State Space Identification Algorithm based on Multivariable Impulse Response

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Abstract: In this paper the definitions of multivariable discrete impulse and multivariable discrete impulse response are clearly stated and explored. From these definitions, two methods to determine the Markov parameters of a multivariable linear system from input and output data are described. Combining any of the methods to a known method to determine the state space model matrices from the Markov parameters, a practical algorithm to determine state space models from input and output data is obtained. The algorithm is then implemented and compared to a known subspace identification algorithm. The main contributions of this paper are the explicit definitions of multivariable discrete impulse and multivariable discrete impulse response, the discussion of these concepts and the application of them to solve the multivariable linear system identification problem.

1 INTRODUCTION

It is possible to characterize a time invariant discrete causal linear system from its infinite sequence of discrete impulse response. For Single Input-Single Output (SISO) systems, which in this paper are referred as monovariable systems, the discrete impulse and the discrete impulse response are well defined concepts that are discussed in many standard textbooks such as (Oppenheim and Schafer, 2010) and (Bottura, 1982).

For Multiple Input-Multiple Output (MIMO) systems, referred in this paper as multivariable systems, the multivariable discrete impulse and the multivariable discrete impulse response are tricky concepts that are worth of some more discussions and attention. In this paper it is clearly stated that the multivariable discrete impulse is in fact a set of signals and the multivariable discrete impulse response is actually the concatenation of the system responses to each one of the signals in that set. The impulse response is also defined as an infinite set of Markov parameters. Obviously, those parameters are scalars for monovariable systems and matrices for multivariable systems.

Although an infinite set of Markov parameters characterizes a linear system, it is not usual to use this set to that end because of at least three reasons: The first one is that, even if just the significant elements of the infinite set are used, a compact representation of the system is difficult to achieve. The second one is that, if this representation is used to simulate the response of the system to a given input, the necessary number of operations is greater than the number of operations that would be used if the system was described by more compact forms. The third is that, with this representation, it is not possible to determine the values of the system internal states.

There are many ways to compact the discrete representation of a system, such as the use of the difference equations, the discrete transfer functions or the discrete state space equations, as proposed in (Kalman, 1963). The state space representation is extremely useful because it provides an efficient algorithm to evaluate the system states and the outputs, as described in (Kalman, 1960). For this reason, many methods were developed to determine matrices of the state space representation of a linear system, given by the following equations:

$$\begin{cases} \mathbf{X}(k+1) = A\mathbf{X}(k) + B\mathbf{U}(k) \\ \mathbf{Y}(k) = C\mathbf{X}(k) + D\mathbf{U}(k) \end{cases} \quad (1)$$

in which $\mathbf{X}(k) \in \mathbb{R}^n$ is the state vector, $\mathbf{U}(k) \in \mathbb{R}^m$ is the input and $\mathbf{Y}(k) \in \mathbb{R}^l$ is the system output. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ e $D \in \mathbb{R}^{l\times m}$ are the matrices to be determined.

There are at least two approaches to deal with the problem of finding a discrete state space representation of a linear system. The first one is the solution of the minimal partial realization problem and the se-

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cond one is the use of subspace identification algorithms.

In the first approach, a finite set of Markov parameters is assumed to be known, and the state space matrices are evaluated from this set using a singular value decomposition (SVD). This approach is explored in the references (Ho and Kalman, 1966), (Tether, 1970), (Inouye, 1983). The problem that this approach solves is defined as the minimal partial realization problem, since it aims to find a mathematical realization of the system with the minimal possible number of states and is based on a finite set of Markov parameters, that is a partial information from the infinite set of Markov parameters that in fact represents the system.

Usually the papers that discuss the minimal partial realization problem do not detail how to determine the set of Markov parameters from the system input and output data. One of the contributions of this paper is to explore the determination of the set of Markov parameters from the input and output data, demonstrating that a practical identification algorithm can be obtained from this.

The second approach to deal with the problem of finding a discrete state space representation of a linear system is based on organizing the system input and output data in matrices with a special structure, that can be used to determine the state space model matrices. The algorithms that apply this approach are known as the subspace methods for identification, and there are at least three of them, as discussed in the sequence:

The first algorithm, known as Multivariable Output Error State Space (MOESP), was proposed in (Verhaegen and Dewilde, 1992a), (Verhaegen and Dewilde, 1992b). In this algorithm, the system matrices are obtained from the decomposition of special matrices, assembled with system input and output data. In the second algorithm, the system states are obtained from decompositions of a special structure of data matrices and, after that, the system matrices are obtained with a simple application of the least squares methods. This approach is used in the Numerical Algorithm for Subspace Identification (N4SID), introduced in (Overschee and de Moor, 1994). The third algorithm is an extension of the application of the Canonical Correlation Analysis (CCA) to the time series realization problem considering exogenous inputs. The application of the CCA to the time series realization problems was introduced in (Akaike, 1974) and (Akaike, 1975). This consists on finding an optimal predictor to the future data based on projections on the subspace spanned by the past data. The extension of this algorithm for systems with exogenous inputs was introduced in (Katayama and Picci, 1999) and is based on finding the projection of the future outputs on the space of past inputs, outputs and future inputs.

In this article, an algorithm for multivariable discrete system state space identification is discussed. The algorithm is based on the determination of the Markov parameters from the input and output data and then on the solution of the minimal partial realization problem by the method proposed in (Ho and Kalman, 1966). To develop that, the explicit definition of the multivariable impulse response is explored in section 2. In the same section a procedure to identify the Markov parameters from discrete impulse input is also shown. Then, in section 3, a practical method to find the Markov parameters from input and output data mentioned in (Katayama, 2005) is described. The section 4 is devoted to explain how to solve the minimal partial realization problem. The combination of the procedures described in sections 2 or 3 with the one described in section 4 is referred in this paper as the Multivariable Discrete Impulse Response Identification (MDIRI) algorithm. In section 5 the results of an identification problem with the MOESP and with the MDIRI are shown. Finally, in section 6 the conclusions are presented.

2 DETERMINATION OF IMPULSE RESPONSE MATRICES FROM THE MULTIVARIABLE DISCRETE IMPULSE

In this section, the determination of the impulse response matrices of a linear multivariable system using the multivariable discrete impulse is developed. Initially, the noiseless case is discussed and then, the analysis for an output corrupted by noise is presented.

Let m be the number of inputs and l the number of outputs of a multivariable time invariant discrete causal linear system. There is a set of $l \times m$ matrices that relate the l-dimensional system output at the instant k to the m-dimensional inputs on that time and on the past. Those parameters are denoted as $\mathbf{G}(k)$, $k = 0 \ldots \infty$. With this definition, denoting the $l \times 1$ output vector at the instant k as:

$$\mathbf{Y}(k) = \begin{bmatrix} y_1(k) & y_2(k) & \cdots & y_l(k) \end{bmatrix}^T \tag{2}$$

and the $m \times 1$ input vector at instant k as:

$$\mathbf{U}(k) = \begin{bmatrix} u_1(k) & u_2(k) & \cdots & u_m(k) \end{bmatrix}^T \tag{3}$$

the following relation is valid:

$$\mathbf{Y}(k) = \sum_{i=0}^{\infty} \mathbf{G}(i)\mathbf{U}(k-i) \tag{4}$$

where:

$$\mathbf{G}(k) = \begin{bmatrix} g_{11}(k) & g_{12}(k) & \dots & g_{1m}(k) \\ g_{21}(k) & g_{22}(k) & \dots & g_{2m}(k) \\ \vdots & \vdots & \ddots & \vdots \\ g_{l1}(k) & g_{l2}(k) & \dots & g_{lm}(k) \end{bmatrix} \tag{5}$$

Thus, if all elements $g_{ij}(k)$, $i = 1 \dots l$, $j = 1 \dots m$, $k = 0 \dots \infty$ are determined, it will be a realization of the multivariable system.

For stable systems, just a finite set of matrices $\mathbf{G}(k)$ is significant. If the number of significant matrices $\mathbf{G}(k)$ is denoted as n_{IR} and if the initial state of the system is assumed zero, the equation (4) can be rewritten as:

$$\mathbf{Y}(k) = \sum_{i=0}^{\min(k,n_{IR})} \mathbf{G}(i)\mathbf{U}(k-i) \tag{6}$$

where $\min(\alpha,\beta)$ denotes the smaller number between α and β.

One of the ways to determine the elements of the n_{IR} significant matrices $\mathbf{G}(k)$ is to perform m experiments. In each one of those experiments, the system response for an input sequence with $u_j(0) = 1$, $j = 1 \dots m$, and all other elements null, is observed. The set of m input signals used in those experiments is defined in this paper as the multivariable discrete impulse.

It is important to notice that, differently from the monovariable case, it is not possible to define only one input signal as the multivariable discrete impulse. Because of that, the definition of multivariable discrete impulse is used simply as an analogous to the monovariable case, and not as a definition of only one signal.

As an example, let $j = 1$. In this experiment the following input will be applied to the system:

$$\mathbf{U}(k) = \begin{cases} \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}^T & k = 0 \\ \mathbf{0}_m & k \neq 0 \end{cases} \tag{7}$$

in which $\mathbf{0}_m$ is the null vector at $\mathbb{R}^m$. Consequently, from (6), the outputs will be:

$$y_i(k) = g_{i1}(k), \quad i = 1 \dots l \tag{8}$$

thus, applying the input defined in (7), it is possible to determine the first column of the matrices $\mathbf{G}(k)$, $k = 0 \cdots n_{ir}$.

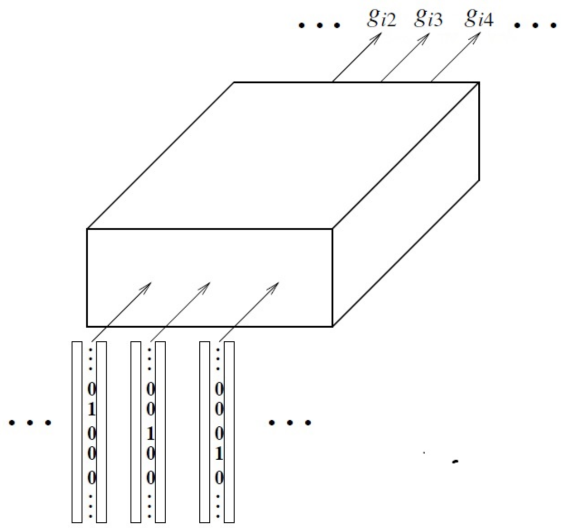

Figure 1: Hypothetical multivariable discrete impulse applied to a system.

With a similar procedure, it is possible to notice that in each one of the m experiments, one of the columns of the matrices $\mathbf{G}(k)$ will be determined, and by the end of those experiments, there will be an algebraic model that is a realization of the system under study. Since the matrices $\mathbf{G}(k)$ are the system responses to the multivariable discrete impulse, they are defined as the system multivariable discrete impulse response. Those matrices are also defined as the Markov parameters of the deterministic system. In the figure 1, a representation of the application of the m inputs to the system, resulting into the determination of its multivariable impulse response, is shown.

Now consider the case where the system outputs are corrupted by an arbitrary Gaussian colored noise with dimension l denoted by:

$$\mathbf{V}(k) = \begin{bmatrix} v_1(k) & v_2(k) & \cdots & v_l(k) \end{bmatrix}^T \tag{9}$$

From the partial stochastic realization theory (Akaike, 1974), (Faurre, 1976), (Giesbrecht and Bottura, 2016), it is known that any Gaussian noise can be generated as the output of a filter that has as input a zero mean uncorrelated multivariable Gaussian white noise $\mathbf{E}(k)$ with dimension p given by:

$$\mathbf{E}(k) = \begin{bmatrix} e_1(k) & e_2(k) & \cdots & e_p(k) \end{bmatrix}^T \tag{10}$$

In (Giesbrecht and Bottura, 2017) more details about the white noise can be found.

Thus, considering the system initial state null, it is possible to find a set of matrices $\mathbf{H}(k) \in \mathbb{R}^{l \times p}$ such as:

$$\mathbf{V}(k) = \sum_{i=0}^{\min(k,n_c)} \mathbf{H}(i)\mathbf{E}(k-i) \tag{11}$$

in which n_c is the number of significant matrices $\mathbf{H}(k)$.

Consequently, the output of a multivariable system subjected to an arbitrary colored noise $\mathbf{V}(k)$ will be given by the following expression:

$$\mathbf{Y}(k) = \sum_{i=0}^{\min(k,n_{IR})} \mathbf{G}(i)\mathbf{U}(k-i) + \sum_{i=0}^{\min(k,n_c)} \mathbf{H}(i)\mathbf{E}(k-i) \tag{12}$$

If the multivariable discrete impulse is applied to the system corrupted by noise, the outputs of the m experiments will not be exactly the columns of the matrices $\mathbf{G}(k)$. In fact, returning to the example where $j = 1$, in which the input is defined in (7) and the noiseless output is given by (8), from (12) a noisy output will be given by:

$$y_I(k) = g_{I1}(k) + \sum_{i=0}^{\min(k,n_c)} \sum_{j=1}^{p} h_{Ij}(i)e_j(i) \tag{13}$$

$I = 1 \cdots l$

Besides the corrupted outputs, it is still possible to determine the Markov parameters by the multivariable impulse if a considerable great number $\mathcal{N}$ of tests is performed for each one of the m inputs. Since by the white noise definition:

$$E[e_j(k)] = \lim_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} e_j(k) = 0 \quad \forall j,k, \tag{14}$$

Consequently, the expected value of the outputs after a considerably great number of experiments will be the desired Markov parameters.

3 DETERMINATION OF IMPULSE RESPONSES FROM OTHER INPUTS

In practical problems, it might be difficult to apply a set of multivariable discrete impulses to a system and observe the outputs. To solve this issue, in this section it is shown that it is possible to determine the impulse response matrices of a multivariable system from any persistent exciting input with a simple application of the least squares method.

The equation (4) can be rewritten as:

$$\begin{aligned}
\mathbf{Y}(k) &= \mathbf{G}(0)\mathbf{U}(k) + \mathbf{G}(1)\mathbf{U}(k-1) + \cdots \\
\mathbf{Y}(k+1) &= \mathbf{G}(0)\mathbf{U}(k+1) + \mathbf{G}(1)\mathbf{U}(k) + \cdots \\
\mathbf{Y}(k+2) &= \mathbf{G}(0)\mathbf{U}(k+2) + \mathbf{G}(1)\mathbf{U}(k+1) + \cdots \\
&\vdots
\end{aligned} \tag{15}$$

The transposition of both sides of the equation above results in:

$$\begin{aligned}
\mathbf{Y}(k)^T &= \mathbf{U}(k)^T\mathbf{G}(0)^T + \mathbf{U}(k-1)^T\mathbf{G}(1)^T + \cdots \\
\mathbf{Y}(k+1)^T &= \mathbf{U}(k+1)^T\mathbf{G}(0)^T + \mathbf{U}(k)^T\mathbf{G}(1)^T + \cdots \\
\mathbf{Y}(k+2)^T &= \mathbf{U}(k+2)^T\mathbf{G}(0)^T + \mathbf{U}(k+1)^T\mathbf{G}(1)^T + \cdots \\
&\vdots
\end{aligned} \tag{16}$$

which can be rewritten as:

$$\mathbf{Y_{bl}}^T = \mathbf{U_{bl}}^T \mathbf{G_{bl}}^T \tag{17}$$

in which $\mathbf{Y_{bl}}$ and $\mathbf{G_{bl}}$ are the following block matrices:

$$\mathbf{Y_{bl}} = \begin{bmatrix} \mathbf{Y}(k) & \mathbf{Y}(k+1) & \mathbf{Y}(k+2) & \cdots \end{bmatrix} \tag{18}$$

$$\mathbf{G_{bl}} = \begin{bmatrix} \mathbf{G}(0) & \mathbf{G}(1) & \mathbf{G}(2) & \cdots \end{bmatrix} \tag{19}$$

and $\mathbf{U_{bl}}^T$ is the following Toeplitz block matrix:

$$\mathbf{U_{bl}}^T = \begin{bmatrix} \mathbf{U}(k)^T & \mathbf{U}(k-1)^T & \cdots \\ \mathbf{U}(k+1)^T & \mathbf{U}(k)^T & \cdots \\ \mathbf{U}(k+2)^T & \mathbf{U}(k+1)^T & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{20}$$

From the relations above, if a finite persistent exciting input is applied to the system, the output will be finite and it will be possible to determine a finite number of system impulse response matrices with the following expression, that is a direct application of the least-squares method:

$$\mathbf{G_{bl}} = (\mathbf{U_{bl}}^{T\dagger} \mathbf{Y_{bl}}^T)^T \tag{21}$$

in which $\mathbf{U_{bl}}^{T\dagger}$ is the pseudo-inverse of the matrix $\mathbf{U_{bl}}^T$.

For practical problems, the number of blocks of $\mathbf{G_{bl}}$ can be limited to n_{IR}, that as previously defined, is the number of significant impulse responses of the system. To obtain the desired number of Markov parameters, the sizes of the data matrices $\mathbf{U_{bl}}$ and $\mathbf{Y_{bl}}$ must be adequate. Obviously, for greater n_{IR}, bigger data matrices will be needed.

If the output is corrupted by noise, it is possible to demonstrate from the optimal properties of the least squares solution that (21) gives an estimate of $\mathbf{G_{bl}}$ that minimizes the quadratic error between the system outputs and the model outputs for the same input.

4 DETERMINATION OF SYSTEM MATRICES FROM IMPULSE RESPONSE MATRICES

Once the multivariable impulse response matrices are determined either by the method described in the section 3 or by the one described in the section 2, the state space model matrices can be determined by the method introduced in (Ho and Kalman, 1966) and summarized in this section:

From (1) it is possible to demonstrate that the following relation between the impulse responses and the state space model matrices is valid:

$$\mathbf{G}(k) = \begin{cases} D & k = 0 \\ CA^{k-1}B & k \neq 0 \end{cases} \tag{22}$$

Supposing that the impulse response matrices are known, the following finite Hankel matrix can be built:

$$H = \begin{bmatrix} \mathbf{G}(1) & \mathbf{G}(2) & \dots & \mathbf{G}(k3) \\ \mathbf{G}(2) & \mathbf{G}(3) & \dots & \mathbf{G}(k3+1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}(k3) & \mathbf{G}(k3+1) & \cdots & \mathbf{G}(2k3-1) \end{bmatrix} \tag{23}$$

in which $k3$ is the number of line blocks of the matrix H. To solve the partial realization problem, this number must be chosen such as $2k3 - 1 = n_{IR}$, where n_{IR} is the number of significant impulse responses.

Defining the observability matrix O and the reachability matrix $\mathcal{C}$ as:

$$O = \begin{bmatrix} C^T & (CA)^T & (CA^2)^T \end{bmatrix}^T \tag{24}$$

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B & \dots \end{bmatrix} \tag{25}$$

and using (22) and (23), it is possible to write:

$$H = O\mathcal{C} \tag{26}$$

so, to determine the matrices O and $\mathcal{C}$, a singular value decomposition of H can be made, as indicated below:

$$O\mathcal{C} = U\Sigma V^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{27}$$

So it is possible to define:

$$O = U_1 \Sigma_{11}^{1/2} \qquad \mathcal{C} = \Sigma_{11}^{1/2} V_1^T \tag{28}$$

Let, by definition $H_\uparrow$ be the matrix H with one line block shifted upwards and a new line block, following the structure presented in (23), added to its end. It is possible to notice that $H_\uparrow = OA\mathcal{C}$. Thus, if H, O and $\mathcal{C}$ are known, it is possible to determine A as indicated below:

$$H_\uparrow = OA\mathcal{C} \Rightarrow A = O^\dagger H_\uparrow \mathcal{C}^\dagger \tag{29}$$

From the first column block of H, denoted as $H(:,1)$, it is possible to determine the matrix B with the following expression:

$$H(:,1) = OB \Rightarrow B = O^\dagger H(:,1) \tag{30}$$

In the same way, from the first line block of H, denoted as $H(1,:)$, it is possible to obtain C:

$$H(1,:) = C\mathcal{C} \Rightarrow C = H(1,:)\mathcal{C}^\dagger \tag{31}$$

and the matrix D is obtained from (22) as the impulse response at $k = 0$.

As shown above, it is possible to determine the matrices A, B, C and D from the system Markov parameters, that can be determined either by the method described in the section 2 or by the described in the section 3. The combination of the methods is defined as the MDIRI algorithm. The results of that algorithm are presented in the next section

5 IDENTIFICATION RESULTS

To evaluate the performance of the algorithms, experiments were executed using the software Matlab running on a desktop computer equipped with a processor Intel Core i3-4160T CPU @ 3.10GHz, 8GB RAM.

The benchmark to be identified is the following state space model:

$$\mathbf{X}(k+1) = \begin{bmatrix} 0.0002 & -0.5 \\ 1 & -1 \end{bmatrix} \mathbf{X}(k) + \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{U}(k) \tag{32}$$

$$\mathbf{Y}(k) = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \mathbf{X}(k) + \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \mathbf{U}(k) + \mathbf{E}(k) \tag{33}$$

In each experiment, the response of the benchmark to random inputs with $N = 100$ samples was collected and the set of inputs and outputs was used in the identification algorithms to be tested.

5.1 MOESP Algorithm Results

To test the MOESP algorithm (Verhaegen and Dewilde, 1992a), (Verhaegen and Dewilde, 1992b), the number of line blocks in data matrices, denoted as k_1, was varied from 2 to 20 in steps of 1. This limits were adopted because it was observed that for the number of samples used, if $k_1 > 20$, the LQ decomposition seldom succeeded, and the identified model was not stable. For each k_1, at least 10000 experiments were tested. The instability problems also happened for some experiments in the range adopted for the number of line blocks in data matrices. The results of those were not taken into account and the experiments were repeated until 10000 of them resulted in stable systems. To avoid infinite loops, a limit of 4x the desired number of experiments was defined. If after this limit the desired number of successful experiments was not reached, it was considered that it was not possible to determine the results with the MOESP for that k_1.

In each experiment, a new input with $N = 100$ samples was generated with a bi-dimensional Gaussian distribution with zero mean and variance equal to the identity. The input was applied to the benchmark defined in (32) and (33) and the outputs were summed to a noise $\mathbf{E}(k)$ with the same statistical characteristics presented by the input, what means that the noise presents the same order of magnitude that the true signal does, which represents a challenging identification problem. The resultant signal was considered as the output of the system to be identified.

For each one of the 10000 valid experiments for each $k1$, the MOESP algorithm was applied to identify the matrices A, B, C and D from the input and output data. After the identification, the input signal of the experiment was applied to the model determined with the MOESP algorithm and the quadratic error E_q defined as:

$$E_q = \frac{1}{N}\sum_{k=1}^{N}\sqrt{(\mathbf{Y}(k)-\mathbf{Y}_{est}(k))^T(\mathbf{Y}(k)-\mathbf{Y}_{est}(k))} \quad (34)$$

was calculated, where $\mathbf{Y}(k)$ is the benchmark output calculated with (33) and $\mathbf{Y}_{est}(k)$ is the output of the model determined with the MOESP algorithm. The times taken to determine the state space model matrices and the the Markov parameters were also measured for each one of the experiments. After that, for each $k1$, the average error and the average execution times were calculated.

Table 1: MOESP algorithm results for $k1 = 20$.

Average error	Time to obtain Markov parameters	Time to obtain state space matrices
0.0412676	0.012461 s	0.012449 s

The average errors of the application of the MOESP algorithm to the 10000 experiments for each $k1$ are shown in the figure 2 and the average execution times in the figure 3. From the figure 2 it is possible to see that the variation of the average error is not significant. The worst result is obtained for $k1 = 2$, what can be explained by the small number of line blocks in the data matrices. A small increase in $k1$ decreases the error significantly, probably because the data matrix to be decomposed is better conditioned than the one obtained for $k1 = 2$. The smallest error is measured for $k1 = 20$. The results for that $k1$ are summarized in the table 1.

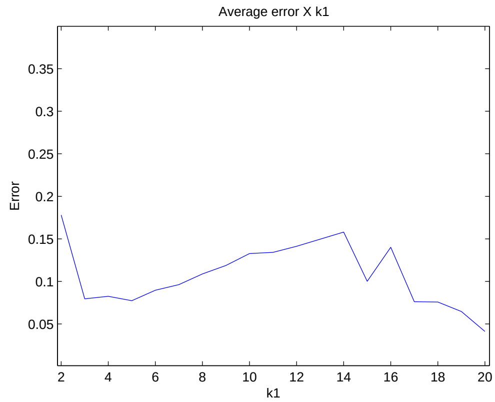

Figure 2: Average errors for the MOESP algorithm as function of $k1$.

From the figure 3 it is possible to notice that as $k1$ increases, the execution time also increases. This happens because this variable is related to the dimension of the matrices that will be decomposed in the algorithm. From that figure it is also possible to notice that the differences of times to obtain the state space matrices and to obtain the Markov parameters are relatively small. This happens because the determination of the Markov parameters from the system matrices involves just products between matrices with relatively small dimensions.

5.2 MDIRI Algorithm Results

The MDIRI algorithm was tested with a similar procedure. For n_{IR} varying from 5 to 49 in steps of 2, the identification experiments with the MDIRI were repeated until 10000 of them resulted in stable systems. For each n_{IR} the average values of errors and the execution times were calculated.

In the figure 4 the average errors found for each n_{IR} adopted are shown. From the figure it is possible to see that, for $n_{IR} < 20$, the error is considerably below 0.05, that is approximately the best value found with the MOESP algorithm. It is also possible to notice that, if the number of impulse responses to be considered increases until $n_{IR} = 32$, the error also increases. This is expected since if n_{IR} increases, the number of block lines in the matrix $\mathbf{U_{bl}}^T$ decreases, and less data is available to estimate the Markov parameters with (21). For $n_{IR} > 32$ the average error sur-

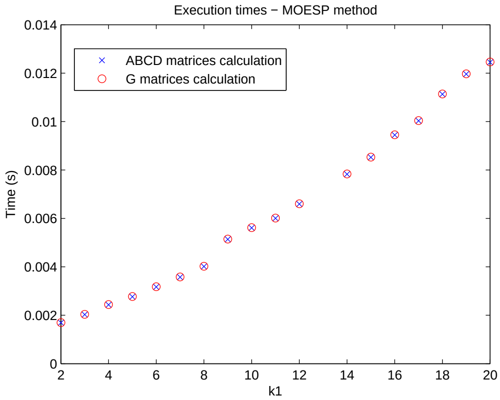

Figure 3: Average execution times to obtain the state space matrices and the Markov Parameters with the MOESP algorithm.

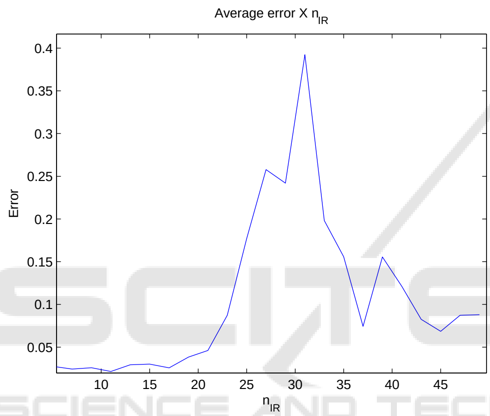

Figure 4: Average errors for the MDIRI algorithm as function of n_{IR}.

prisingly decreases. Probably this happens because for that n_{IR}, the input data is closer to a square matrix, what makes the estimation of the $\mathbf{G_{bl}}$ more accurate since the linear system (21) is better conditioned. The smallest error for the MDIRI algorithm was found for $n_{IR} = 11$. The results for this case are summarized in the table 2.

Table 2: MDIRI algorithm results for $n_{IR} = 11$.

Average error	Time to obtain Markov parameters	Time to obtain state space matrices
0.021473	0.002038 s	0.002625 s

In the figure 5 the average times to find the Markov parameters and the state space matrices are shown. Since in this algorithm the first step is to determine the Markov parameters from the equation (21) and then to apply the procedure described in the section 4 to determine the state space model matrices, the time to obtain the first values is considerably smaller than the time to obtain the second values. From

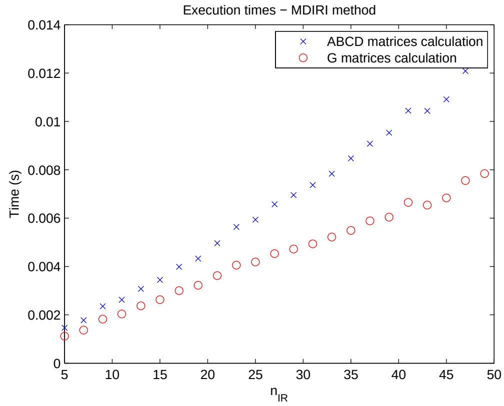

Figure 5: Average execution times to obtain the state space matrices and the Markov Parameters with the MDIRI algorithm.

the same figure it is noticed that as n_{IR} increases, the difference between the time to obtain the state space model matrices and the time to obtain the Markov parameters gets bigger. This is explained by the increase in the computational costs to perform the decompositions presented in the section 4.

Making a general comparison between the figures 2, 3, 4 and 5 it is possible to notice that accurate results were obtained with both algorithms within a reasonable time. By comparing the results presented in the tables 1 and 2 it is possible to notice that, specifically for the benchmark studied in this paper, the MDIRI algorithm resulted in a estimation with almost half of the average error obtained with the best MOESP solution in a execution time that was almost 6 times smaller. This indicates that the algorithm that is based on the multivariable impulse response may be advantageous to solve practical problems with higher accuracy and smaller execution time.

6 CONCLUSIONS

In this paper the definitions of multivariable discrete impulse and the multivariable discrete impulse response were clearly stated and explored. From those definitions, two methods to determine the system Markov parameters from input and output data were described. Combining the described methods to a known method to determine the state space model matrices from the Markov parameters, a practical algorithm to determine state space models from input and output data was explored.

The results of the implementation of the MDIRI algorithm showed that accurate models could be obtained in a reasonable amount of time. Those results were compared to the ones obtained with the

MOESP algorithm and the conclusion was that, for the tested benchmark, the algorithm described in this paper is faster and more accurate.

A future research is to compare the MDIRI algorithm to N4SID and CCA concerning the execution time and average error. Another future direction is to test the MDIRI performance if the number of samples get smaller and compare to the performance of other algorithms. This kind of analysis is important for the development of future algorithms to identify multivariable discrete time variant systems based on windowed data, such as the one proposed in (Tamariz. et al., 2005), or to initialize evolutionary algorithms for online time variant system identification, such as the one proposed in (Giesbrecht and Bottura, 2015).

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