

# Hyperspectral Compressive Sensing Imaging Via Spectral Sparse Constraint

Qi Wang<sup>1</sup>, Hong Xu<sup>2</sup>, Lingling Ma<sup>1,\*</sup>, Chuanrong Li<sup>1</sup>, Yongsheng Zhou<sup>1</sup> and Lingli Tang<sup>1</sup>

<sup>1</sup>Key Laboratory of Quantitative Remote Sensing Information Technology, Academy of Opto-Electronics,  
Chinese Academy of Sciences, Beijing, 100094, China

<sup>2</sup>High Technology Research and Development Center, Ministry of Science and Technology, Beijing 100862, China

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**Abstract:** The existing algorithms to reconstruct hyperspectral compressive sensing images mainly use the sparse property of spatial information and some simple non-adaptive spectral constraint such as the low-rank property. However, these strategies cannot remove the spectral redundancy efficiently and a new method to make full use of the abundant redundancy of spectral information and improve the quality for hyperspectral CS reconstruction is necessary. A new CS sampling and reconstruction model based on spectral sparse representation was proposed in this paper. The spectral sparse dictionary was constructed from training samples to enhance the effect of sparse representation and the total variation constraint of spatial images was also considered to further enhance the precision during the reconstruction. The experiment to reconstruct AVIRIS hyperspectral images of 200 bands show that the hyperspectral image was almost perfectly reconstructed at 25% sampling rate and the spatial and spectral precision was higher than traditional methods which only adopt the spatial sparsity and simple non-adaptive spectral constraint in the same condition.

## 1 INTRODUCTION

Hyperspectral remote sensing technique has the ability to acquire and analyze the physical and chemical properties of land surface objects. Over the past 30 years it has witnessed a great progress in various fields such as atmosphere monitoring, ocean monitoring, mineral exploration, precision agriculture and target detection. However, hyperspectral images have much larger amount of data which makes data transmission, storage and onboard processing more difficult. Furthermore, the optics system will also become more complicated and costly with the improvement of resolution. As far as the traditional imaging system based on Nyquist sampling theory is concerned, it is almost impossible to conquer the contradiction between high efficiency and high resolution demand. Therefore, new theory is expected to be developed so as to promote more efficient application of hyperspectral remote sensing.

As a novel theory in signal processing, compressive sensing (CS) theory which integrates compression and sampling process has drawn much

attention in many fields including remote sensing imaging (Donoho, 2006). By concerning the sparse characteristic of the object, images can be reconstructed from very few measurements. Therefore the size and complexity, as well as the on-orbit computational cost of the CS imaging system is much lower than conventional systems, making it a very promising application in the remote sensing (Li, 2014). A. Wagadarikar (2008), Thomas A. Russell (2012), and J. Wu (2014) have developed various CS spectral imaging systems respectively.

The images of CS imaging system are reconstructed from compressive measurements by solving specific optimization problem. The sparse characteristics of target signal is the foundation of CS reconstruction: the better the signal is sparsely represented, the higher reconstruction precision would be obtained. The total variation (TV) model based on the sparsity of two dimensional discrete gradient was widely used to reconstruct spatial images (Combettes, 2014). However, in the hyperspectral remote sensing imaging which demands for higher compression rate, the

reconstruction effect of non-adaptive total variation model is usually dissatisfied. There is strong correlation among the bands of hyperspectral image. Regarding to this characteristic, several constraint models using spectral correlation had been proposed. Zhang et. al. extended the traditional two-dimensional TV model to 3 dimensions in order to reconstruct hyperspectral images (Zhang, 2014). Feng et. al. (2012) used the current reconstructed band as the reference to predict the following band. Liu et. al. (2011) presented the hyperspectral reconstruction algorithm based on the prediction of the residual vector. Golbabaee et. al. (2012) applied the low-rank property of hyperspectral data bands and reconstructed the image by adding the nuclear norm to the constraint model. Jia et. al. (2014) put forward a structural-relation-based model via researching the spectral statistical correlation of hyperspectral image. These spectral correlation models do enhance the reconstruction precision to some extent, but the spectral constraint models are relatively simple and non-adaptive, thus the detailed spectral information is difficult to be reconstructed correctly at a low sampling rate. This shortage is more notable for the hyperspectral scenes with a wider spectral range and more bands of data.

Actually, most of the earth surface spectrum possesses piecewise smooth property with a large amount of redundant information. The spectral information redundancy is usually more abundant than that of the complex spatial information and is not well removed using the available methods such as neighboring band prediction or low-rank minimization of the HSI data. On the other hand, the sparse representation theory suggests that, the signals can be precisely represented by very few atoms from some kinds of dictionaries that can highly reduce the information redundancy. Recently some researchers applied the sparse representation theory in the hyperspectral fields such as spectral classification, unmixing, and reconstruction (Zare, 2012, Charles, 2011, Wang, 2013). By exploiting the spectral sparse property, researchers obtained better results than conventional methods, making it an effective way to process HSI data.

On such basis, the sparse representation theory is introduced to reconstruct compressive sensing hyperspectral images in this paper and a new hyperspectral sampling model is proposed to precisely reconstruct the images in the CS scheme. In Section II, the sparse representation theory and the method of spectral sparse dictionary learning are discussed. In Section III, the principles of spectral compressive sensing model and the hyperspectral

image reconstruction algorithm based on the spectral sparse dictionary are presented. In Section IV, the experiment of sampling and reconstructing the AVIRIS scene is conducted, and our method as well as three different hyperspectral reconstruction methods is applied in comparison. In Section V, the content of this paper is concluded and the problems and prospects are analyzed.

## 2 SPECTRAL SPARSE DICTIONARY LEARNING

Suppose  $x$  is a signal of length  $N$  and  $\mathbf{D}=[\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_K]$  is a set of basis (also called as dictionary) in the  $N$ -dimensional Euclidean space. If  $x$  can be linearly represented by the  $L$  atoms in the dictionary (usually  $L \ll K, N$ ), then we declaim that  $x$  has a sparse representation in the dictionary  $\mathbf{D}$ , the sparsity level is  $L$ . That is

$$x = \sum_{i=1}^L \alpha_i d_{\delta_i} + \varepsilon, \quad \delta_i \in 1, 2, \dots, K \quad (1)$$

where  $\varepsilon$  is the residual error of the sparse representation. (1) can be rewritten in the matrix form:

$$x = \mathbf{D}s + \varepsilon \quad (2)$$

In which  $s$ , called as the sparse coefficient, is the coordinate vector with only  $L$  non-zero elements. The process to calculate  $s$  is called as sparse coding and it solves the minimization problem:

$$\min \|s\|_0, \quad s.t. \quad x = \mathbf{D}s \quad (3)$$

where  $\|\cdot\|_0$  denotes the number of non-zero elements and (3) can be solved by several algorithms.

A more important question of the sparse representation is the way to construct the dictionary. In the area of signal and image processing, DCT dictionary, wavelet dictionary and Gabor dictionary are usually adopted. The atoms in these dictionaries are fixed and hard to be adjusted adaptively, which results in the limitation that more efficient and accurate decomposition is needed for hyperspectral CS reconstruction. In recent years the adaptive dictionary training method from the characterized sample set is developed and used in the sparse dictionary construction by many researchers. The dictionary learning method is an adaptive approach for complex signals and usually achieves a better result than conventional dictionaries such as DCT and wavelet. The main idea of dictionary training is to find a matrix that ensures each sample in the training

set a sparse representation by such matrix. The sparsity level and the residual error in the representation are the optimization goal in the calculation. For the hyperspectral dictionary construction in this paper, the spectral sparse dictionary model is presented as

$$\min_{D, X} \|W - DS\|_2 \quad s.t. \quad \forall i \|s_i\|_0 \leq L \quad (4)$$

where the matrix  $\mathbf{W}$  is composed by the column vectors of spectral sample set that are extracted from hyperspectral scenes,  $\mathbf{D}$  and  $\mathbf{S}$  are the sparse dictionary and the coefficients to be solved respectively, and  $L$  is the sparsity level control parameter. The solving process of (4) consists of two key steps called as sparse coding and dictionary updating. The sparse coding step is to solve the sparse coefficients  $\mathbf{S}$  under the dictionary  $\mathbf{D}$  and the updating step is to adjust  $\mathbf{D}$  according to the new coefficients. The two steps are executed alternatively until convergence. Based on the different approach to realize dictionary updating, a series of dictionary learning algorithms such as KSVD, MOD and RLS are presented (Aharon, 2006, Mairal, 2009, Skretting, 2010). Here we choose the KSVD algorithm as the dictionary learning algorithm and fast OMP as the sparse coding algorithm for their relatively high accuracy and efficiency (Azimi, 2014).

### 3 HYPERSPECTRAL SAMPLING AND RECONSTRUCTION

Suppose the matrix  $\mathbf{X}$  represents a hyperspectral data cube, with  $N=n_1*n_2$  spatial pixels and  $B$  spectral bands. The compressive sampling of hyperspectral object is conducted in the spatial region usually.  $x_i$  denotes for the column vector with length  $N$  expanding from the  $i$ -th band image, and  $y_i$  denotes for the corresponding measurements. To sample the scene by the linear mixing matrix  $\mathbf{P}$  in each band, we have

$$[y_1, y_2, \dots, y_B] = P[x_1, x_2, \dots, x_B] \quad (5)$$

The usual way to solve (5) is to reconstruct each spatial image in the constraint of total variation and adjust the solution based on the spectral correlation model. As pointed before, the spectral sparse property with a large amount of information redundancy is not exploited sufficiently in this model. Therefore, we propose a new sampling scheme in the spectral region.  $\lambda_i$  denotes the spectral data vector with length  $B$  of the  $i$ -th spatial pixel and  $y_i$  denotes the corresponding compressive measurements. The sampling process is

applied in the whole spatial region, that is

$$[y_1, y_2, \dots, y_N] = P[\lambda_1, \lambda_2, \dots, \lambda_N] \quad (6)$$

And the hyperspectral scene is reconstructed based on (6).

In the spectral sparse model of Section II, the spectral signal in (6) can be decomposed as the product of sparse dictionary and coefficients, that is

$$\lambda_i = Ds_i, \quad i = 1, 2, \dots, N \quad (7)$$

Define matrix  $\mathbf{A}=\mathbf{PD}$  and put (7) into (6):

$$y_i = PDs_i = As_i \quad (8)$$

And the sparse coefficients  $s_i$  is determined from solving the optimization problem

$$\min \|s_i\|_0, \quad s.t. \quad y_i = As_i \quad (9)$$

As the sparse representation error is unavoidable, (9) is presented as an unconstrained optimization problem with the regularizer  $\beta_0$ :

$$\min_{s_i} \frac{1}{2} \|y_i - As_i\|_2^2 + \beta_0 \|s_i\|_0 \quad (10)$$

The optimization of the  $l_0$  norm is a NP hard problem. Therefore we use smooth Gaussian function to approximate the  $l_0$  norm and convert the problem to classical convex optimization problem which can be solved via gradient descent pursuit algorithm. The detailed steps are presented in (Mohimani, 2007).

Solve (10) for the spectrum in each pixel and calculate the reconstructed spectrum via (7) and arrange all the spectrum into a three dimensional matrix to construct the raw reconstructed hyperspectral data cube  $\mathbf{X}_0$ . The accuracy of spatial information of the reconstructed image is hard to guarantee with the spectral constraint only. Therefore the raw reconstructed data is revised in the constraint of total variation to solve the optimization problem with the regularizer  $\beta_1$ :

$$\min_{x'_j} \frac{1}{2} \|x_j - x'_j\|_2^2 + \beta_1 \|x'_j\|_{TV} \quad (11)$$

in which  $x_j$  is the  $j$ -th band spatial image of  $\mathbf{X}_0$  and  $x'_j$  is the revised image in the constraint of total variation. The TV norm represents for

$$\|x\|_{TV} = \sum_i |x_{i+1} - x_i| + |x_{i+n_1} - x_i| \quad (12)$$

Solve (12) for each band in  $\mathbf{X}_0$  to construct the data cube  $\mathbf{X}'$  by applying the algorithm in (Chambolle, 2004) and further revise the spectrum via

solving the optimization problem which is similar to (10):

$$\min_{s_i} \frac{1}{2} \|y_i - As_i\|_2^2 + \frac{1}{2} \|s_i - s'_i\|_2^2 + \beta_2 \|s_i\|_0 \quad (13)$$

Calculate (11) and (13) alternatively to revise spatial and spectral solution of the reconstructed scene until the convergence is reached:

$$\frac{\|X - X'\|_2^2}{\|X\|_2^2} \leq 0.001 \quad (14)$$

#### 4 EXPERIMENTS AND RESULTS

*Spectral Dictionary Learning:* The spectral data from the remote sensing images acquired by AVIRIS hyperspectral imagery at the range of 0.4-2.5  $\mu\text{m}$  is selected as the dictionary learning samples. The sample set contains 5000 spectrums from 38 types of ground objects in the scene *Indian pines, Salinas, Cuprite and Kennedy Space Center*. Since some bands are influenced by water vapor absorption, 24 bands are discarded and the data of 200 bands are used eventually. The spectral data is normalized to avoid the difference of the intensity in the different scene and some types of training samples are shown in Fig. 1. In the dictionary training algorithm we set the sparsity level  $L=5$ , the iteration times  $T=30$ , and the number of atoms  $K=400$ . The parameters are selected from many experiments considering both the efficiency and precision of the algorithm, while the influence of the parameter changing on the dictionary and the method to decide the best parameter is yet to be further studied.

*Experiment 1:* The experiment scene to be reconstructed is selected from the area of Indian Pines containing 128\*128 pixels (the test area is not included in the training set). The compressive sampling is conducted to the scene by the random Bernoulli matrix that has good incoherence property and is sustainable by hardware. The sampling rate is 25%, and Gaussian noise of 40dB SNR is added to the measurements. The original data of 200 bands are compressed to 40 bands after sampling.

The algorithm presented in Section III is applied to reconstruct the hyperspectral scene from the compressive measurements. The regularizers of  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  are set to 0.4, 0.1, and 0.1 respectively. In contrast, three different hyperspectral CS reconstruction algorithms are also tested including TV algorithm in (Combettes,2004), TVSS algorithm in (Liu,2011) and TVNU algorithm in

(Golbabaee,2012). Fig. 2 shows the reconstructed image of the 100th band via the four algorithms in the same experimental condition. The quality of the reconstructed image by the proposed method is significantly better than other algorithms and it effectively avoids the over-smoothing problem blurring the image details by TV constraint algorithms.

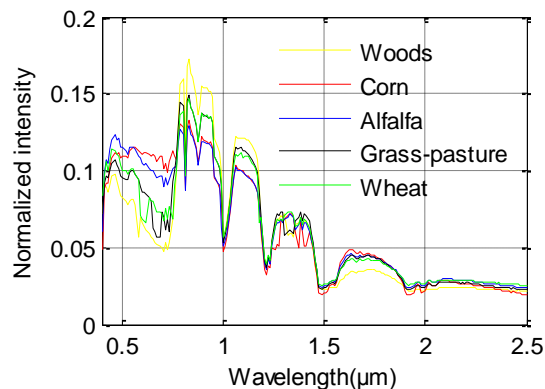


Figure 1: Some types of spectrum in the training set extracted from AVIRIS data containing 200 valid bands. The spectral intensity is normalized.

The spectral curve of corn in the scene is compared before and after reconstruction in Fig.3 via different algorithms. The reconstructed spectrum by the proposed method achieves very high accuracy with little deviation. In contrast, the TV algorithm fails at some bands and makes the spectrum discontinuous due to the lack of constraint in the spectral region. The TVSS and TVNU algorithm with spectral constraint maintain the continuity of the reconstructed spectrum to some extent, but there are apparent errors in some details especially at the wavelength 0.5-1.0 $\mu\text{m}$  where the spectrum has a wide fluctuation.

*Experiment 2:* Change the sampling rate and reconstruct the hyperspectral scene via different algorithms. The sampling rate is set from 10% to 50%. To assess the reconstruction precision, two parameters represented for space and spectrum respectively are calculated for each reconstructed hyperspectral scene. One is the mean value of the root square error (RMSE) of each spatial band and the other is the mean value of the spectral angle (MSA) between the reconstructed spectrum and the original one. The result is shown in Fig.4. The precision of the reconstructed scene via the proposed method is better than other algorithms in both spatial region and spectral region. The advantage of our method is more dominant especially at low sampling rate due to the precise sparse constraint in the spectral region.



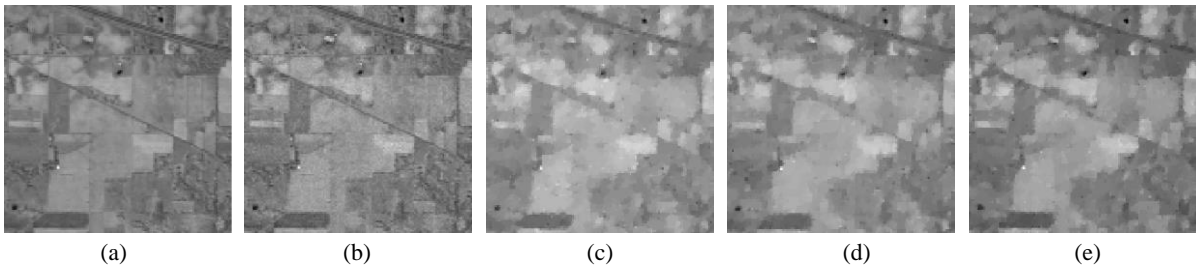


Figure 2: The 100th band reconstructed image from the AVIRIS scene *Indian pines* via different algorithms from 25% sampling measurements. (a)Original image. (b) Our method. (c)TV algorithm. (d)TVSS algorithm. (e)TVNU algorithm.

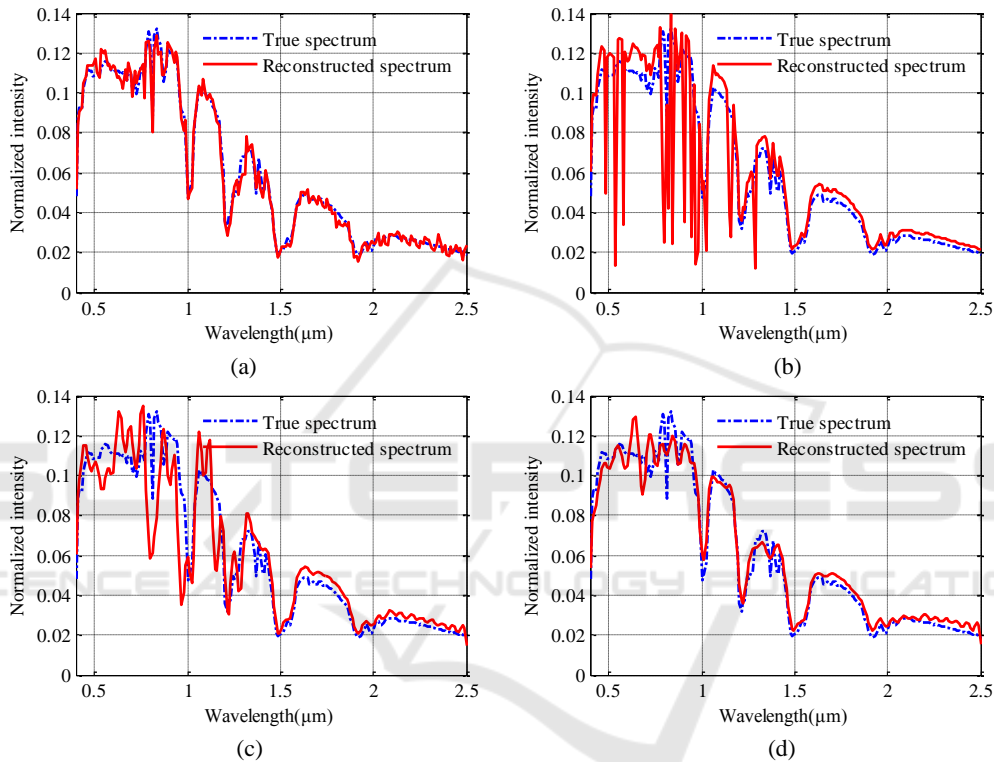


Figure 3: The corn spectrum in the AVIRIS scene *Indian pines* before and after reconstruction via different algorithms from 25% sampling measurements. (a) Our Method. (b)TV algorithm. (c)TVSS algorithm. (d)TVNU algorithm.

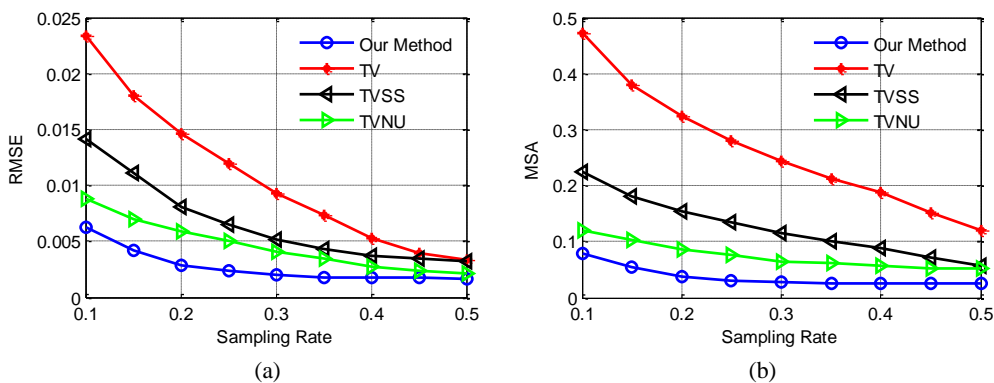


Figure 4: The quality assessment parameters of the reconstructed hyperspectral image at different sampling rate varying from 0.1 to 0.5 in the same CS experiment. (a) The RMSE value to assess spatial reconstruction quality. (b)The MSA value to assess spectral reconstruction quality.

## 5 CONCLUSIONS

Aiming at the problem to utilize the spectral sparse property in the hyperspectral CS remote sensing imaging, this paper presents a new sampling and reconstruction method based on the spectral sparse representation. By learning the spectral sparse dictionary to constrain the spectral region in the reconstruction and optimizing the spatial precision via total variation constraint, the AVIRIS hyperspectral scene is reconstructed in very high quality from 25% compressive measurements, which provides a new idea to enhance the hyperspectral sampling efficiency. Compared with other presented hyperspectral CS reconstruction algorithms, the reconstruction precision in spatial and spectral region of our method has a significant superiority in the same experimental condition.

However, there still exist some problems to be further studied in order to better apply the new theory. One is the method of the spectral training sample construction and dictionary learning. In the experiment it is found that if the training samples are extracted from the sensor or the type of ground object with a great difference from that of the reconstructed area, the effect of the spectral sparse representation is significantly affected and the reconstruction precision decreases. The other one is the realization of spectral random coding in hardware, for the spectral sampling scheme is more difficult to realize than spatial sampling.

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