Chaos Engineering and Control in Mobile Robotics Applications

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Abstract: This article briefly summarizes the theory of chaos and its applications. Firstly, we begin by describing chaos as an aperiodic bounded deterministic motion, which is sensitive to initial states and therefore unpredictable after a certain time. Then, fundamental tools of the chaos theory, used for identifying and quantifying chaotic dynamics, are shared. The paper covers a main numerical approach to identify chaos such as the Lyapunov exponents. Many important applications of chaos in several areas such as chaos in electrical and electronic engineering and chaos applications in robotics have been presented. An analysis of the reviewed publications is presented and a brief survey is reported as well.

1 INTRODUCTION

During the 20th century, three great revolutions occurred: quantum mechanics, relativity and chaos. The theory of chaos, also called dynamical systems theory, is the study of unstable aperiodic behavior in deterministic dynamical systems, which show a sensitive dependence on initial conditions (Vaidyanathan, 2013). The sensitive dependence on initial conditions implies that arbitrary initial conditions follow trajectories that move away from one another after a certain time (Moon, 2008), as shown in figure 1. Due to determinism (Morrison, 2012), chaos is predictable for the short time; but it is unpredictable in the long run due to sensitivity to initial conditions. Chaos is characterized by a large sensitive dependence to the initial state, by its inability to predict future consequences, by the Lyapunov exponent (Kuznetsov, 2016), by its fractal dimension, and so on.

The nonlinear dynamics and chaos terms have become known to most scientists and engineers over the past few decades. Nonlinearities occur in feedback processes (Gaponov-Grekhov and Rabinovich, 2011), in systems containing interacting subsystems, and in systems interacting with the environment.

This scenario is qualitatively and quantitatively distinct from the situations where the perturbations develop linearly. Thanks to the availability of high-speed computers and new analytical techniques, it has become clear that the chaotic phenomenon is of a universal nature and has transverse consequences in various areas of human endeavour.

The devices of the fire fighting and floor cleaning have been developed by exploiting autonomous mobile robots as useful tools in activities that put the integrity of humans in danger, such as monitoring and exploring of terrains for explosives or dangerous materials and such as intrusion patrols at military installations. This has driven to the development of intelligent robotic systems (Martins-Filho and Macau, 2007). Therefore, the unpredictability of a trajectory is also a crucial factor for the mission success for such an autonomous mobile robot. To meet this challenge, Sekiguchi and Nakamura suggested a strategy in 2001 to solve the problem of path planning based on chaotic systems (Nakamura and Sekiguchi, 2001).

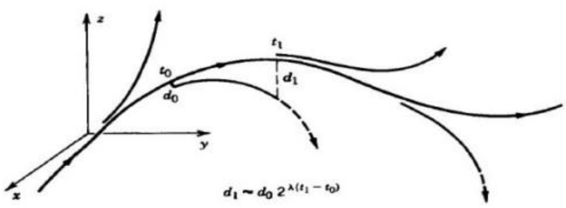

Figure 1: Two trajectories that start close to each other but diverge within a few tens of seconds (Moon, 2008).

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A key property of chaos is that simple dynamical systems can often engender complex dynamics. These systems can be implemented using simple analogue hardware (Aihara, 2012).

In order to explore the applications of chaos in engineering and robotics, this paper is organized as follows. Section 2 gives an overview of the chaos theory. In section 3, we review the research on applications of chaotic dynamics in electrical and electronic engineering. Chaos synthesis in robotics is presented as an application of chaotic systems for motion planning of autonomous mobile robots in section 4. The conclusion is presented in section 5.

2 CHAOS THEORY: AN OVERVIEW

The movements of several natural or engineering systems can be governed by a set of equations derived from natural laws such as the Newton's laws or the Euler equation. The equations that describe a dynamic system can be algebraic or differential. The set of equations, mathematically defined as a dynamic system, gives the temporal evolution of the state of a system from the knowledge of its previous history. Therefore, the state at any time can be determined by the governing equations and the initial states. In modern science, the term chaos is used to describe a type of motion resulting from a dynamic system that appears to be disordered and extremely complex under detailed examination. Complicacy and disorder are due to the reasons that chaos is a recurrent aperiodic motion. Hence, chaos can be defined as a bounded steady-state response that is not an equilibrium state, a periodic motion, or a quasi-periodic motion.

Chaotic movements are also characterized by sensitivity to initial states; i.e, a simple variation in the initial conditions can quickly produce enormous differences in response. The long-term prediction of chaos is impossible, due to such sensitivity. In other words, chaos is unpredictable after some time because a small difference in initial conditions beyond their precision will result in a rapid growing of the movement.

A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence, and sensitive dependence on initial conditions (Azar and Vaidyanathan, 2015, Azar and Vaidyanathan, 2014). The Chaos theory investigates the qualitative and numerical study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems.

In 1963, Lorenz (Lorenz, 1963) discovered a 3-D chaotic system when he was studying a 3-D weather model for atmospheric convection. After a decade, Rössler (Rössler, 1976) discovered a 3-D chaotic system, which was constructed during the study of a chemical reaction. These classical chaotic systems paved the way to the discovery of a lot of 3-D chaotic systems such as the Arneodo system (Arneodo et al., 1981), the Chen system (Chen and Ueta, 1999), the Lü-Chen system (Lü and Chen, 2002), the Tigan system (Tigan and Opriş, 2008), etc.

Some other scientists have tried to propose new dynamic systems. In (Leonov and Kuznetsov, 2013), Leonov gave a survey on the relationship between hidden oscillators and hidden chaotic attractors. Snchez-Lpez et al. (Sánchez-López et al., 2010) proposed a method to generate a multi-scroll chaotic attractor based on Saturated Nonlinear Function Series. In (Zhang and Tang, 2012), Zhang and Tang introduced a new chaotic system with four components that can generate chaos, hyperchaos, and periodic and quasi-periodic behaviors. Sun et al. (Sun et al., 2014) presented a finite-time combination scheme of four chaotic systems and solved the problem of the synchronization of two systems. Bouallegue (Bouallegue, 2015b) found a method to generate a new class of chaotic attractors that possessed a multi-fractal scroll based on a fractal process, as illustrated in figure 2. Another work by Kais Bouallegue (Bouallegue, 2015a) and Salah Nasr (Salah NASR, 2015) put forward a new behavior of chaotic attractors with separated scrolls using the combination between the fractal process and the chaotic attractors as depicted in figure 3.

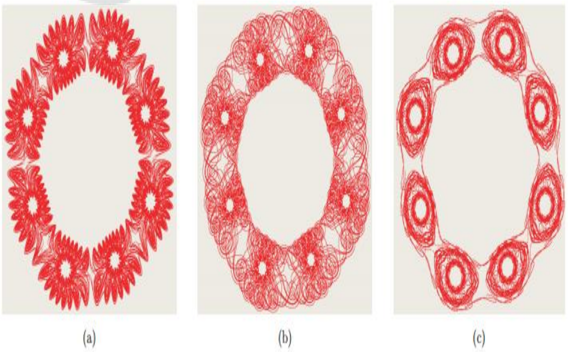


Figure 2: Chaotic attractor with fractal and multifractal scrolls. (a) Multichaotic attractor with Lorenz system, (b) multichaotic attractor with Chua system and (c) multichaotic attractor with Rössler system (Bouallegue, 2015b).

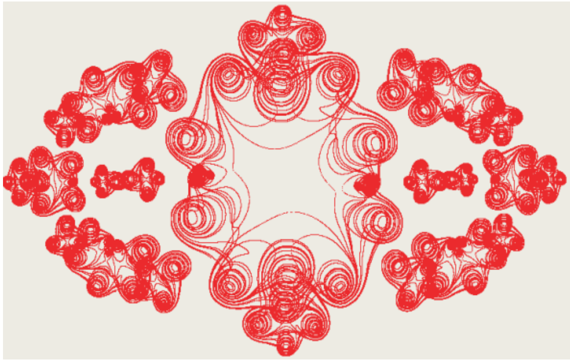


Figure 3: Chaotic attractors with four behavior forms (Salah NASR, 2015).

2.1 Identification of Chaos

The identification of chaos is the set of diagnostic tests to determine if chaotic behavior occurs in a specific system. To identify chaos, some numerical characteristics associated with the motion of a system can be used. These characteristics include the Lyapunov exponents, power spectra and entropies.

2.2 Lyapunov Exponent

To estimate and compute the Lyapunov dimension, Leonov (Kuznetsov et al., 2016, Kuznetsov, 2016) proposed an analytical approach in 1991. A Lyapunov exponent may be positive or negative. A positive Lyapunov exponent implies the divergence in the corresponding direction. A negative exponent implies the constriction in the corresponding direction. Hence, most dynamic systems can be characterized by their Lyapunov exponents (Kaygisiz et al., 2011). For a chaotic attractor, a two-dimensional system must have a positive and a negative Lyapunov exponents. If the system has two zero Lyapunov exponents and two negative ones then, the system is quasi-periodic. If the four-dimensional system is hyperchaotic, it should have two positives, a zero one and a negative one.

3 CHAOS ENGINEERING APPLICATIONS

Over the last few decades, the terms chaos and nonlinear dynamics are known to most engineers and scientists. Nonlinearities occur in feedback processes, in systems containing interacting subsystems and in systems interacting with the environment. It is striking to note that simple devices, such as a double pendulum, and a very complex event such as time follow the same dynamics, which can only be predicted for short time horizons (Kaygisiz et al., 2011). Thanks to the existence of high-speed computers, new analytical techniques and sophisticated experiments, it has become clear that the chaotic phenomenon is of a universal nature and has transverse consequences in various fields of human endeavour.

Recently, chaos theory is found to have important applications in several areas such as biology (Garfinkel, 1992, May, 1976, Vaidyanathan, 2015a, Vaidyanathan, 2015b), memristors (Pham et al., 2015, Volos et al., 2015a), electrical circuits (Volos et al., 2015b), robotics (Nakamura and Sekiguchi, 2001), etc.

A lot of practical applications of deterministic chaos have been developed in various fields of engineering and technology. Actually, studies of nonlinear dynamics in engineering disciplines have been steadily progressing over a half century. Among these disciplines, we are going to deal with electrical and electronic engineering, and synchronization of chaotic systems.

3.1 Electrical and Electronic Engineering

A lot Electrical and electronic circuits are typical fields in which rich chaotic phenomena have been reported. In 1927, Van der Mark and Van der Pol heard noise corresponding to deterministic chaos in an electrical system composed of a resistor, a neon glow lamp, a variable condenser, and D.C. and A.C. power sources (Van der Pol and Van der Mark, 1927). Since then, chaos has been detected both experimentally and numerically in many electrical and electronic circuits like the Chua's circuit, the Shinriki's circuit (Chua et al., 1993, van Wyk and Steeb, 1997) and the series RLC circuit with a varactor diode (Testa et al., 1982). It should be noted that both the Duffing equation and the Duffing-Van Der Pol mixed type equation are also models of electrical circuits with a nonlinear inductance and a nonlinear resistance, respectively.

In order to create controlled chaotic outputs, the work presented in (Hanias and Tombras, 2009, Hanias et al., 2010) makes the study of simple circuits which are composed of simple transistors triggered externally be able to give significant results for the possibility of creating its chaotic outputs. These results are very important, especially in telecommunication systems where the need for safe signal transmission is imperative. In their work, the authors presented the chaotic behavior of three forms of these circuits as well as corresponding theoretical methodology. Next, in the study of simple chaotic circuits, we consider the case of an optoelectronic chaotic circuit which is based on an optocoupler device and which can be used as a controlled optoelectronic chaotic signal generator, presented in (Hanias, 2010).

These studies have confirmed that chaos can be actually and easily generated in nonlinear electrical and electronic circuits.

3.2 Synchronization of Chaotic System

In parallel with the great advances in the chaos theory, the prospects of utilizing chaos in various applications, particularly in telecommunication, have motivated researchers to study the possibility of synchronizing chaos. The synchronization of non-linear oscillators is a phenomenon which has attracted the attention of researchers since the discovery and description of this phenomenon by Huygens in 1673, in an example of two coupled mechanical systems. The phenomenon of synchronization is manifested when two dynamic systems evolve in an identically as a function of time.

Since this innovative discovery, different synchronization regimes have been distinguished such as identical synchronization (Pecora and Carroll, 1990) and generalized synchronization (Rulkov et al., 1995). In (Jemaâ-Boujelben and Feki, 2016, Feki, 2004), the authors suggested a simple Multi-input Multi-output (MIMO) adaptive control based on a Sliding Mode Observer (SMO) to synchronize chaos in the PMSM.

Similar to the integration of order chaotic systems, the synchronization of fractional order chaotic systems has interested several researchers (Wang and Song, 2009, Tang and Fang, 2010). In (Kiani-B et al., 2009), the synchronization of fractional chaotic systems using the fractional extended Kalman filter has were presented with an application to secure communication. To synchronize uncertain fractional order chaotic systems, the authors used the adaptive fuzzy sliding mode control in (Lin et al., 2011), and in (Senejohnny and Delavari, 2012) the authors employed a combination of a classical sliding observer and an active observer. The generalized synchronization was also addressed in the context of fractional order chaotic systems.

4 CHAOTIC APPLICATIONS IN ROBOTICS

We hope time is ripe for reviewing the application of chaos in robotics. The community of robotics is trying to emulate these natural behaviours by investigating humanoids, bio-robots and the biologically inspired systems such as swarms. These systems confront problems like vibration, noise-sensing and robot environment interactions leading to chaos.

As a very interesting topic, Robotics and particularly their applications have emerged in various activities. In the military field, robotic systems should have an interesting feature as target identification and perception as well as positioning robots on the ground. Moreover, the greatest challenge for those successful robot missions is the optimal path planning. Such nonlinearities have led researchers to utilize chaotic trajectory planning techniques for autonomous mobile robots to ensure a rapid search of the whole workspace(Kaygisiz et al., 2011) . The aim of the employment of chaotic signals for autonomous mobile robots is to benefit from coverage areas got through their movement paths.

To meet this challenge Sekiguchi and Nakamura suggested a strategy in 2001 to solve the problem of path planning based on chaotic systems (Nakamura and Sekiguchi, 2001). In that work, the chaotic behavior of the Arnold dynamical system was imparted to the mobile robot's motion control.

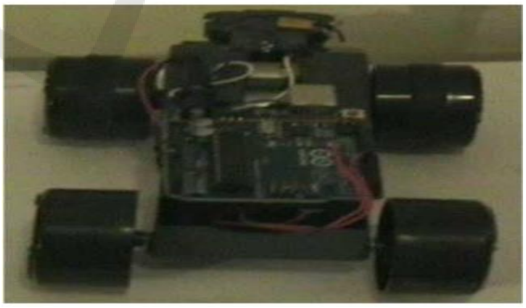
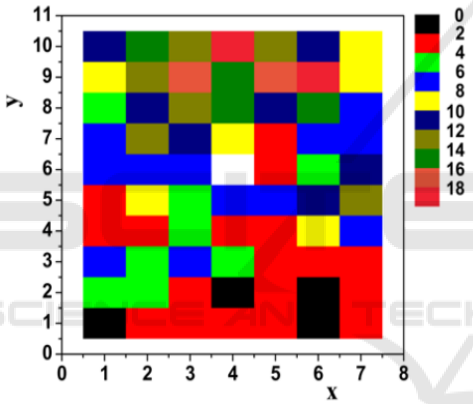


Figure 4: The chaotic autonomous mobile robot (Volos et al., 2013a).

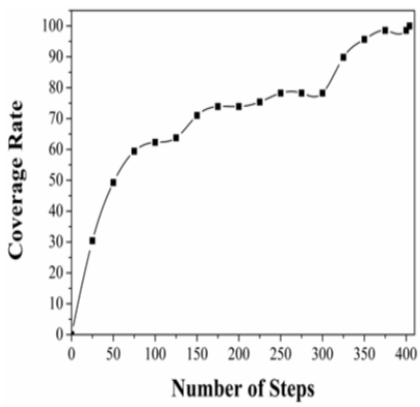
Since then, a lot of relative researches have been presented by many research teams because the chaotic movement of the robot ensures the scanning of the entire workspace without previous knowledge of the terrain map. In (Volos et al., 2013a), a new navigation strategy by designing a controller was experimentally investigated, which ensured a chaotic motion to an autonomous mobile robot wich is presented in figure 4. The proposed controller produced an unpredictable trajectory by imparting the system's chaotic behavior to the two independent active wheels of the mobile robot.

In figure 5 a, the diagram shows the color scale map of the terrain's cells versus the times of visiting. There are cells, wherein the robot has visited from 1 to up 18 times in the 24 minutes of operation. In addition, the coverage rate versus the number of visited cells, for the robot with the proposed chaotic motion controller, is presented in figure 5 b.

Finally, according to the experimental results, the high unpredictability of the robot's trajectory, which is very crucial in many tasks, is confirmed by utilizing the suggested chaotic motion controller. Furthermore, other two crucial tasks are succeeded such as the complete and fast scanning of the workspace.



(a)

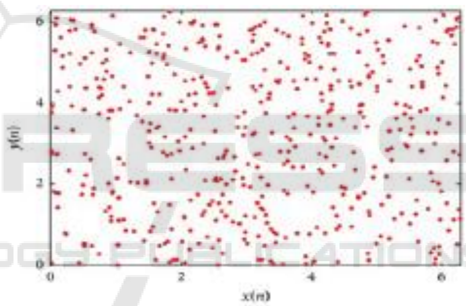


(b)

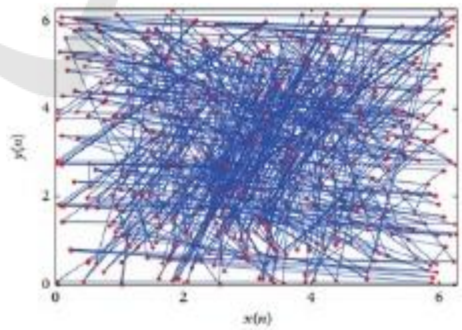
Figure 5: (a) Color scale map of the terrain's cells versus the times of visiting, (b) Coverage rate versus number of motion commands, for operation of 16 minutes. (Volos et al., 2013a).

Similarly, a chaotic path planning generator for a mobile robot was implemented by Volos et al. to cover the overall workspace in an erratic and swift manner. Therefore, the authors presented a Khepera, which is popular in the robotics community, integrating a behaviour-based control (Volos et al., 2012a, Volos et al., 2012b). The chaotic generator used three different chaotic systems producing a double-scroll chaotic attractor.

Furthermore, Volos et al. put forward a motion control strategy for humanoid and mobile robots utilizing a chaotic truly random bit generator (Volos et al., 2012d, Volos et al., 2012c). They implemented an autonomous robot on an experimental platform called the "Magician Chassis" using this generator. This technique ensured highly unpredictable robot trajectories that are random from an observer's viewpoint (Volos et al., 2013a, Volos et al., 2013b). The numerical simulations demonstrated the efficiency of this strategy and the statistical tests also proved the randomness of the planned motions.



(a)



(b)

Figure 6: Intermediate iteration points distribution in the phase space and their trajectories: (a) Iteration points; (b) Iteration trajectories (Li et al., 2015).

Other proposals such as the studies of Caihong et al. introduced a chaotic path planner based on a logistic map. This deterministic and simple system was characterized by a random behaviour, allowing large workspace coverage (Li et al., 2013). Morever, they suggested a fusion strategy to develop a chaotic path planner for mobile robots, based on the standard map (Li et al., 2015). Figure 6 analyzes the iteration points distribution of the standard map in the phase space and the adjacent qualitative and quantitative trajectories. Using the original Standard map of the chaotic state as a robot path planner generator to perform the surveillance mission is a good choice except for its large distance of the chaotic trajectories between the iteration adjacent intermediate points.

To accomplish boundary surveillance missions for mobile robots, Curiac and Volosencu proposed an improved chaotic path planning technique (Curiac and Volosencu, 2014). This study underpinned the testing and design of a chaotic controller incorporating a known chaotic equation. While performing monitoring and research tasks, the chaotic trajectories for autonomous robots had a great enhancement over other methods. Security patrol, fire fighting and cleaning were included in the proposed applications of chaotic mobile robots. As depicted in figure 7, the authors started with the periodic motion on a closed contour of a reference frame in which the Henon chaotic system would evolve. They proved that the compound trajectories obtained in the fixed frame were also chaotic. Based on this result, they developed an original method to create chaotic trajectories in the proximity of any arbitrary boundary shape.

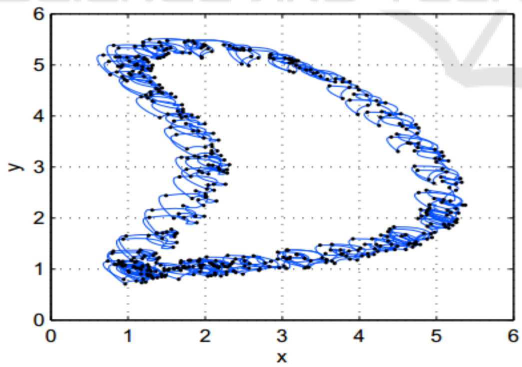


Figure 7: Chaotic robot path in the proximity of a closed curve (two complete laps along the guiding line) (Curiac and Volosencu, 2014).

These efforts show that the application of the chaotic behaviour of dynamic systems for the planning of mobile robot movements is a fascinating interdisciplinary research field.
Unlike other path planning methods, chaotic path planning does not require a map of the workspace and it is more efficient than the random walk algorithm. With various chaos equations, a robot could exhibit a range of motion paths. The chaotic controllers can be implemented by embedding simple chaotic circuits into the robots. Chaotic trajectories are generated using state variables of dynamical systems, which are used as input for the wheels of differential-drive robots.

5 CONCLUSIONS

This paper presents the concept of chaos, which leads to several understandings of chaos. The theory of chaos reveals our inability to make long-term predictions of these deterministic dynamic systems. Chaotic dynamics can be explained, measured and categorized using this theory. The growth of research on chaos is highlighted by the interdisciplinary nature of the field. Due to several applications in electrical appliances, the application of deterministic chaos has attracted a lot of attention. The deterministic chaos has been perceived as unpredictable and unstable, and therefore worthless. Over the past decades, its usefulness and application have been recognized. Like other fields of science and technology, chaotic dynamics have been discovered and implemented in various fields of robotics.

In addition, some efforts for uncovering the chaotic behaviour of various robots are presented. This part provides basic knowledge about the common methods and processes involved in finding the evidence for the existence of chaos in robotic motion, which can help in better applications of chaos to real robots.

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