

Many-Objective Nurse Scheduling using NSGA-II based on Pareto Partial Dominance with Linear Subset-size Scheduling

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Abstract: This paper describes a nurse scheduling in Japanese standard general hospitals. In the standard general hospital in Japan basically three shift system is adopted for nurses working in there. We have compiled evaluations of the monthly nurse schedule into twelve penalty functions in the past work. These twelve penalty functions are translated to twelve objective functions in this paper. The nurse scheduling with twelve objective functions is solved as a multi-objective optimization problem by means of NSGA-II. The optimization is insufficient when NSGA-II is applied to such an optimization problem with four or more objective functions, known as a many-objective optimization problem. One method for reducing this problem is a technique based on Pareto partial dominance. In this technique, the partial non-dominated sorting is executed by using a subset selected from all objective functions. In the conventional technique, the schedule of subset size over optimization has to be prepared beforehand in the form of a list. Moreover, the selection list brings a great influence on the result of optimization. Creating such a selection list is a heavy burden for the user. This paper proposes a technique of NSGA-II based on Pareto partial dominance with a linear subset-size scheduling. By embedding the subset-size scheduling into the algorithm, the user, namely the chief nurse, is released from the designing of the selection list.

1 INTRODUCTION

General hospitals consist of several sections such as the internal medicine department and the pediatrics department. Each section is organized by about fifty to thirty nursing staffs. A section manager constitutes a roster, or a shift schedule, of all nurses of her/his section every month. In our interviewing research to the actual hospitals (Ohki et al., 2006; Uneme et al., 2008), we found that the manager considers more than fifteen requirements for the scheduling. Such the schedule arrangement, in other words, the nurse scheduling, is a very complex task. In the interview, even a veteran manager usually spends 1 or 2 weeks to complete nurse scheduling. Moreover, the schedule obtained after such hardships is not always satisfactory. This means a great loss of work force. Therefore, computer software for the nurse scheduling has been recently come to require in the general hospitals (Goto et al., 1993; Berrada et al., 1996; Ikegami, 2001; Burke et al., 2001a; Burke et al., 2001b; Kawanaka et al., 2002; Itoga et al., 2003; Ohki et al., 2006; Uneme et al., 2008; Ohki, 2012).

In fact, the nurse schedule is still made by the hand of a manager or a chief nurse in many general hospitals. In our survey (Ohki et al., 2006; Ohki, 2012), many general hospitals does not do the nurse scheduling using a commercial software, even when they have purchased it at high price. This is because the purchased software gives unsatisfactory optimization results and that it is very complicated to set and difficult to use. So that many interactions to correct the schedule are also very complex for the user.

As a result of interviewing at many general hospitals, twelve evaluation functions are defined for the nurse scheduling. That is, the nurse scheduling is a multi-objective optimization problem (MOP). This paper discusses a creation of the nurse schedule and an optimization of it by means of multi-objective evolutionary algorithm (MOEA). When creating the nurse schedule, the only one constraint condition for this problem is the degree of satisfaction of the number of nurses in each work shift. This constraint is invalidated by restricting a crossover operator and a mutation operator proposed in this paper. Thus, the

nurse scheduling is defined as an unconstrained multi-objective optimization problem.

Although NSGA-II is well-known as one of effective optimizing frameworks for MOP, it is also known that the ability of optimization significantly decreases for the problem with 4 or more objective functions (Aguirre and Tanaka, 2007; Tsuchida et al., 2009; Sato et al., 2006), in other words, many-objective optimization problem (MaOP)(Hughes, 2005). This paper focuses NSGA-II based on Pareto partial dominance (Brockhoff and Zitzler, 2006; Sato et al., 2010) for MaOP. Since NSGA-II based on Pareto partial dominance requires a scheduling list of the size of the target subset selected from all objective functions beforehand. Since the selection list for Pareto partial dominance largely affects the quality of the optimization, it is very difficult to prepare by the user. Therefore, this paper proposes a technique to automatically schedule the subset size for Pareto partial dominance. By means of the proposed technique the user is not only released from creation of the selection list, but also obtain an effectively optimized nurse schedule.

2 MANY OBJECTIVE NURSE SCHEDULING

Many nurse organizations of the general hospitals in Japan are operated according to the three shift system, including a day shift, an evening shift and night shift. About twenty to thirty nurses belong to each department of the general hospital. The chief of each department creates a monthly nurse schedule as shown in Fig1. The schedule shown in Fig1 is treated as an individual to be optimized with NSGA-II in this paper. The training, the meeting and the requested holiday is called a fixed work and treated as a shift that can not be modified.

	d_1	d_2	d_3	...	d_j	...	d_D
n_1	D	N	E	...	E	...	H T
n_2	N	H	D	...	E	...	M R
\vdots	\vdots				\vdots		\vdots
n_i	R	R	E	...	N	...	M D
\vdots	\vdots				\vdots		\vdots
n_N	D	D	H	...	H	...	M R

Figure 1: A monthly nurse schedule, where n_i denotes the i -th nurse, d_j denotes the j -th day, and D, E, N, T, M, H and R denotes the daytime shift, the evening shift, the night shift, a training, a meeting, a holiday and a requested holiday.

The schedule, in other words, the individual of NSGA-II, is evaluated by the following penalty functions.

p_{1i} : work load to the nurse i given by evaluation of each consecutive three shifts

p_{2i} : concentration to the nurse i of the evening shift and the night shift.

p_{3i} : appearance of specific prohibited shift patterns for the nurse i .

p_{4i} : excess or shortage of the holiday number of the nurse i .

p_{5i} : excess or shortage of the evening shift and the night shift for the nurse i .

p_{6i} : excess number of days of consecutive shifts for the nurse i .

p_{7j} : nursing level in the day shift of the day j .

p_{8j} : nursing level in the evening shift of the day j .

p_{9j} : nursing level in the night shift of the day j .

p_{10j} : compatibility among nurses during the night shift of the day j .

p_{11j} : the number of new nurses at the night shift of day j .

p_{12j} : absence of more than veteran at the day shift and the night shift of day j .

These penalty functions are detailed in (Ohki, 2012). Objective functions for MOP are defined by the following equation by using these penalty functions.

$$f_k = \begin{cases} \frac{1}{1 + \sum_{i=1}^N P_{ki}} & , \quad k = 1, 2, \dots, 6 \\ \frac{1}{1 + \sum_{i=1}^D P_{kj}} & , \quad k = 7, 8, \dots, 12 \end{cases} \quad (1)$$

In general, MOP is a problem which finds solutions that maximizes the objective function vector, where the solution \mathbf{x} satisfies constraint conditions, \mathbf{S} , as follows.

$$\begin{cases} \text{maximize} & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})]^T \\ \text{subject to} & \mathbf{x} \in \mathbf{S} \end{cases} \quad (2)$$

Satisfaction of the number of necessary nurses for each shift on each day is the only constraint condition in the nurse scheduling treated in this paper. By performing initial population generation and mating operators as described below, the number of nurses

assigned to each shift on each day is unchanged during the optimization. Therefore, the constraint condition is invalidated, and the nurse scheduling problem is defined as the following unconstrained MOP.

$$\text{maximize } \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})]^T \quad (3)$$

where $m = 12$.

First of all in the initialization, the fixed works are placed onto the schedule. When the individuals of the initial population is generated, the meeting and the training is transformed to the day shift, and the requested holiday is transformed to the holiday. Taking a look each day, in consideration of these fixed work arrangements, one of the day shift, the evening shift nor the night shift is randomly assigned to the nurses unassigned with the fixed work so as to satisfy the number of nurses assigned for each shift.

In the mating, a crossover operator and a mutation operator are performed as shown in Figs.2 and 3. For each day, the crossover operator decides which parent to extract the shift from. Since the fixed works are assigned to the same place in either parent, they are copied to a child. Then the shift of the nurse who is not assigned fixed work is selected from a decided parent and copied to a child. On the other hand, the mutation operator selects two nurses and a day. Now,

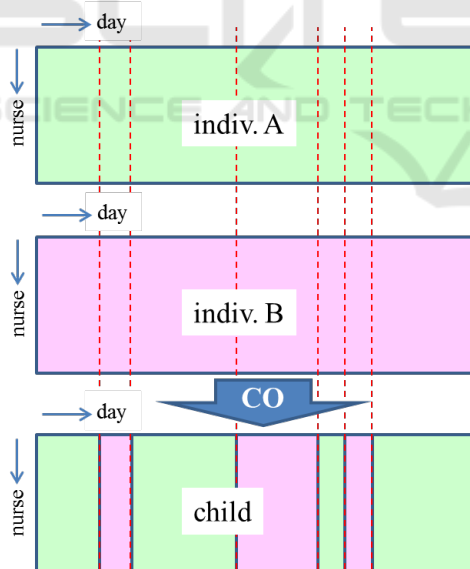


Figure 2: Crossover operator.

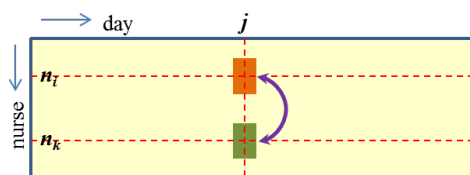


Figure 3: Mutation operator.

two shifts have been selected. If one of them or both have the fixed work, two nurses and a day is selected again. If both are not fixed work, both are exchanged. In the mating, few child individuals are generated with a low probability using only the mutation operator, and remaining most child individuals are generated using the crossover operator. Furthermore, the mutation operator is applied with a low probability to child individuals generated by the crossover operator. These mating operators always generate the individual satisfying the number of necessary nurses for each shift on each day. Therefore, the nurse scheduling shown in this paper has no constraint condition.

Now, we are facing to the nurse scheduling problem (NSP), which is a multi-objective symbol matrix optimization problem, with twelve objective functions. First, we consider applying NSGA-II as shown in Fig.4 to this MOP. NSGA-II applies non-dominated sorting (ND sorting) to the population \mathbf{Q} , and the individuals are classified to several ranked subsets, $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \dots$. While not exceeding the size of the parent set \mathbf{P} , the individuals of each subset are moved to the parent set in order. Individuals of the subset that exceeds the size of the parent set is sorted using crowding distance (CD sorting) and moved to the parent set. The individuals not selected are culled. The mating operators generates the child set \mathbf{C} from the parent set \mathbf{P} by using the crossover and mutation operators.

3 SUB-SET SIZE SCHEDULING FOR PARETO PARTIAL DOMINANCE

Although NSGA-II, shown in Fig.fig:NSGA-II, successfully solves the multi-objective optimization problem with three or fewer objective functions, as the objective functions increases to four or more, the performance of the optimization gradually decreases. When NSGA-II is applied to the optimization problem having many objective functions, since the number of the objective functions is large, the superiority / inferiority relationship is difficult to decide among the individuals of the population. As a result, most individuals belong to the 1st front set, \mathbf{F}_1 . Then, the individuals are ranked only by CD sorting, not ND sorting, and the selection pressure drops remarkably.

In this paper, NSGA-II based on Pareto partial dominance (Sato et al., 2010) (PPD-NSGA-II) is applied to the nurse scheduling having twelve objective functions. PPD-NSGA-II takes out from all m objective functions and performs ND sorting only with

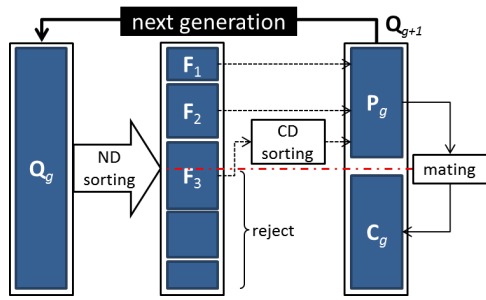
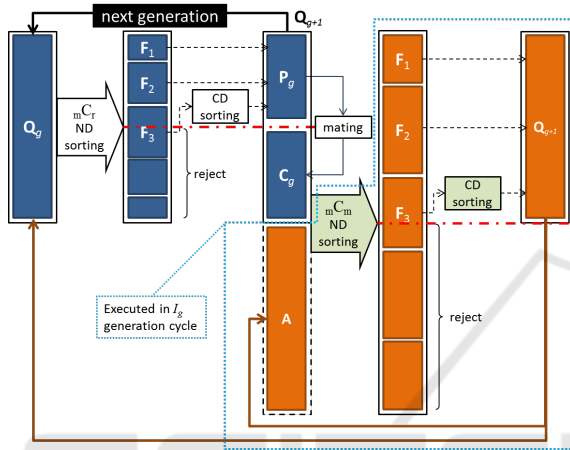

 Figure 4: NSGA-II, where g denotes a generation cycle.


Figure 5: NSGA-II based on Pareto partial dominance.

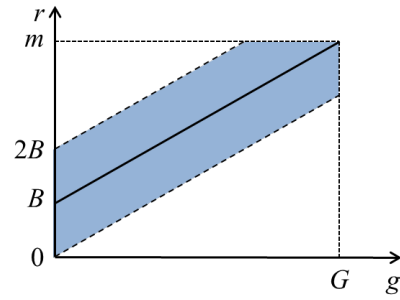
these r objective functions as shown in Fig.5. PPD-NSGA-II takes r objective functions out from all m objective functions and performs ND sorting only with these r objective functions. This is an mC_r ND sorting in Fig.5. In contrast, ND sorting using all m objective functions is shown as mC_m ND sorting. The mC_m ND sorting is performed in I_g generation cycle.

PPD-NSGA-II requires a selection list for selecting r objective functions. The contents of the selection list greatly influence the optimization result. On the other hand, the creation of the selection list is a very troublesome and difficult task for the user, namely the chief nurse. In order to solve this difficulty, we propose a technique to simply schedule the parameter r , the subset size. PPD-NSGA-II treated in this paper does not use the selection list for each I_g generation cycle, randomly selects the subset using the parameter r given by the following equations.

$$r' = \frac{g \cdot m}{G} + \text{rand_int}(2B + 1) - B$$

$$r = \begin{cases} B & , r' < B \\ r' & , B \leq r' < m \\ m & , m \leq r' \end{cases} \quad (4)$$

where $\text{rand_int}(\cdot)$ denotes a function returns a random integer less than the argument, B denotes an integer


 Figure 6: A simple subset-size scheduling for PPD-NSGA-II. The possible value of the subset size r , which takes a value in the colored range according to the generation g .

parameter less than $m/2$, and G denotes the final generation. Therefore, the parameter r is decided as shown in Fig.6.

In PPD-NSGA-II, individuals having the same contents as an individual already existing in the children, C_t , or the archive set, A , are generated and stored by the mating, so that duplicates of individuals given good evaluation increases within the population. If the problem is relatively simple, individuals with the same content frequently appear during the optimization. The second improvement is killing such the individuals having the same contents of an individual already existing in the children and the archive set after the mating. By setting the value of all objective functions of such the individual to 0, the individual are killed.

4 NURSE SCHEDULING IN ACTUAL HOSPITAL

In order to verify the effectiveness of the proposed technique, the nurse schedule is created and optimized based on the same information as the actual hospital. This information represents the standard hospital nurse organization in our country among the information obtained by our survey (Ohki et al., 2006; Uneme et al., 2008). In the nurse scheduling, the number of nurses is 23, the number of days is 28. The nurse schedule is optimized for 1,000,000 generations by NSGA-II. In this paper, we try to apply two techniques, the conventional NSGA-II and the improved PPD-NSGA-II, to the nurse scheduling.

In this NSP handled in this paper, several objective functions has saturated to a maximum number of 1. These objective functions do not always saturate.

Fig.7 shows individual distribution on the f_1 — f_{11} plane at the final generation. This figure shows an example where PPD-NSGA-II has given dominant

results compared to the conventional NSGA-II. This means that focusing on these two objective functions shows that PPD-NSGA-II has given dominant result. The same results are obtained also in Figs.8 and 9.

Fig.10 shows individual distribution on the f_2 — f_3 plane at the final generation. In this figure, several advantageous points overlap. In other words, when focusing on these two objective functions, the conventional NSGA-II and PPD-NSGA-II have given nearly equivalent results. The same results are obtained also in Figs.11 and 12.

Fig.13 shows individual distribution on the f_1 — f_7 plane at the final generation. This figure shows an example where the conventional NSGA-II has given dominant results compared to PPD-NSGA-II. This means that focusing on these two objective functions shows that the conventional NSGA-II has given dominant result. The same results are obtained also in Fig.14.

Fig.15 shows individual distribution on the f_9 — f_{11} plane at the final generation. In this figure, the conventional NSGA-II is advantageous for the objective function f_{11} while PPD-NSGA-II is advantageous for the objective function f_9 . Thus, we could find many cases that superiority or inferiority can not be distinguished by the objective function of interest.

Since 12 objective functions are defined in NSP handled in this paper, as shown above, 2 objective functions are necessary to select from 12 in order to visualize the individual distribution. These 12 objective functions can be classified into 6 objective functions related to the shift of each nurse and 6 objective functions based on evaluation on each work day. Therefore, partial sums of the objective functions are defined as follows.

$$s_1 = \sum_{i=1}^6 f_i \tag{5}$$

$$s_2 = \sum_{i=7}^{12} f_i \tag{6}$$

By using these partial sums, the transition of the individual distribution is observed on the s_1 — s_2 plane as shown in Figs.16—24. In the optimization up to the 600 thousand generation, we can observe that the population given by the both techniques gradually moves in the upper right direction. Especially in PPD-NSGA-II, the population converges to the upper right direction more quickly. After that, both populations concentrate on the upper right part in the generation of about 700 thousand. The reason for concentrating like this is unknown. Although the population given by the conventional NSGA-II then spreads again to

a wide range, the population given by PPD-NSGA-II remains relatively dense afterwards. In the case of using the conventional NSGA-II, no significant progress is observed in the optimization after the 800 thousand generation. On the other hand, in the case of using PPD-NSGA-II, the optimization progressed thereafter, and gives several excellent solutions in the 900 thousand generation. However, in the final generation of PPD-NSGA-II, each solution of the population has been deteriorated. As one of the reason for this result, it can be considered that the value of r of PPD-NSGA-II is almost equal to m at the end of the optimization.

Finally, in order to numerically compare the performance of both, we consider on Norm (Sato et al., 2006) value and Maximum Spread (MS) (Zitzler, 1999) value. These values are obtained by the following equations.

$$\text{Norm}(\mathcal{POS}) = \frac{\sum_{j=1}^{|\mathcal{POS}|} \sqrt{\sum_{i=1}^m f_i(\mathbf{x}_j)^2}}{|\mathcal{POS}|}, \tag{7}$$

$$\text{MS}(\mathcal{POS}) = \sqrt{\sum_{i=1}^m \left(\max_{j=1}^{|\mathcal{POS}|} f_i(\mathbf{x}_j) - \min_{j=1}^{|\mathcal{POS}|} f_i(\mathbf{x}_j) \right)^2}, \tag{8}$$

where \mathcal{POS} denotes the individuals of the first front set at the final generation. These values obtained by both are summarized in Table.1. PPD-NSGA-II dominates to the conventional NSGA-II in terms of the convergence property of the population to POS, or Norm value. On the other hand, the conventional NSGA-II is somewhat predominant with regard to the diversity of the population at the final generation, or MS value.

Table 1: Comparison of Norm value and MS value of the population at the final generation.

	NSGA-II	PPD-NSGA-II
Norm	1.5589	2.1795
MS	2.9126	2.3970

5 CONCLUSION

This paper has treated the nurse scheduling in Japanese standard general hospitals. Twelve objective functions have been defined for the nurse scheduling. The nurse scheduling problem is solved as a multi-objective optimization problem by means of NSGA-II in this paper. It is known that optimization is insufficient when NSGA-II is applied to an optimization

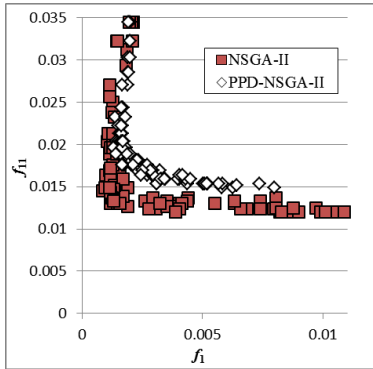


Figure 7: Individual distribution on the f_1 — f_{11} plane at the final generation.

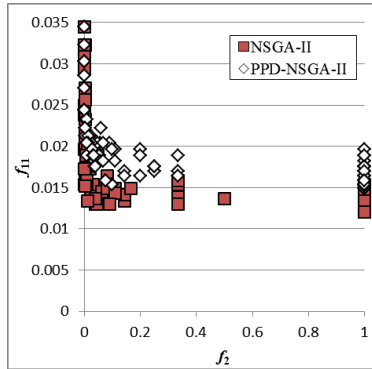


Figure 8: Individual distribution on the f_2 — f_{11} plane at the final generation.

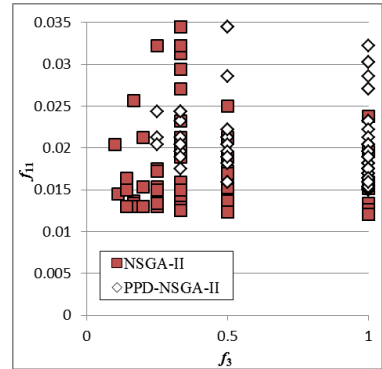


Figure 9: Individual distribution on the f_3 — f_{11} plane at the final generation.

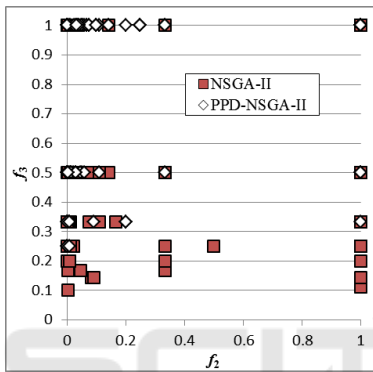


Figure 10: Individual distribution on the f_2 — f_3 plane at the final generation.

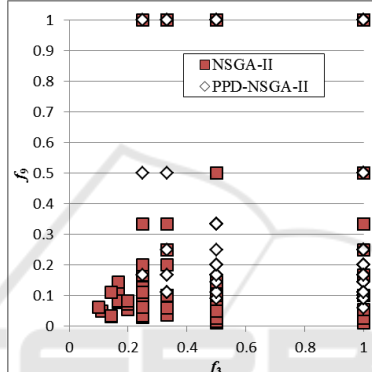


Figure 11: Individual distribution on the f_3 — f_9 plane at the final generation.

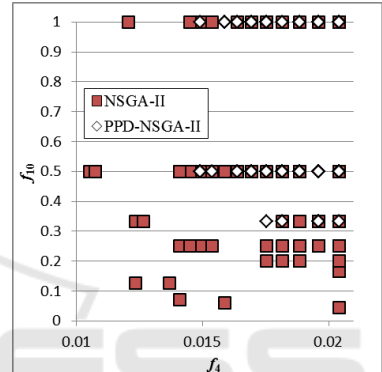


Figure 12: Individual distribution on the f_4 — f_{10} plane at the final generation.

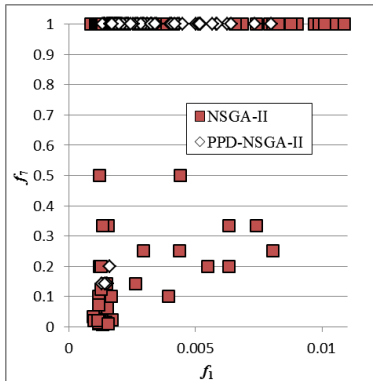


Figure 13: Individual distribution on the f_1 — f_7 plane at the final generation.

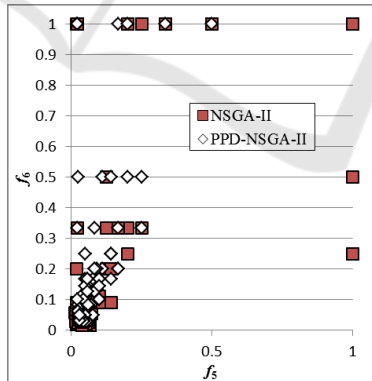


Figure 14: Individual distribution on the f_5 — f_6 plane at the final generation.

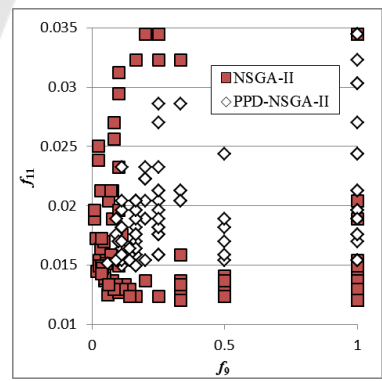


Figure 15: Individual distribution on the f_9 — f_{11} plane at the final generation.

problem with four or more objective functions. One method for reducing this problem is a technique based on Pareto partial dominance. PPD-NSGA-II is applied to the nurse scheduling treated in this paper.

PPD-NSGA-II requires to receive the number r of objective functions, or the subset size, to be selected from all m objective functions in the form of a list in

advance. Moreover, this selection list has a great influence on the result of optimization. Creating such a selection list is a heavy burden for the user and is also substantially impossible. This paper has proposed a technique of PPD-NSGA-II with linear subset-size scheduling. By embedding the subset-size scheduling into the algorithm, the user, namely the chief

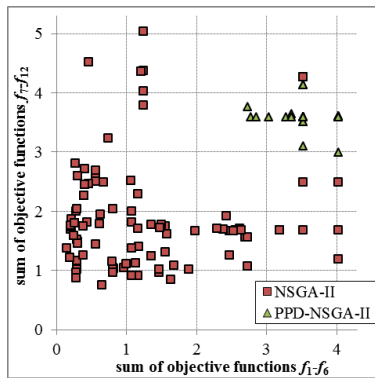


Figure 16: Individual distribution on the s_1-s_2 plane at the 100k-th generation.

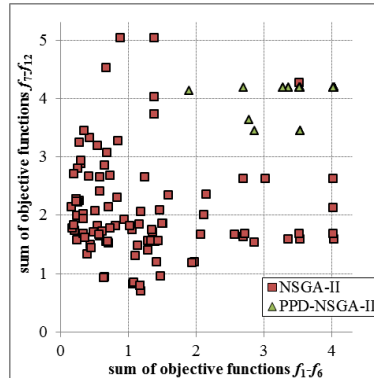


Figure 17: Individual distribution on the s_1-s_2 plane at the 200k-th generation.

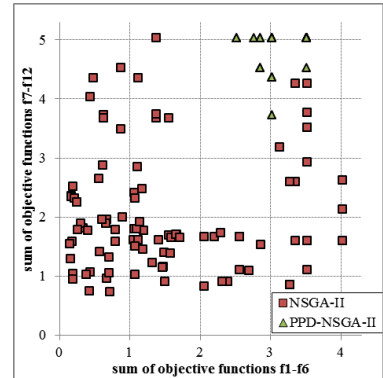


Figure 18: Individual distribution on the s_1-s_2 plane at the 400k-th generation.

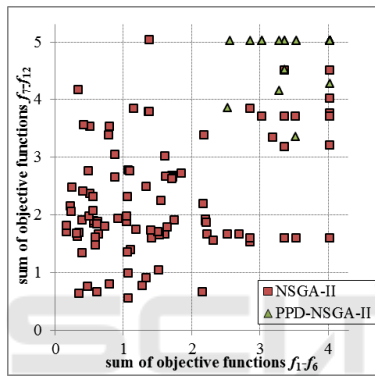


Figure 19: Individual distribution on the s_1-s_2 plane at the 500k-th generation.

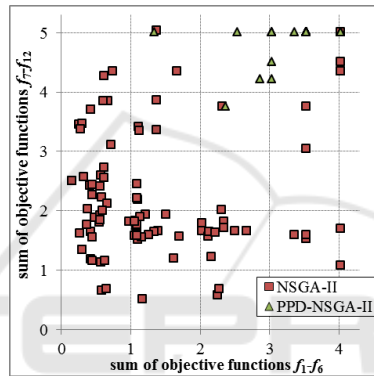


Figure 20: Individual distribution on the s_1-s_2 plane at the 600k-th generation.

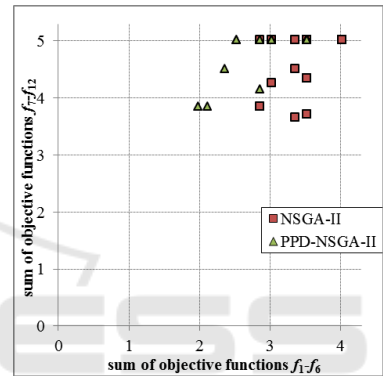


Figure 21: Individual distribution on the s_1-s_2 plane at the 700k-th generation.

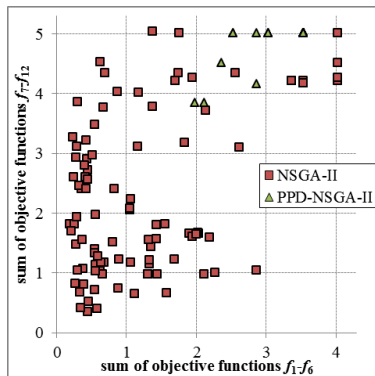


Figure 22: Individual distribution on the s_1-s_2 plane at the 800k-th generation.

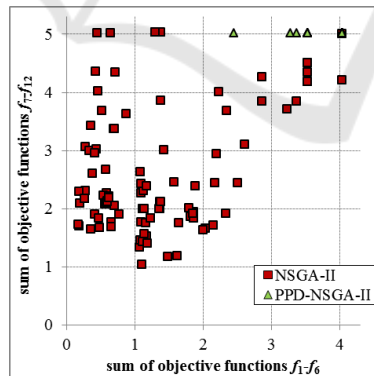


Figure 23: Individual distribution on the s_1-s_2 plane at the 900k-th generation.

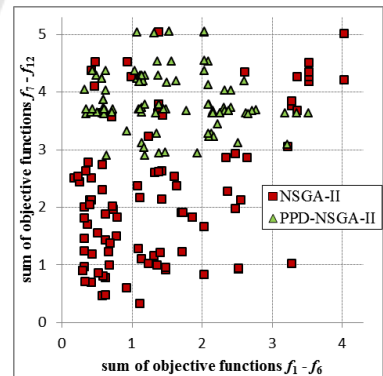


Figure 24: Individual distribution on the s_1-s_2 plane at the final generation.

nurse, is released from the designing of the selection list.

In order to verify the effectiveness, the proposed technique and the conventional NSGA-II have been applied to the nurse scheduling problem. Although the proposed method is somewhat inferior on the di-

versity of the population as compared with the conventional NSGA-II, it is significantly effective on the convergency of the population to the Pareto optimal solution set.

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