}$$

where $\mathbf{R}$ denotes a set of r indexes selected from $\mathbf{M}$. Since conditions satisfying Pareto partial dominance are relaxed as compared with the conventional dominance using all m objective functions, the population is easier to rank finely in MaOP with large m.

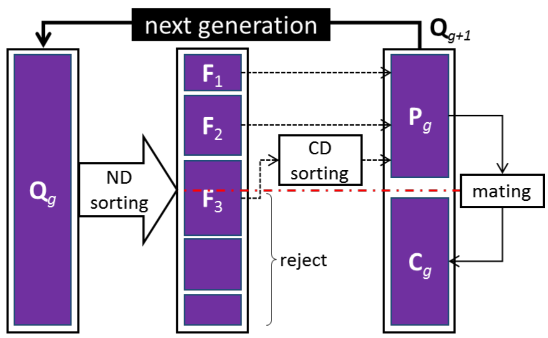

Figure 1: The conventional NSGA-II.

In NSGA-II based on Pareto partial dominance, first of all, given r, the number of objective functions to be considered in the partial ND sorting, a combination list of ${}_mC_r$ selections is prepared beforehand. For each I_g generations, the combination of the objective functions to be considered for Pareto partial dominance is changed, and $\mathbf{R}_{g+1}$ is selected with performing ND sorting on $\mathbf{P}_g + \mathbf{C}_g + \mathbf{A}$ using all m objective functions and copied to the archive set $\mathbf{A}$, where $+$ denotes the direct sum.

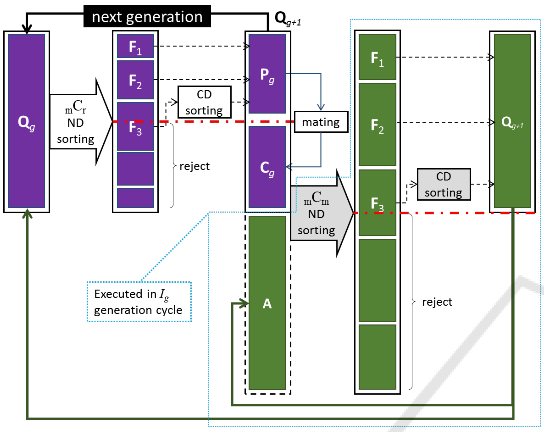

Figure 2: NSGA-II with Pareto partial dominance.

3 IMPROVEMENT OF NSGA-II BASED ON PARETO PARTIAL DOMINANCE

NSGA-II based on Pareto partial dominance has the following three problems. The first problem is that the subset size of the objective functions to be used for Pareto partial dominance is required to beforehand specify before the optimization in a form of a list, or the combination list. The second one is that an appropriate value of the subset size according to the complexity of the problem is unknown. The contents of the combination list greatly influence the optimization result. On the other hand, the creation of the combination list is a very troublesome and difficult task for the user. NSGA-II based on Pareto partial dominance performs ND sorting using all objective functions at a specific generation cycle, and preserves parents as an archive set for the next generation. This process generates child individuals having the same contents as the already existing individual in the archive set in some cases. As a result, the same individuals increases in the first front set, which disturbs effective ranking in the front selection. This is the third problem. In order to avoid these problems, this paper proposes two improvements. A block chart of the improved NSGA-II based on Pareto partial dominance is shown in Fig.3.

As the first improvement, a subset size scheduling is proposed for NSGA-II based on Pareto partial dominance. NSGA-II based on Pareto partial dominance treated in this paper does not use the combination list for each I_g generation cycle. The parameter r is given by the following equations.

$$q = \frac{g \cdot m}{G} + \text{rand\_int}(2B+1) - B, \tag{6}$$

$$r = \begin{cases} B, & q < B \\ q, & B \leqq q < m \\ m, & q \geqq m \end{cases} \tag{7}$$

where m denotes the number of the objective functions, rand_int$(\cdot)$ denotes a function returns a random integer less than the argument, B denotes an integer parameter larger than 1 and less than $m/2$, and G denotes the end generation. Fig.4 shows the possible value of the selection number, r.

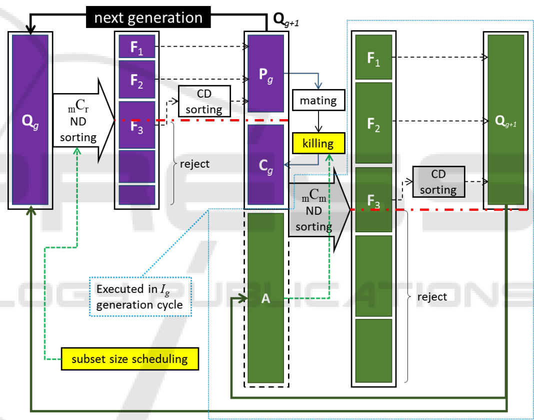

Figure 3: Improved NSGA-II with Pareto partial dominance.

In NSGA-II based on Pareto partial dominance, several individuals having the same contents as an individual already existing in the children, $\mathbf{C}_t$, or the archive set, $\mathbf{A}$, are generated and stored by the mating. If the optimization proceeds while sustaining such the individuals having relatively good evaluation, duplicates of them increases within the population. If the problem to be optimized is relatively simple, individuals with the same content arefrequently generated during the optimization. The second improvement is killing such the individuals having the same contents of an individual already existing in the children, $\mathbf{C}_g$, and the archive set, $\mathbf{A}$, after the mating. Since the optimization problem treated in this paper is the maximizing problem, by setting the value of all objective functions of such the individual to 0, the individual are killed. The same content individual become the worst individual. After killing the same content individual, the mating does not reproduce the individual.

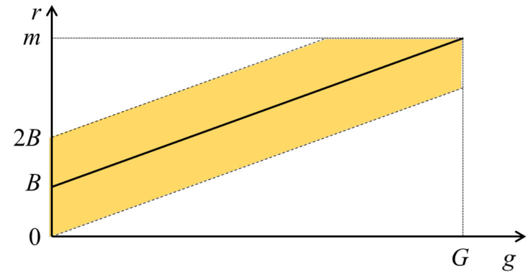

Figure 4: The selection number, r, probablistically takes a value on the colored range according to the generation g, where rand_int$(\cdot)$ denotes a function returns a random integer less than the argument, B denotes an integer parameter larger than 1 and less than $m/2$, and G denotes the final generation.

4 MANY-OBJECTIVE 0/1 KNAPSACK PROBLEM

In order to verify the effectiveness of the improved technique, a many-objective 0/1 knapsack Problem (MaOKSP) is performed. MaOKSP composed of m knapsacks and j items. The capacity of the i-th knapsack is c_i. The weight and the price of the j-th item are w_{ij} and p_{ij} respectively in the i-th knapsack. Let an individual $\mathbf{x} \in 0,1^n$ be the n dimensional vector that selects the items. MaOKSP is defined by the following formula.

$$\begin{cases} \text{max.} & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x})]^T \\ \text{s.t.} & \sum_{j=1}^{n} w_{ij} \cdot x_j \leqq c_i \end{cases} \tag{8}$$

$$f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} \cdot x_j \quad \text{for } i = 1, 2, \cdots, m \tag{9}$$

$\mathcal{POS}$ obtained by the optimization is evaluated by using Maximum Spread (MS)(Zitzler, 1999) and Norm(Sato et al., 2006).

MS expresses a measure showing the spread of $\mathcal{POS}$ distribution. On the other hand, Norm shows a measure of the convergence to the optimal Pareto front of $\mathcal{POS}$. These values are obtained by the following equations.

$$\text{Norm}(\mathcal{POS}) = \frac{\sum_{j=1}^{|\mathcal{POS}|} \sqrt{\sum_{i=1}^{m} f_i(\mathbf{x}_j)^2}}{|\mathcal{POS}|} \tag{10}$$

$$\text{MS}(\mathcal{POS}) = \sqrt{\sum_{i=1}^{m} \left(\max_{j=1}^{|\mathcal{POS}|} f_i(\mathbf{x}_j) - \min_{j=1}^{|\mathcal{POS}|} f_i(\mathbf{x}_j) \right)^2} \tag{11}$$

The conventional NSGA-II, NSGA-II based on Pareto partial dominance when $r = 3$, $r = 6$ and $r = 8$, NSGA-II based on Pareto partial dominance in the case of giving the combination list shown in Table1 and the improved technique are carried out for the verification. The optimization is performed by setting the objective number to $m = 4, 6, 8, 10$ and the iterative generations to $G = 1,000,000$.

Fig.5 shows transition of the number of individuals of the first-front according to the generation in the case that $m = 10$ and $I_g = 500$. In the figure, "NSGA-II" denotes the results by the conventional NSGA-II, "PPD(r=*)" denotes the results by NSGA-II based on Pareto partial dominance with the constant value of $r = *$, "PPD(list)" denotes the results by NSGA-II based on Pareto partial dominance with the combination list shown in Table1, and "Improved" denotes the results by the algorithm proposed in this paper. The conventional NSGA-II and NSGA-II based on Pareto partial dominance in $r = 8$ has given large number of the individuals of the first-front set throughout the optimization. NSGA-II based on Pareto partial dominance with $r = 6$ has given the number next to them. At the end of the optimization, the improved technique has caught up with these values. NSGA-II based on Pareto partial dominance with the combination list is also similar.

Fig.6 shows Norm values values after the optimization to the objective number m in the case that $I_g = 500$. In any technique, the convergence to $\mathcal{POS}$ increases as the number of objectives increases. Although, regarding to the convergence, NSGA-II based on Pareto partial dominance in the case that $r = 3$, NSGA-II based on Pareto partial dominance with the combination list and the improved technique have given almost equivalent results, the conventional NSGA-II has given relatively poor results.

Fig.7 shows MS values values after the optimization to the objective number m in the case that $I_g = 500$. The MS value, or the diversity of $\mathcal{POS}$, given by NSGA-II based on Pareto partial dominance in

Table 1: The combination list for NSGA-II based on Pareto partial dominance.

	generation range			
	0 −500k	500k −900k	900k −1M	
m	r			
4	2	3	4	
6	3	5	6	

	generation rage			
	0 −300k	300k −600k	600k −900k	900k −1M
m	r			
8	3	5	7	8
10	3	6	8	10

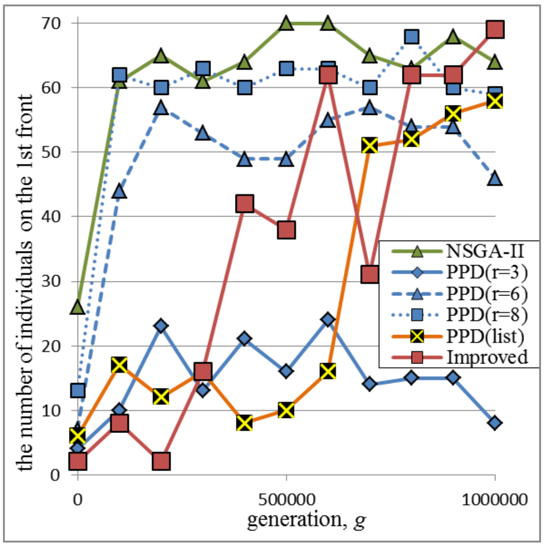

Figure 5: Transition of the number of individuals of the first-front according to the generation.

the case that $r = 3$ decreases as the objective number increases, whereas it increases with the other three techniques. In the improved technique, since r increases as the generation progresses, the superiority/inferiority relationship of solutions becomes difficult to decide by Pareto partial dominance at the end of the optimization, and many individuals belong to the first-front set. As a result, since most individuals of the parents are ranked by the CD sorting, and it is considered that diversity has increased. NSGA-II based on Pareto partial dominance with the combination list has shown diversity equal to or less than that of the improved technique. The reason that sufficient diversity has not been obtained by NSGA-II based on Pareto partial dominance in the case that $r = 3$ is considered as because partial dominance by using all objectives has not been performed only between $900,000$-1 million generations. Regarding the diversity of solutions, the conventional NSGA-II has given the highest value.

Fig.8 shows Norm values to the generation g in the case that $m = 10$ and $I_g = 500$. In NSGA-II based on Pareto partial dominance, the convergence to $\mathcal{POS}$ tends to decrease as the value of the parameter r increases. In this technique, when r approaches m, the solutions are hard to dominated by the partial dominance, so a large number of individuals are selected as the first-front set. As a result, sufficient ranking is not made in the non-dominated sorting, and the convergence has deteriorated. On the other hand, although the improved technique has shown the highest convergence at the beginning of the optimization, the convergence has declined at the final stage. In the improved technique, since the value of r increases as the generation progresses, the solutions become hard to dominated by the partial dominance. As a result, sufficient ranking is not made in the non-dominated sorting, and the convergence has deteriorated in the final stage.

Fig.9 shows MS values to the generation g in the case that $m = 10$ and $I_g = 500$. Although the diversity in the cases of the conventional NSGA-II and NSGA-II based on Pareto partial dominance in $r = 8$, maintains a high value throughout, the convergence is low as shown in Fig.8, so it is not necessary to pay attention to them. On the other hand, the diversity is rising as the optimization progress in the case of the improved technique. Moreover, the improved technique

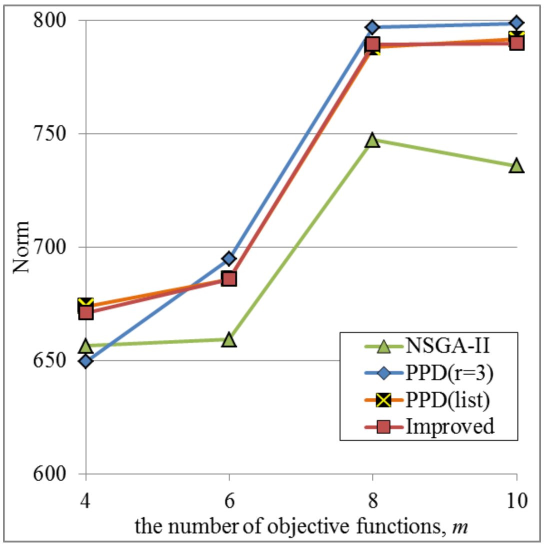

Figure 6: Comparison of Norm values to the object number m.

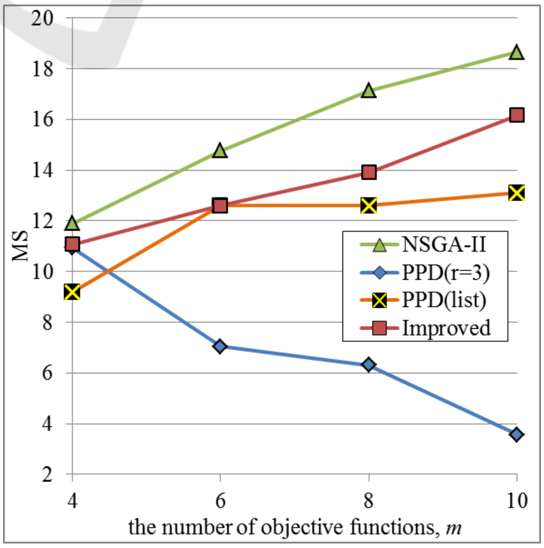

Figure 7: Comparison of MS values to the object number m.

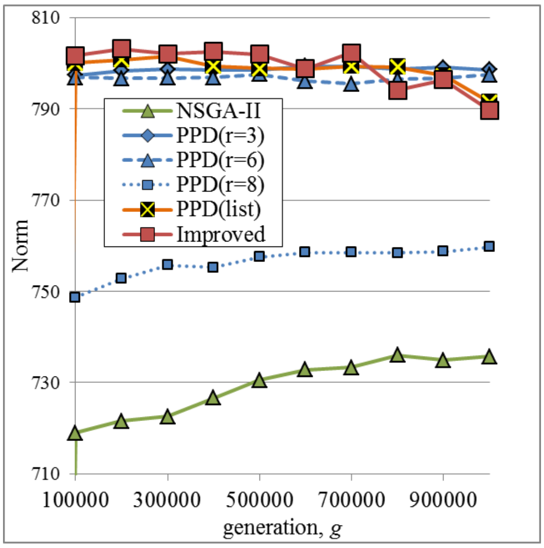

Figure 8: Comparison of the Norm values to the generation g.

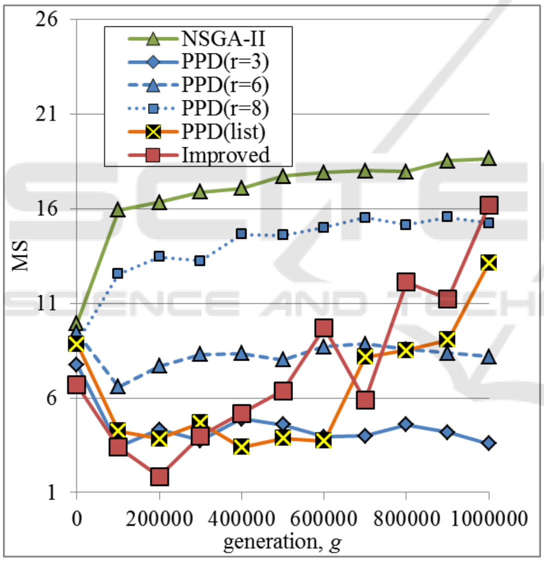

Figure 9: Comparison of the MS values to the generation g.

brings relatively high convergence as shown in Fig.8, so that the superiority of the improved technique is shown overall.

5 CONCLUSION

In this paper, the improvement of NSGA-II based on Pareto partial dominance has been proposed with the aim of improving the solution search performance of MOEA for MaOP. In the improvement, we have proposed the simple scheduling of the number r of the objective functions for Pareto partial dominance and killing the individuals of the archive set, where the individual has the same contents as the individual created by the mating. The improved technique and other conventional techniques are applied to the many-objective 0/1 knapsack problem for verification of the effectiveness. The improved technique has given the higher diversity than other techniques as the number of the objective functions of the problem increases. On the other hand, the improved technique has given the convergence equal to or higher than the other techniques even when the number of the objective functions becomes large. By means of the proposed simple scheduling of the parameter r, sufficient convergence has been obtained in the early generations with the smaller r, and the diversity has been supplemented in the generations with the larger r at the end of the optimization.

Since the improved technique still has given insufficient results in terms of the diversity, we need to improve this point while maintaining the current convergence. Although each technique has been applied to the relatively simple many-objective 0/1 knapsack problem in this paper, we need to apply to more complicated problems and verify the effectiveness. And we also need to pursue an optimal combination list for NSGA-II based on Pareto partial dominance with the selection list.

ACKNOWLEDGEMENTS

This research work has been supported by JSPS KAKENHI Grant Number JP17K00339.

The author would like to thank to her families, the late Miss Blackin', Miss Blanc, Miss Caramel, Mr. Civita, Miss Marron, Miss Markin', Mr. Yukichi and Mr. Ojarumaru, for bringing her daily healing and good research environment.

REFERENCES

Aguirre, H. E. and Tanaka, K. (2007). Working principles, behavior, and performance of moeas on mnk-landscapes. *European Journal of Operational Research*, 181(3):1670–1690.

Coello, C. A. C., Lamont, G. B., Van Veldhuizen, D. A., et al. (2007). *Evolutionary algorithms for solving multi-objective problems*, volume 5. Springer.

Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*, volume 16. John Wiley & Sons.

Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T. (2000). A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: Nsga-ii. In *Interna-*

tional Conference on Parallel Problem Solving From Nature, pages 849–858. Springer.

Hughes, E. J. (2005). Evolutionary many-objective optimisation: many once or one many? In *Evolutionary Computation, 2005. The 2005 IEEE Congress on*, volume 1, pages 222–227. IEEE.

Purshouse, R. C. and Fleming, P. J. (2003). Conflict, harmony, and independence: Relationships in evolutionary multi-criterion optimisation. In *International Conference on Evolutionary Multi-Criterion Optimization*, pages 16–30. Springer.

Sato, H., Aguirre, H. E., and Kiyoshi, T. (2010). Effects of moea temporally switching pareto partial dominance on many-objective 0/1 knapsack problems. *Transactions of the Japanese Society for Artificial Intelligence*, 25:320–331.

Sato, M., Aguirre, H. E., and Tanaka, K. (2006). Effects of δ-similar elimination and controlled elitism in the nsga-ii multiobjective evolutionary algorithm. In *Evolutionary Computation, 2006. CEC 2006. IEEE Congress on*, pages 1164–1171. IEEE.

Tsuchida, K., Sato, H., Aguirre, H. E., and Tanaka, K. (2009). Analysis of nsga-ii and nsga-ii with cdas, and proposal of an enhanced cdas mechanism. *JACIII*, 13(4):470–480.

Zitzler, E. (1999). *Evolutionary algorithms for multiobjective optimization: Methods and applications*. Citeseer.

Zitzler, E., Laumanns, M., and Thiele, L. (2001). Spea2: Improving the strength pareto evolutionary algorithm. *TIK-report*, 103.

Zitzler, E. and Thiele, L. (1998). Multiobjective optimization using evolutionary algorithms comparative case study. In *international conference on parallel problem solving from nature*, pages 292–301. Springer.