Pipeline Pattern for Parallel MCTS

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Abstract: In this paper, we present a new algorithm for parallel Monte Carlo tree search (MCTS). It is based on the pipeline pattern and allows flexible management of the control flow of the operations in parallel MCTS. The pipeline pattern provides for the first structured parallel programming approach to MCTS. The Pipeline Pattern for Parallel MCTS algorithm (called 3PMCTS) scales very well to a higher number of cores when compared to the existing methods. The observed speedup is 21 on a 24-core machine.

1 INTRODUCTION

In recent years there has been much interest in the Monte Carlo tree search (MCTS) algorithm. In 2006 it was a new, adaptive, randomized optimization algorithm (Coulom, 2006; Kocsis and Szepesvári, 2006). In fields as diverse as Artificial Intelligence, Operations Research, and High Energy Physics, research has established that MCTS can find valuable approximate answers without domain-dependent heuristics (Kuipers et al., 2013). The strength of the MCTS algorithm is that it provides answers with a random amount of error for any fixed computational budget (Goodfellow et al., 2016). Much effort has been put into the development of parallel algorithms for MCTS to reduce the running time. The efforts are applied to a broad spectrum of parallel systems; ranging from small shared-memory multicore machines to large distributed-memory clusters. In the last two years, parallel MCTS played a major role in the success of AI by defeating humans in the game of Go (Silver et al., 2016; Hassabis and Silver, 2017).

The general MCTS algorithm has four operations inside its main loop (see Algorithm 1). This loop is a good candidate for parallelization. Hence, a significant effort has been put into the development of parallelization methods for MCTS (Chaslot et al., 2008a; Yoshizoe et al., 2011; Fern and Lewis, 2011; Schaefers and Platzner, 2014; Mirsoleimani et al., 2015b). To implement these methods, the computation associated with each iteration is assumed to be independent (Mirsoleimani et al., 2015a). Therefore, we can assign a chunk of iterations as a separate task to each parallel thread for execution on separate processors (Chaslot et al., 2008a; Schaefers and Platzner, 2014; Mirsoleimani et al., 2015a). This type of parallelism is called *iteration-level* parallelism (ILP). Close analysis has learned us that each iteration in the chunk can also be decomposed into separate operations for parallelization. Based on this idea, we introduce *operation-level* parallelism (OLP). The main point is to assign each operation of MCTS to a separate processing element for execution by separate processors. This leads to flexibility in managing the control flow of operations in the MCTS algorithm. The main contribution of this paper is introducing a new algorithm based on the *pipeline pattern for parallel MCTS* (3PMCTS) and showing its benefits.

The remainder of the paper is organized as follows. In section 2 the required background information is briefly described. Section 3 provides necessary definitions and explanations for the design of 3PMCTS. Section 4 gives the explanations for the implementation the 3PMCTS algorithm, Section 5 shows the experimental setup, and Section 6 gives the experimental results. Finally, in Section 7 we conclude the paper.

2 BACKGROUND

Below we discuss MCTS in Section 2.1, in Section 2.2 the parallelization of MCTS is explained.

2.1 The MCTS Algorithm

The MCTS algorithm iteratively repeats four steps or operations to construct a search tree until a predefined computational budget (i.e., time or iteration constraint) is reached (Chaslot et al., 2008b; Coulom,

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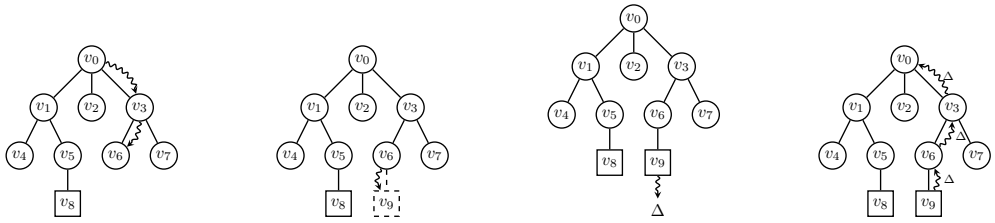


(a) SELECT (b) EXPAND (c) PLAYOUT (d) BACKUP

Figure 1: One iteration of MCTS.

2006). Algorithm 1 shows the general MCTS algorithm.

In the beginning, the search tree has only a root (v_0) which represents the initial state in a domain. Each node in the search tree resembles a state of the domain, and directed edges to child nodes represent actions leading to the succeeding states. Figure 1 illustrates one iteration of the MCTS algorithm on a search tree that already has nine nodes. The non-terminal and internal nodes are represented by circles. Squares show the terminal nodes.

1. SELECT: A path of nodes inside the search tree is selected from the root node until a non-terminal leaf with unvisited children is reached (v_6). Each of the nodes inside the path is selected based on a predefined *tree selection policy* (see Figure 1a).
2. EXPAND: One of the children (v_9) of the selected non-terminal leaf (v_6) is generated randomly and added to the tree together with the selected path (see Figure 1b).
3. PLAYOUT: From the given state of the newly added node, a sequence of randomly simulated actions (i.e., RANDOMSIMULATION) is performed until a terminal state in the domain is reached. The terminal state is evaluated using a utility function (i.e., EVALUATION) to produce a reward value Δ (see Figure 1c).
4. BACKUP: For each node in the selected path, the number $N(v)$ of times it has been visited is incremented by 1 and its total reward value $Q(v)$ is updated according to Δ (Browne et al., 2012). These values are required by the tree selection policy (see Figure 1d).

Algorithm 1: The general MCTS algorithm.

```
1 Function MCTS(s_0)
2     v_0 := creat root node with state s_0;
3     while within search budget do
4         < v_l, s_l > := SELECT(v_0, s_0);
5         < v_l, s_l > := EXPAND(v_l, s_l);
6         Δ := PLAYOUT(v_l, s_l);
7         BACKUP(v_l, Δ);
8     end
9     return action a for the best child of v_0
```

As soon as the computational budget is exhausted, the best child of the root node is returned (e.g., the one with the maximum number of visits).

The purpose of MCTS is to approximate the game-theoretic value of the actions that may be selected from the current state by iteratively creating a partial search tree (Browne et al., 2012). How the search tree is built depends on how nodes in the tree are selected (i.e., tree selection policy). In particular, nodes in the tree are selected according to the estimated probability that they are better than the current best action. It is essential to reduce the estimation error of the nodes' values as quickly as possible. Therefore, the tree selection policy in the MCTS algorithm aims at balancing exploitation (look in areas which appear to be promising) and exploration (look in areas that have not been well sampled yet) (Kocsis and Szepesvári, 2006).

The Upper Confidence Bounds for Trees (UCT) algorithm addresses the exploitation-exploration dilemma in the selection step of the MCTS algorithm using the UCB1 policy (Kocsis and Szepesvári, 2006). A child node j is selected to maximize:

$$UCT(j) = \overline{X}_j + 2C_p\sqrt{\frac{2\ln(N(v))}{N(v_j)}} \qquad (1)$$

where $\overline{X}_j = \frac{Q(v_j)}{N(v_j)}$ is an approximation of the game-theoretic value of node j. $Q(v_j)$ is the total reward of all playouts that passed through node j, $N(v_j)$ is the number of times node j has been visited, $N(v)$ is the number of times the parent of node j has been visited, and $C_p \geq 0$ is a constant. The left-hand term is for exploitation and the right-hand term is for exploration (Kocsis and Szepesvári, 2006). The decrease or increase in the amount of exploration can be adjusted by C_p in the exploration term (see Section 6).

2.2 Parallelization of MCTS

Parallelization of MCTS consists of a precise arrangement of tasks and data dependencies. In section 2.2.1 we explain how to decompose MCTS into tasks. In section 2.2.2 we investigate what types of data dependencies exist among these tasks. In Section 2.2.3 the existing parallelization methods for MCTS are discussed.

2.2.1 Decomposition into Tasks

The first step towards parallelizing MCTS is to find concurrent tasks in MCTS. As stated above, there are two levels of **task decomposition** in MCTS.

1. *Iteration-level tasks (ILT)*: In MCTS the computation associated with each iteration is independent. Therefore, *these are candidates* to guide a task decomposition by mapping a chunk of iterations onto a task.
2. *Operation-level tasks (OLT)*: The task decomposition for MCTS occurs inside each iteration. Each of the four MCTS operations can be treated as a separate task.

2.2.2 Data Dependencies

The second step is dealing adequately with the data dependency. When a search tree is shared among multiple parallel threads, there are two levels of **data dependency**.

1. *Iteration-level dependencies (ILD)*: Strictly speaking, in MCTS, iteration j has a *soft dependency* to its predecessor iteration $j-1$. Obviously, to select an optimal path, it requires updates on the search tree from the previous iteration.[1] A parallel MCTS can ignore ILD and then simply suffers from the *search overhead*.[2]
2. *Operation-level dependencies (OLD)*: Each of the four operations in MCTS has a *hard dependency* to its predecessor.[3] For example, the EXPAND operation cannot start until the SELECT operation has been completed.

[1] i.e., a violation of ILD does not impact the correctness of the algorithm.

[2] It occurs when a parallel implementation in a search algorithm searches more nodes of the search space than the sequential version; for example, since the information to guide the search is not yet available.

[3] i.e., a violation of OLD yields an incorrect algorithm.

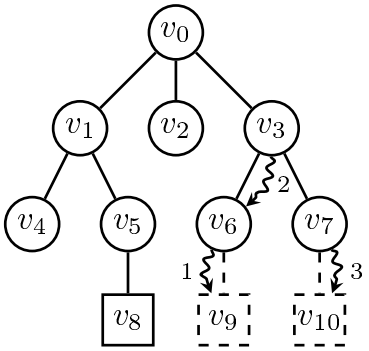


Figure 2: Tree parallelization. The curly arrows represent threads. The rectangles are terminal leaf nodes.

2.2.3 Parallelization Methods

There are three parallelization methods for MCTS (i.e., *root parallelization*, *leaf parallelization*, and *tree parallelization*) that belong to two main categories: (A) parallelization with an ensemble of trees, and (B) parallelization with a single shared tree.

The root parallelization method belongs to category (A). It creates an ensemble of search trees (i.e., one for each thread). The trees are independent of each other. When the search is over, they are merged, and the action of the best child of the root is selected.

The leaf parallelization and tree parallelization methods belong to category (B). In the leaf parallelization, the parallel threads perform multiple PLAYOUT operations from a non-terminal leaf node of the shared tree. These PLAYOUT operations are independent of each other, and therefore there is no data dependency. In tree parallelization, parallel threads are potentially able to perform different MCTS operations on a same node of the shared tree (Chaslot et al., 2008a). Figure 2 shows the tree parallelization where three threads simultaneously perform different MCTS operations on the tree. This is the topic of our research.

The existing parallel implementations for tree parallelization are based on ILP (Chaslot et al., 2008a; Enzenberger and Müller, 2010; Mirsoleimani et al., 2015b; Mirsoleimani et al., 2015a). The tasks are of the type of ILT, and the only dependency that exists among them is ILD. The potential concurrency is exploited by assigning a chunk of iterations to separate parallel threads and having them work on separate processors. Our proposed algorithm allows implementation of tree parallelization based on OLP.

3 DESIGN OF 3PMCTS

In this section, we describe our proposed method for parallelizing MCTS. Section 3.1 describes how the pipeline pattern is applied in MCTS. Section 3.2 provides the 3PMCTS algorithm.

3.1 Pipeline Pattern for MCTS

Below we describe how the pipeline pattern is used as a building block in the design of 3PMCTS. Figure 3 shows two types of pipelines for MCTS. The inter-stage buffers are used to pass information between the stages. When a stage of the pipeline completes its computation; it sends a path of nodes from the search to the next buffer. The subsequent stage picks a path from the buffer and starts its computation. Here we introduce two possible types of pipelines for MCTS.

1. *Pipeline with sequential stages*: Figure 3a shows a pipeline with sequential stages for MCTS. The idea is to map each MCTS operation to pipeline stages so that each stage of the pipeline computes one operation. Figure 4 illustrates how the pipeline executes the MCTS operations over time. Let C_i represent a multiple-step computation on path i. $C_i(j)$ is the jth step of the computation in MCTS (i.e., $j \in$ {S, E, P, and B}). Initially, the first stage of the pipeline performs $C_1(S)$. After the step has been completed, the second stage of the pipeline receives the first path and computes $C_1(E)$ while the first stage computes the first step of the second path, $C_2(S)$. Next, the third stage computes $C_1(P)$, while the second stage computes $C_2(E)$ and the first stage $C_3(S)$. Each stage of the pipeline takes the same amount of time to do its work, say T. Figure 4 shows that the expected execution time for 4 paths in an MCTS pipeline with four stages is approximately $7 \times T$. In contrast, the sequential version takes approximately $16 \times T$ because each of the 4 paths must be processed one after another. The pipeline pattern works best if the operations performed by the various stages of the pipeline are all about equally computationally intensive. If the stages in the pipeline vary in computational effort, the slowest stage creates a bottleneck for the aggregate throughput. In other words, when there are sufficient processors for each pipeline stage, the speed of a pipeline is approximately equal to the speed of its slowest stage. For example, Figure 5 shows the scheduling diagram that occurs when the PLAYOUT stage takes $2 \times T$ units of time while others take T units of time. Figure 5 shows that the expected execution time for 4 paths is approximately $11 \times T$.

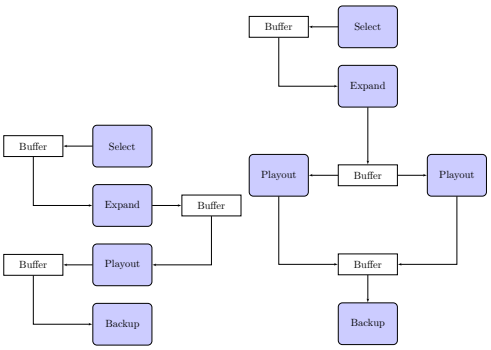


Figure 3: (3a) Flowchart of a pipeline with sequential stages for MCTS. (3b) Flowchart of a pipeline with parallel stages for MCTS.

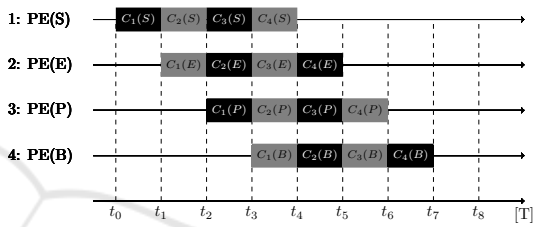


Figure 4: Scheduling diagram of a pipeline with sequential stages for MCTS. The computation for stages are equal.

2. *Pipeline with parallel stages*: Figure 3b shows a pipeline for MCTS with two parallel PLAYOUT stages. Using two PLAYOUT stages in the pipeline results in an overall speed of approximately T units of time per path as the number of paths grows. Figure 6 shows that the MCTS pipeline is perfectly balanced by using two PLAYOUT stages. The expected execution time for 4 paths is approximately $8 \times T$. Therefore, introducing parallel stages improves the scalability of the MCTS pipeline.

3.2 Pipeline Construction

The pseudocode of MCTS is shown in Algorithm 1. Each operation in MCTS constitutes a stage of the pipeline in 3PMCTS. In contrast to the existing methods, our 3PMCTS algorithm is based on OLP for parallelizing MCTS. The pipeline pattern can satisfy the operation-level dependencies (OLDs) among the iteration-level tasks (OLTs).

The potential concurrency is also exploited by assigning each stage of the pipeline to a separate processing element for execution on separate processors. If the pipeline has only sequential stages then the speedup is limited to the number of stages.

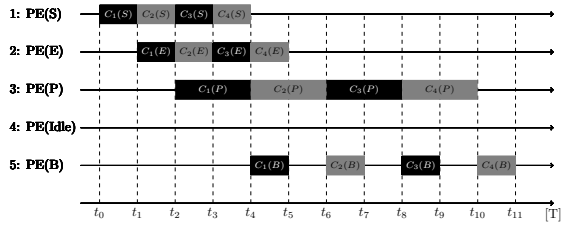


Figure 5: Scheduling diagram of a pipeline with sequential stages for MCTS. The computation for stages are not equal.

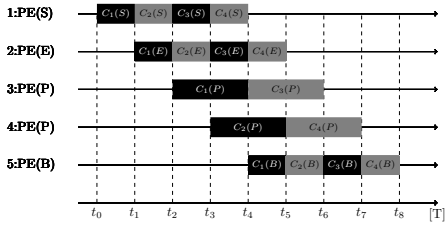


Figure 6: Scheduling diagram of a pipeline with parallel stages for MCTS. Using parallel stages create load balancing.

[4] However in MCTS, the operations are not equally computationally intensive, e.g., the PLAYOUT operation (random simulations plus evaluation of a terminal state) could be more computationally expensive than other operations. Therefore, our 3PMCTS algorithm uses a pipeline with parallel stages. Introducing parallel stages makes 3PMCTS more scalable.

Figure 7 depicts one of the possible pipeline constructions for 3PMCTS. We split the PLAYOUT operation into two stages to achieve more parallelism (See Section 2.1). The five stages run the MCTS operations SELECT, EXPAND, RANDOMSIMULATION, EVALUATION, and BACKUP, in that order. The SELECT stage and BACKUP stage are serial. The three middle stages (EXPAND, RANDOMSIMULATION, and EVALUATION) are parallel and do the most time-consuming part of the search. A serial stage does process one token at a time. A parallel stage is able to process more than one token. Therefore, it needs more than one in-flight *token*. A token represents a path of nodes inside the search tree during the search.

The pipeline depicted in Figure 7 is one of the possible constructions for 3PMCTS. Each of the five stages could be either serial or parallel. Therefore, 3PMCTS provides a great level of flexibility. For example, a pipeline could have a serial stage for the SELECT operation and a parallel stage for the BACKUP operation. In our experiments we use this construction (see Section 6).

[4]This holds when the operations performed by the various stages are all about equally computationally intensive.

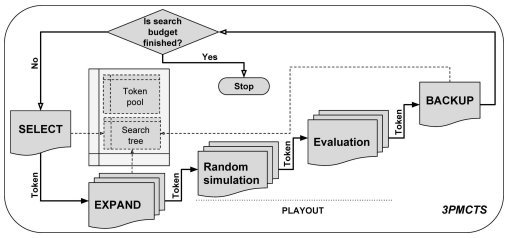


Figure 7: The 3PMCTS algorithm with a pipeline that has three parallel stages (i.e., EXPAND, RANDOMSIMULATION, and EVALUATION).

4 IMPLEMENTATION

We have implemented the proposed 3PMCTS algorithm in the **ParallelUCT** package (Mirsoleimani et al., 2015a). The ParallelUCT package is an open source tool for parallelization of the UCT algorithm.[5] It uses *task-level parallelism* to implement different parallelization methods for MCTS. We have also used an algorithm called *grain-sized control parallel MCTS* (GSCPM) to measure the performance of ILP for MCTS. The GSCPM algorithm creates tasks based on the *fork-join pattern* (McCool et al., 2012). More details about this algorithm can be found in (Mirsoleimani et al., 2015a). Both 3PMCTS and GSCPM are implemented by TBB parallel programming library (Reinders, 2007) and they are available online as part of the ParallelUCT package. In our implementation for 3PMCTS, we can specify the number of in-flight tokens. This is equal to the number of tasks for GSCPM algorithm.

5 EXPERIMENTAL SETUP

The performance of 3PMCTS is measured by using a High Energy Physics (HEP) expression simplification problem (Kuipers et al., 2013; Ruijl et al., 2014). Our setup follows closely (Kuipers et al., 2013). We discuss the case study in 5.1, the hardware in 5.2, and the performance metrics in 5.3.

5.1 Case Study

Our case study is in the field of Horner's rule, which is an algorithm for polynomial computation that reduces the number of multiplications and results in a computationally efficient form. For a polynomial in one variable

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0, \qquad (2)$$

[5]https://github.com/mirsoleimani/paralleluct/

the rule simply factors out powers of x. Thus, the polynomial can be written in the form

$$p(x) = ((a_n x + a_{n-1})x + ...)x + a_0. \quad (3)$$

This representation reduces the number of multiplications to n and has n additions. Therefore, the total evaluation cost of the polynomial is $2n$.

Horner's rule can be generalized for multivariate polynomials. Here, Eq. 3 applies on a polynomial for each variable, treating the other variables as constants. The order of choosing variables may be different, each order of the variables is called a *Horner scheme*.

The number of operations can be reduced even more by performing common subexpression elimination (CSE) after transforming a polynomial with Horner's rule. CSE creates new symbols for each subexpression that appears twice or more and replaces them inside the polynomial. Then, the subexpression has to be computed only once.

We are using the HEP(σ) expression with 15 variables to study the results of 3PMCTS. The MCTS is used to find an order of the variables that gives efficient Horner schemes (Ruijl et al., 2014). The root node has n children, with n the number of variables. The children of other nodes represent the remaining unchosen variables in the order. Starting at the root node, a path of nodes (variables) inside the search tree is selected. The incomplete order is completed with the remaining variables added randomly (i.e., RANDOMSIMULATION). The complete order is then used for Horners method followed by CSE to optimize the expression. The number of operations (i.e., Δ) in this optimized expression is counted (i.e., EVALUATION).

5.2 Hardware

Our experiments were performed on a dual socket Intel machine with 2 Intel *Xeon* E5-2596v2 CPUs running at 2.4 GHz. Each CPU has 12 cores, 24 hyperthreads, and 30 MB L3 cache. Each physical core has 256KB L2 cache. The peak TurboBoost frequency is 3.2 GHz. The machine has 192GB physical memory. We compiled the code using the Intel C++ compiler with a -*O*3 flag.

5.3 Performance Metrics

One important metric related to performance and parallelism is speedup. Speedup compares the time for solving the identical computational problem on one worker versus that on P workers:

$$speedup = \frac{T_1}{T_P}. \quad (4)$$

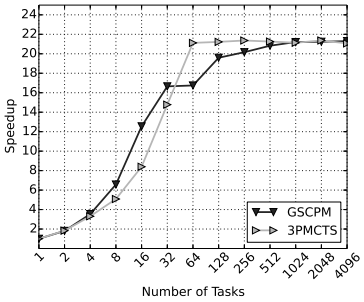


Figure 8: Playout-speedup as function of the number of tasks (tokens). Each data point is an average of 21 runs for a search budget of 8192 playouts. The constant C_p is 0.5. Here a higher value is better.

Where T_1 is the time of the program with one worker and T_p is the time of the program with P workers. In our results we report the scalability of our parallelization as *strong scalability* which means that the problem size remains fixed as P varies. The problem size is the number of playouts (i.e., the search budget) and P is the number of tasks. In the literature this form of speedup is called *playout-speedup* (Chaslot et al., 2008a).

The second important metric is the *number of operations* in the optimized expression. A lower value is desirable when higher number of tasks is used.

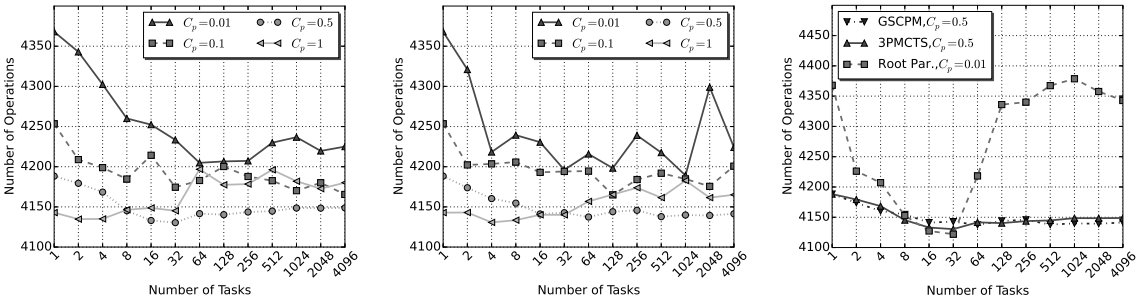
6 EXPERIMENTAL RESULTS

In this section, the performance of 3PMCTS is measured. Table 1 shows the sequential time to execute the specified number of playouts.

Figure 8 shows the playout-speedup for both 3PMCTS and GSCPM, as a function of the number of tasks (from 1 to 4096). The search budget for both algorithms is 8192 playouts. The 3PMCTS algorithm uses a pipeline with five stages for MCTS operations. Four stages are parallel; the SELECT stage is chosen to be serial (see the end of Section 3.2). A playout-speedup close to 21 on a 24-core machine is observed for both algorithms. From our results, we may provisionally conclude that the 3PMCTS algorithm (a) for 4 to 32 parallel tasks, shows a speedup less than GSCPM and (b) for 64 to 512 parallel tasks, shows a better speedup than the GSCPM algorithm (see Figure 8). At the same time, 3PMCTS also allows flexible control of the parallel or serial execution of MCTS

Table 1: Sequential time in seconds when $C_p = 0.5$.

Processor	Num. Playouts	Time (s)
CPU	8192	215.72 ± 4.12



(a) 3PMCTS (b) GSCPM (c) Root Parallelization

Figure 9: Number of operations as function of the number of tasks (tokens). Each data point is an average of 21 runs for a search budget of 8192 playouts. Here a lower value is better.

operations (e.g., the SELECT stage is sequential and the BACKUP stage is parallel in our case), something that GSCPM cannot provide.

Figure 9a and 9b show the results of the optimization in the number of operations in the final expression for both algorithms. These results show consistency with the findings in (Kuipers et al., 2013; Ruijl et al., 2014). From our results, we may arrive at three conclusions. (1) When MCTS is sequential (i.e., the number of tasks is 1), for small values of C_p, such that MCTS behaves exploitively, the method gets trapped in local minima, and the number of operations is high. For larger values of C_p, such that MCTS behaves exploratively, lower values for the number of operations is found. (2) When MCTS is parallel, for small numbers of tasks (from 2 to 8), it turns out to be good to choose a high value for the constant C_p (e.g., 1) for both 3PMCTS and GSCPM. With higher numbers of tasks, a lower value for C_p in the range [0.5; 1) seems suitable for both algorithms. Figure 9c also shows that 3PMCTS can find lower number of operations for 8, 16, and 32 tasks when $C_p = 0.5$. (3) When both algorithms find the same number of operations, the one with higher speedup is better. For instance, the 3PMCTS algorithm finds the same number of operations compared to GSCPM for 64 tasks, but it has higher speedup when $C_p = 0.5$. Note that these values hold for a particular polynomial and that different polynomials give different optimal values for C_p and number of tasks.

A comparison to root parallelization is illustrated in Figure 9c. Both 3PMCTS and GSCPM belong to the category of tree parallelization. For $C_p = 0.01$, root parallelization finds a lower number of operations for both 16 and 32 tasks compared to the two other methods. However, increasing the number of tasks causes root parallelization to provide a much higher number of operations. From these results, we may conclude that root parallelization could also be a feasible choice in this domain.

Kuipers et al. remarked that tree parallelization would give a result that is statistically a little bit inferior to a run with sequential MCTS with the same number of playouts due to the violation of iteration-level dependency (ILD) that produces search overhead (Kuipers et al., 2015). It is clear from our results that the effectiveness of any parallelization method for MCTS depends heavily on the choice of three parameters: (1) the C_p constant, (2) the number of playouts, and (3) the number of tasks. If we select these parameters carefully, it is possible to overcome the search overhead to some extent. Furthermore, the 3PMCTS algorithm provides the flexibility of managing the execution (serial or parallel) of different MCTS operations that helps us even more to achieve this goal.

7 CONCLUSION AND FUTURE WORK

Monte Carlo Tree Search (MCTS) is a randomized algorithm that is successful in a wide range of optimization problems. The main loop in MCTS consists of individual iterations, suggesting that the algorithm is well suited for parallelization. The existing parallelization methods, e.g., tree parallelization, straight forwardly fans out the iterations over available cores.

In this paper, a new parallel algorithm based on the pipeline pattern for MCTS is proposed. The idea is to break-up the iterations themselves, splitting them into individual operations, which are then parallelized in a pipeline. Experiments with an application from High Energy Physics show that our implementation of 3PMCTS scales well. Scalability is only one issue, although it is an important one. The second issue is the flexibility of task decomposition in parallelism. These higher levels of flexibility allow fine-grained manag-

ing of the execution of operations in MCTS which provide different pipeline constructions. We consider the flexibility an even more important characteristic of 3PMCTS. For future work, we will study the effectiveness of 3PMCTS with regards to the different pipeline constructions.

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