

# Particle based Waterfall Simulation with Spray Cloud Emerging from Basin

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**Abstract:** One of the most challenging issues is to simulate and visualize physical phenomena such as thunder, aurora, avalanche and so forth. Among them, simulation of liquid behavior is difficult but most familiar to us. Then, there are many researches related to water; however, there are not so many studies on waterfall. Some researchers visualized waterfall behavior, which, however, was not based on physical simulations. Other papers on physics based simulations demonstrated part of waterfall instead of the whole behavior. Then, we have tried to visualize the whole behavior of waterfall with a physics based particle method. In order to simulate the whole behavior of waterfall, huge amount of particles are needed so that our model divides the whole of waterfall into three parts: water stream, splashing spray, and spray cloud. In the previous works, we have been able to visualize the whole behavior of waterfall, where water stream was translated to splashing spray that was also changed into spray cloud. Then, the previous method had splashing spray and spray cloud emerging from water stream, while real waterfalls have spray cloud that appears from the basin instead of water stream. Therefore, this paper proposes a model to generate spray cloud emerging from the basin instead of water stream.

## 1 INTRODUCTION

In recent years, we can visualize almost everything with computer graphics technology, and some studies focus on the reality and others emphasize the accuracy. The reality based visualization is important for games or movies; however, the accuracy is important for the simulation. Unless there are physics based expressions, the reality is a little bit odd. Then, there are many kinds of physics based simulations and the visualizations. Among them, one of the most challenging issues is to visualize liquid behavior, including ocean, rivers, bubbles, spray and so on. However, there were not so many studies related to waterfall. In addition, some of them did not perform physics based simulations, and others simulated part of waterfall instead of the whole behavior.

Then, we have been researching how the whole behavior of waterfall could be visualized since huge amount of calculation resources are needed for physics based simulations. In order to simulate the whole behavior of waterfall with a normal PC, we divide the model of waterfall into three parts: water stream, splashing spray, and spray cloud. In addition, water stream particles that flow from the lip of the

waterfall are translated into splashing spray particles through isolated particles, and splashing spray particles are changed into spray cloud. Thus, spray cloud emerges from water stream, while real waterfalls have spray cloud appearing from the basin instead of water stream.

Therefore, in this paper, we propose a waterfall model that outputs spray cloud from the basin instead of water stream. In the simulation, we employ a hybrid method of particle and grid for the efficient use of calculation resources, because the area of water stream and splashing spray is limited, while spray cloud disperses broadly. In addition, we employ appropriate methods and the governing equations for each part: particle based method with Navier-Stokes equation, equation of motion for small particles, and grid based method with Navier-Stokes equation.

## 2 RELATED WORKS

(Mould and Yang, 1997) surveyed some previous studies related to water modeling, and indicated that some were based on hydrodynamic theory, while others presented the context based on the observation

of real phenomena. (Iglesias, 2004) also researched papers on realistic modeling, rendering, and animation of water, which were published during 1980s and 1990s. In addition, (Darles et al., 2011) presented a survey on ocean simulation and rendering in computer graphics field, and described that there were two types of works: physically based methods using Navier-Stokes equations, and oceanographic researches based on empirical laws.

Related to the presentation of ocean, (Hinsinger et al., 2002) proposed interactive animation methods of ocean waves that were located far from the coast, and (Cui et al., 2004) presented another real-time simulation method for irregular long crest waves. In addition, (Dupuy and Bruneton, 2012) demonstrated a scalable method of vast ocean scene. These methods rendered ocean waves that were basically continuous surfaces, and mesh models were used for many researches. However, mesh models require re-meshing if the topology change such as river stream breaking occurs.

Then, (Müller et al., 2003) used a particle method called SPH (Smoothed Particle Hydrodynamics), which solved Navier-Stokes equations with surface tension. They visualized water pouring into a glass with point splatting and surfaces generated by marching cubes. (Kipfer and Westermann, 2006) also used SPH for realistic rendering of rivers that flowed from a rock and filled the lake below.

For the simulation of liquid behavior, there are basically two types of methods: Eulerian (grid based) and Lagrangian (particle based) methods. (Chentanez and Müller, 2011) proposed an Eulerian simulation method and optimized the grid for GPU (Graphics Processing Unit) implementation, and (Nishino et al., 2012) used a grid based method to present freezing ice that had air bubbles.

On the other hand, (Foster and Fedkiw, 2001) employed semi-Lagrangian method to handle viscous liquids that interacts with 3D objects, and (Busaryev et al., 2012) proposed a particle based algorithm to simulate bubble behavior, where bubble particles were generated with Voronoi diagram.

In addition, some researchers used hybrid methods of Eulerian and Lagrangian approaches. For instance, (Hong et al., 2008) combined a particle based method for bubbles with a grid based one for the background that was composed of large amount of water and air, and (Chentanez and Müller, 2010) presented a hybrid water simulation method to visualize spray, splash and foam. (Miller, 1989) also introduced a method for animating viscous fluids by considering collision between particles and obstacles, and (Sims, 1990) proposed a parallel particle rendering system

allowing to treat different shapes, sizes, colors and transparencies. (Greenwood and House, 2004) presented a particle level-set fluid simulation algorithm to generate various kinds of bubble shapes. Moreover, (Geiger et al., 2006) used a particle level set algorithm to present the main body of fluid and fine splash particles, and (Kim et al., 2007) employed a level set method to produce bubbles in liquid and gas interaction. (Losasso et al., 2008) proposed a two-way coupled simulation framework using both particle level set for dense liquid volume and particle method for diffuse regions.

In terms of waterfalls, (Mallinder, 1995) proposed an idea of string texture that was continuous stream since huge number of particles were needed if each particle was independent. On the other side, (Foster and Metaxas, 1997) developed a system for animators to be able to control fluid animations without knowledge on governing equations of fluid. (Howes and Forrest, 1997) also developed a simple yet flexible simulation system that generated visually convincing rendering. In addition, (Sakaguchi et al., 2007) created tools for generating a variety of animations from underwater to waterfall, and the tools was used for a movie of "Pirates of the Caribbean 3". (Hardie, 2007) generated a waterfall with a particle method, which works was accepted for ACM SIGGRAPH Art Gallery. The aim of these methods was for animators to generate realistic water related behavior.

On the other hand, (Takahashi et al., 2003) proposed a new method called CIP (Cubic Interpolated Propagation) to visualize dynamic behavior of fluids with splashes and foam. (Hoetzlein and Höllerer, 2009) presented a method to extract surfaces from particles. (Miyashita and Funahashi, 2012) also developed VR (Virtual Reality) learning system, where the envelope surface was rendered around the key particles in real time. In addition, (Nielsen and Østerby, 2013) used an Eulerian two-continua simulation of air and water to visualize waterfall behavior.

In fact, waterfall are composed of three parts: water stream, splashing spray, and spray cloud; however, the works mentioned above did not propose methods to present the whole behavior of waterfall by physics based simulation. Then, (Mukai et al., 2014) proposed a method to visualize the whole behavior of a waterfall including water stream, splashing spray, and spray cloud, and (Nishibe et al., 2015) also visualized spray cloud that was soaring from the basin. In addition, (Mukai et al., 2016) presented various kinds of spray cloud behavior that changes according to the environments. These results, however, showed spray cloud emerging from water stream, while spray cloud appears from the basin instead of water stream in real

waterfalls. Therefore, this paper proposes a waterfall model how spray cloud emerges from the basin.

### 3 WATERFALL MODEL

Figs.1 and 2 show the previous waterfall model, which is divided into three parts: water stream, splashing spray and spray cloud. The waterfall is composed of many particles, and water particles flow from a river into the lip and fall down to the basin with a stream, which is called “water stream” in the method. Particles in water stream are also divided into three kinds: main stream, free surface, and isolated particles, respectively as shown in Fig. 2. These three particles are defined according to the density as follows.

$$\rho_{free} = \alpha_{free} \rho_{main} \quad (1)$$

$$\rho_{iso} = \alpha_{iso} \rho_{main} \quad (2)$$

$$0 < \alpha_{iso} < \alpha_{free} < 1 \quad (3)$$

where,  $\rho_{main}$ ,  $\rho_{free}$  and  $\rho_{iso}$  are the densities of main stream, free surface and isolated particles, respectively.  $\rho_{main}$  keeps a constant value because water is incompressible fluid.

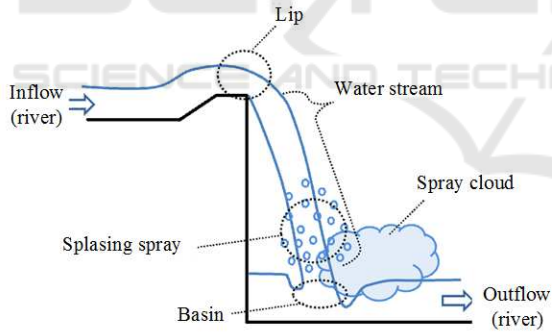


Figure 1: Previous model of waterfall.

In the previous model, isolated particles generate splashing spray particles, which change into spray cloud after a constant time. Then, there are many isolated particles are generated in water stream because it spreads out gradually during falling down to the basin, and the density reduces. As a result, spray cloud emerges around water stream.

In real waterfalls, however, spray cloud emerges from the basin instead of water stream. Then, we consider that some water particles are broken and changed into small splashing spray particles when they collide with the water surface of the basin, and splashing spray particles change into spray cloud that emerges from the basin. Fig. 3 shows this concept.

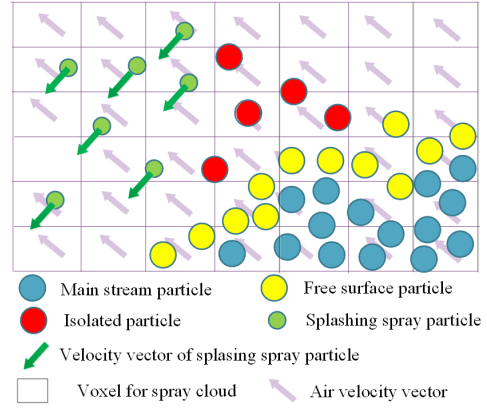


Figure 2: Previous particle model of waterfall.

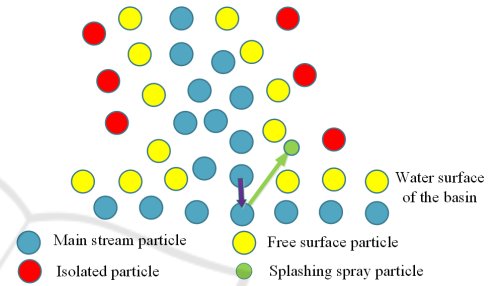


Figure 3: New particle model for splashing spray.

The amount of splashing spray particles depends on the amount and the velocities of main stream particles that collide with the water surface, and the velocity of air. The equation that decides the amount of splashing spray particles is defined as the following.

$$Q_s = \frac{C_a * Q_m |V_m|}{|V_a|} \quad (4)$$

where,  $Q_s$  and  $Q_m$  are the amounts of splashing spray and main stream particles, respectively.  $V_m$  is the velocity of a main stream particle that collides with the water surface, and  $V_a$  is the velocity of air. In addition,  $C_a$  is an attenuation coefficient. The above equation is based on conservation of momentum, and it is supposed that splashing spray particle has the same velocity as that of air. Moreover, wall boundary condition is applied to the velocities of main stream particles that collide with the water surface, which means that the vertical component of the velocity is reversed and the horizontal component is preserved considering the attenuation.

## 4 GOVERNING EQUATIONS

We employ a particle method of SPH (Smoothed particle Hydrodynamics) for main stream, free surface and isolated particles since the simulation area is limited, and these particles obey Navier-Stokes equations. On the other hand, splashing spray particles are very small so that equation of motion for small particle is used as the governing equation. Spray cloud also obeys Navier-Stokes equation, however, it spreads broadly in the simulation space. Then, a grid based method is applied to the simulation. Each method is described briefly as follows.

### 4.1 Water Stream

Navier-Stokes equation for a particle method is written as the following (S. Koshizuka, 2008).

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \quad (5)$$

where,  $\mathbf{u}$  is velocity,  $t$  is time,  $\rho$  is density,  $p$  is pressure,  $\nu$  is viscosity, and  $\mathbf{f}$  is external acceleration. A physical quantity  $\phi(\mathbf{x}_i)$ , its gradient  $\nabla \phi(\mathbf{x}_i)$  and its Laplacian  $\nabla^2 \phi(\mathbf{x}_i)$  at a position  $\mathbf{x}_i$  are defined respectively as follows.

$$\phi(\mathbf{x}_i) = \sum_j m_j \frac{\phi_j}{\rho_j} W(\mathbf{x}_i - \mathbf{x}_j) \quad (6)$$

$$\nabla \phi(\mathbf{x}_i) = \sum_j m_j \frac{\phi_j}{\rho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j) \quad (7)$$

$$\nabla^2 \phi(\mathbf{x}_i) = \sum_j m_j \frac{\phi_j - \phi_i}{\rho_j} \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} \nabla W(\mathbf{x}_i - \mathbf{x}_j) \quad (8)$$

where,  $m_j$ ,  $\rho_j$  and  $\mathbf{x}_j$  is the mass, the density and the position of a particle  $j$ , respectively.  $W$  is called kernel function, and  $W_d$ ,  $W_p$ , and  $W_v$  are the kernel functions of density, pressure, and viscosity, respectively (Müller et al., 2003; Fujisawa, 2013).

In addition, the gravity and the air resistance are considered as the external acceleration  $\mathbf{f}$ , which is defined as follows.

$$\mathbf{f} = \mathbf{g} + \frac{1}{\rho_p V_p} \mathbf{F}_{air} \quad (9)$$

$$\mathbf{F}_{air} = \frac{1}{2} \pi \left( \frac{D_p}{2} \right)^2 \rho_a C_r |\mathbf{u}_p| \mathbf{u}_p \quad (10)$$

where,  $\mathbf{g}$  is gravity, and  $\rho_p$ ,  $V_p$ ,  $D_p$ , and  $\mathbf{u}_p$  are density, volume, diameter and velocity of a particle, respectively.  $\rho_a$  and  $C_r$  are density of air and coefficient of air resistance, respectively, and the air resistance is calculated by approximating the particle as a sphere.

### 4.2 Splashing Spray

Equation of motion for small particle, which is used as the governing equation of splashing spray, is described as the following (Ushijima, 2004).

$$\mathbf{F}_r + \mathbf{F}_b = \rho_p \frac{\pi}{6} D_p^3 \frac{d\mathbf{u}_p}{dt} \quad (11)$$

$$\mathbf{F}_r = \frac{1}{2} \pi \left( \frac{D_p}{2} \right)^2 \rho_a C_r |\mathbf{u}_a - \mathbf{u}_p| (\mathbf{u}_a - \mathbf{u}_p) \quad (12)$$

$$\mathbf{F}_b = (\rho_p - \rho_a) \frac{\pi D_p^3}{6} \mathbf{g} \quad (13)$$

where,  $\rho_a$  and  $\mathbf{u}_a$  are density and velocity of air, respectively and other parameters are the same as those in Eqs.(9) and (10). Splashing spray particles come out of isolated particles, and the initial velocity of splashing spray particle is the same value as that of isolated particle. On the other hand, the velocity of air is the same value as that of spray cloud, which is calculated in the next section.

### 4.3 Spray Cloud

We employ Stable Fluid (Stam, 1999) as the grid based method that guarantees stable behavior of fluid. Navier-stokes equation for spray cloud is written as follows, which is different from Eq.(5) because advection term  $(\mathbf{u} \cdot \nabla) \mathbf{u}$  should be considered in a grid based method.

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \quad (14)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (15)$$

In addition, we consider vapor density for visualization of spray cloud. Vapor density ( $\rho_v$ ) is calculated with the following equation (Kondo, 1994).

$$\rho_v = 1.293 \frac{273.15}{273.15 + T} \frac{p}{1013.25} \left( 1 - 0.378 \frac{e}{p} \right) \quad (16)$$

where,  $1.293 [kg/m^3]$  is the density of dried air,  $273.15 [degree]$  is a coefficient of translation from Celsius temperature to absolute temperature,  $T [degree]$  is Celsius temperature,  $p [hPa]$  is pressure,  $1013.2 [mb]$  is air pressure,  $0.378$  is a coefficient of translation from the gravity of dried air to the gravity of wet air, and  $e [hPa]$  is vapor pressure.

The density difference generates the force that makes spray cloud move up. Then, the following acceleration is added to  $\mathbf{f}$  in Eq.(14) as an external acceleration of the lower grid.

$$\left(\frac{\rho_l - \rho_u}{\rho_l}\right)\mathbf{g} \quad (17)$$

where,  $\rho_l$  and  $\rho_u$  are the densities of lower and upper grids, respectively, and  $\mathbf{g}$  is gravity.

## 5 SIMULATIONS

We have performed the waterfall simulation based on the method described above. Waterfall is composed of three parts: water stream, splashing spray, and spray cloud, and three kinds of methods are applied to each part: Navier-Stokes equation based on particle model, equation of motion for small particle, and Navier-Stokes equation based on grid model.

In the simulation, huge amount of particles are needed. Then, the simulation is divided into two stages: algorithm for water stream, and algorithm for splashing spray and spray cloud. Here, the water stream simulation can be performed independently because it is not affected by the result of splashing spray and spray cloud. However, splashing spray and spray cloud simulation is affected by the result of water stream. Then, it should be performed after the water stream simulation has finished. The simulation algorithm is as follows.

### <A: For water stream>

- A1 Initialize and set parameters.
- A2 Add main stream particles to the lip and remove others from the basin.
- A3 Detect free surface and isolated particles.
- A4 Analyze the behavior of each particle with SPH.
- A5 Calculate position, velocity and density of each particle.
- A6 Continue from A2 to A5.

### <B: For splashing spray and spray cloud>

- B1 Input positions, velocities and densities of main stream, free surface, and isolated particles.
- B2 Generate splashing spray particles and remove them after a constant time.
- B3 Analyze the behavior of splashing spray particles with equation of motion for small particles.
- B4 Generate spray cloud and remove them after a constant time.
- B5 Analyze the behavior of spray cloud with Stable Fluid.
- B6 Update air velocity, which affects splashing spray particles, with the velocity of spray cloud for the next time step.

**B7** Continue from B2 to B6.

## 6 SIMULATION RESULTS

We have performed the waterfall simulation with a normal PC that has Intel Core i7 2.8GHz CPU, 4GB main memory, and GeForce GTX 670 with 4GB memory. The maximum number of particles was 262,144, and the grid size for spray cloud was  $80^3$ . Simulation results are shown in Fig. 4. In the figure, spray cloud is colored in green for easy recognition.

In the previous method, splashing spray particles are generated from isolated particles, which exit in water stream and the basin. Then, spray cloud, which is translated from splashing spray particles, appears along water stream and from the basin as shown in Fig. 4 (a). On the other hand, in the proposed method, splashing spray particles only come out of main stream particles that collide with the water surface of the basin. Thus, spray cloud emerges only from the basin, and it does not appear along water stream and emerges only from the basin as in Fig. 4(b).

In addition, Fig. 5 shows the comparison of the simulation result with a real waterfall called “Kegon no Taki”, which is very famous waterfall in Japan. Water falls down from the lip to the basin along water stream, which spreads out gradually and has some water masses, which are generated by considering air resistance as one of the external forces. From the figure, the simulation result by the proposed method is very similar to the real waterfall.

## 7 CONCLUSIONS

In this paper, we have proposed a model that generates splashing spray particles from the basin, and performed the waterfall simulation. According to the proposed model, splashing spray particles are generated from main stream particles that collide with the water surface in the basin, and the amount of the particles is defined according to the theory based on conservation of momentum. As a result, spray cloud emerged only from the basin and did not appear along water stream. On the comparison of the simulation result with a real waterfall, the simulation result was very similar to the real one with spreading water stream and some water masses in them. However, spray cloud did not soar up because this simulation did not consider the terrain surrounding the waterfall, while the previous simulation used some terrains, which helped spray cloud to soar up. In fact,

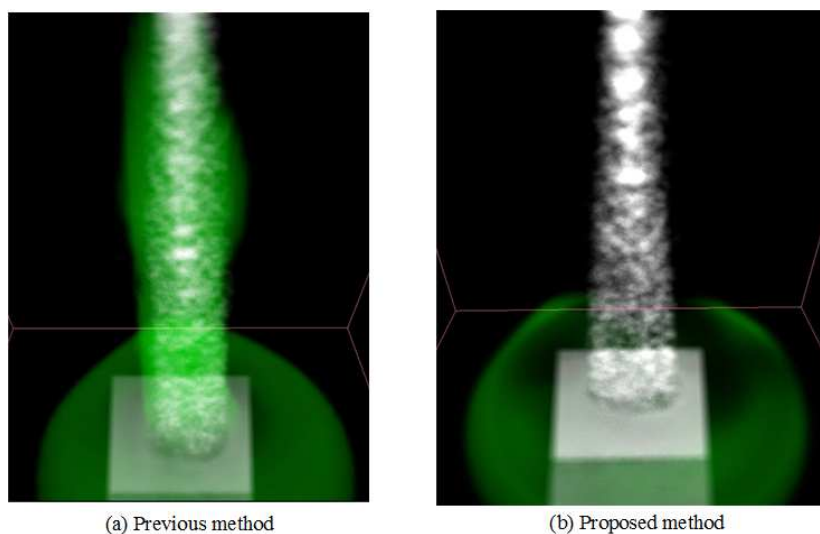


Figure 4: Comparison of the simulation result with the previous method.



Figure 5: Comparison of the generated waterfall with a real one.

some real waterfalls have spray cloud that soars up with the help of the surrounding terrains, while others have spray cloud soaring up from the basin even in vast space. Therefore, in the future, we have to consider the method for spray cloud to soar up without the surrounding terrains by considering the force generated from the density and the temperature difference, and also wind around the waterfall.

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