Tackling Demand Stochasticity by Redistribution among Retailers in a Two-stage Distribution System

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Abstract: This paper considers a two-stage distribution system consisting of a supplier supplying several retailers with stochastic demand rates. Replenishments by the supplier happen cyclically, based on average retailer demand rates. Hence, the considered distribution problem between the suppliers and its retailers is a Cyclic Inventory Routing Problem (CIRP), which we solve with a state-of-the-art heuristic solution method. To cope with the demand stochasticity, lateral transshipments can happen between retailers in each time period. We propose a redistribution policy that determines the quantities being redistributed through lateral transshipments based on the desired level of customer service the retailers want to reach (using the desired fill rate) and their actual inventory levels. The cost of the daily redistribution is determined by solving a Travelling Salesman Problem (TSP) covering the participating retailers. The potential benefits of the collaboration among retailers are analyzed by comparing the distribution costs, the redistribution costs, the inventory holding costs and the costs of lost sales at the retailers in different scenarios.

1 INTRODUCTION

Routing and inventory control often represent a large part of a company's operational costs. Traditionally, operations management focused on optimizing the internal operations of a company. However, researchers and companies realized that operational performance does not only depend on their own decisions, but also on the decisions made by other players in the supply chain. Hence, focus shifted from single-firm optimization to a more global 'supply chain management' perspective to create higher operational efficiency. (Mentzer et al. (2001); Power (2005))

In this paper, a combination of vertical (i.e., among players on subsequent levels of the supply chain) and horizontal (i.e., among players on the same level of the supply chain) collaboration is introduced in a distribution network. We consider a two-echelon supply chain consisting of a supplier and his retailers.

Vertical collaboration between the supplier and his retailers is established through Vendor-Managed Inventory (VMI). The retailers share demand data with the supplier and hand over the responsibility for the replenishment timing and quantity to the supplier. This way, the supplier can coordinate retailer replenishments better and design more efficient routes. The supplier is then faced with an integrated inventory and vehicle routing problem known as the Cyclic Inventory Routing Problem (CIRP). VMI has been extensively studied and comprehensive overviews of the existing literature are given by Moin and Salhi (2007), Andersson (2010) and Coelho (2013).

Horizontal collaboration is established among the retailers through lateral transshipments, which enables them to cope with demand uncertainty. Lateral transshipments were defined by Tagaras (1999) as *"the redistribution of stock from retailers with stock on hand to retailers that cannot meet customer demands or retailers that expect significant losses due to high risk"*. Lateral transshipments can lead to service improvement by preventing stockouts and to reduced inventory holding costs. Previous research made a distinction in redistribution based on the timing of the lateral transshipments. Redistribution can take place at predetermined times before all demand is realized (i.e., proactive transshipments), or at any time to respond to (potential) stockouts (i.e., reactive transshipments). Paterson (2011) gives a review on inventory models with lateral transshipments.

A large part of the literature on lateral transshipments focuses solely on inventory control, assumes a periodic inventory review policy, only considers one

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moment for redistribution within the order cycle or lets lateral transshipments take place between only a limited number of inventory points (Paterson et al. (2011)). This paper contributes to this literature by combining routing and inventory control at the retailers through lateral transshipments. We develop an operational model to determine the costs of redistribution and to analyze the benefits.

2 PROBLEM DESCRIPTION

We consider a two-echelon distribution network consisting of a single supplier S who delivers a single product to a retailer set I. The supplier cyclically replenishes the retailers under a VMI system, based on the retailers' mean demand rates d_i. The actual demand rates at the retailers $a_{i,t}$ (i.e., the demand rate at retailer i in time period t) are assumed to be stochastic with a known probability distribution.

To cope with demand uncertainty, lateral transshipments among the retailers are possible. The quantities that are redistributed are determined by the redistribution policy. This policy is aimed at achieving a target service level at all retailers. Hence, the redistributed quantities are based on the retailers' inventory levels and their fill rates. Since redistribution will only involve relatively small quantities of goods (compared to the supplier deliveries), we assume that redistribution can be performed by a single vehicle.

We develop an operational solution method to design a cyclic distribution plan for the supplier to replenish the retailers and a redistribution policy among the retailers that realizes the potential benefits resulting from lateral transshipments. The solution method determines the distribution cost, the cost of lateral transshipments, the inventory holding cost at the retailers and the cost of lost sales. These costs are compared between the baseline system in which no lateral transshipments occur and the collaborative system with lateral transshipments. Given that redistribution brings along additional costs, these have to be balanced with the potential benefits of lower inventory holding costs and lower lost sales.

3 DISTRIBUTION SOLUTION ALGORITHM FOR THE CIRP

The distribution routes of the supplier are designed using the two-phase route and fleet design CIRP heuristic (Raa and Dullaert (2017)).

In the first phase of the CIRP heuristic, the vehicle routes are designed using a construct-and-improvement heuristic. An initial solution is constructed using a savings-based heuristic based on the Clarke-and-Wright heuristic (Clarke and Wright (1964)), which was adapted for the CIRP. The initial solution is improved by using local search operators (2-opt, relocate and exchange operators). Each local search operator is reiterated until no more improvement is found before moving on the the next operator. A best-accept strategy is used per operator.

The route cycle times are chosen such that the distribution and inventory holding costs are minimized. Every time a route r is made, it incurs a distribution cost F_r consisting of a vehicle loading and dispatch cost, the delivery costs at the subset I_r (i.e., the set of retailers served by route r), and the transportation costs to drive the route (i.e., the TSP tour visiting the depot and the retailers in the route). To reduce distribution costs, the time between iterations of route r, i.e. the *cycle time* T_r, should be as long as possible.

Contrary to the distribution costs, the inventory holding costs at the retailers that are covered by a route increases with the route's cycle time. The delivery quantity of retailer i is $d_i T_r$. Hence, the average inventory level is $\frac{d_i T_r}{2}$ and the inventory holding cost rate is $\frac{\eta_i T_r d_i}{2}$, with η_i the holding cost per item per period. The total cost rate TC_r of route r thus varies with the cycle time T_r:

$$TC_r = \frac{F_r}{T_r} + T_r \sum_{i \in I_r} \frac{\eta_i d_i}{2} \tag{1}$$

The cycle time is however restricted by the vehicle capacity κ and the storage capacity of the retailers κ_i, resulting in a maximum cycle time $T_{\max,r}$ (cfr. formula 2). The retailers in route r are visited each T_r days, so they receive a delivery of $d_i T_r$ items. This delivery quantity should be lower than the storage capacity of the retailers κ_i. Hence, the cycle time must be less than or equal to $\min_{i \in r} \frac{\kappa_i}{d_i}$. Further, when making the route r, the vehicle is loaded with $\sum_{i \in r} d_i T_r$ items. This load should be lower than the vehicle capacity κ, so the cycle time T_r cannot be higher than $\frac{\kappa}{\sum_{i \in r} d_i}$.

$$T_{\max,r} = \left\lfloor \min\left(\frac{\kappa}{\sum_{i \in r} d_i}, \min_{i \in r} \frac{\kappa_i}{d_i} \right) \right\rfloor \tag{2}$$

There is an optimal cycle time T_r^* that balances the holding cost and the route cost and hence minimizes the cost rate.

$$T_r^* = \min\left(\sqrt{\frac{2F_r}{\sum_{i \in r} \eta_i d_i}}, T_{\max,r} \right) \tag{3}$$

The total distribution cost rate equals the sum of all the routes r ∈ R that the supplier has to perform to replenish all his retailers I:

$$TC_S = \sum_{r \in R} TC_r \tag{4}$$

The first phase of the CIRP algorithm thus divides the set of retailers into subsets that are each covered by a separate route and for which the route cycle time is chosen to minimize the cost rate.

The second phase of the CIRP algorithm assigns the routes to vehicles and to specific periods using a construct-and-improvement heuristic. The sequence of the routes stays the same, but their cycle times can be adjusted to minimize the required number of vehicles and thus the fleet costs.

The construction heuristic is a best-fit insertion heuristic that inserts the routes into the schedule in such a way that the cumulative remaining time of the vehicles to which the routes are assigned is minimal. The routes are inserted in the schedule using two cycle time selection rules. Firstly, the routes are inserted with cycle times as close as possible to their optimal cycle time (resulting in minimal route cost rates). Or secondly, they are inserted with cycle times as close as possible to their maximal cycle times (resulting in minimal fleet cost rate). This results in two initial schedules that are passed on to the improvement step. In the improvement step, two local search operators are applied, namely removing any single route from the schedule and reinserting it in the cheapest possible spot, or removing all routes made by the vehicle with the lowest utilization from the schedule en reinserting them in the cheapest possible way. To escape a potential local optimum, the schedule that results from the improvement step is scrambled by shuffling route allocations. Local search is then repeated on the scrambled solution. A predefined number of scrambles is set as a stopping criterion for the improvement step.

The two-phase route and fleet design approach is then reiterated within a metaheuristic framework (Raa and Dullaert (2017)).

4 REDISTRIBUTION STRATEGY AMONG RETAILERS

Once the routes and their allocation to vehicles are known, the supplier can execute his distribution plan. Then, the actual daily demand values $a_{i,t}$ are observed for all retailers in each period. These demand rates are generated using their known probability distribution. Since these demand values deviate from the average demand values, the actual inventory levels and the resulting inventory holding costs will differ from the inventory holding costs resulting from the CIRP solution and have to be recalculated. Hence, in order to know the actually incurred distribution cost rate, the inventory holding costs ($T_r \sum_{i \in I_r} \frac{\eta_i d_i}{2}$) have to be subtracted from the routes' cost rates TC_r.

Further, inventory levels may be lower than anticipated (after periods with higher than average demand) and the risk of stockout may occur. This is when redistribution through lateral shipments is activated.

Given that the cycle times of the routes can differ, the time periods in which the retailers are replenished will also differ from one retailer to another. Hence, we allow redistribution in each time period across a certain planning horizon.

At the start of period t, every retailer i has an inventory level $SI_{i,t}$. The ending inventory of retailer i in that period $EI_{i,t}$ is calculated by subtracting the demand in that period from the start inventory (= $SI_{i,t}$ - $a_{i,t}$). Redistribution is assumed to happen 'overnight' between the periods, so the starting inventory level of the next period also includes any lateral transshipments.

The redistribution quantities are thus determined based on the ending inventory level of the retailers. The retailers are divided into three different groups in each time period. The retailers whose ending inventory is too low to reach a certain desired service level until their next delivery from the supplier, are called *receivers* (1). Retailers whose ending inventory is more than necessary to reach their desired service level are called *contributors* (2). Finally, there are the retailers that do not want to participate to the redistribution in a specific period (3). The group to which a retailer belongs can differ from one period to the next, since his ending inventory will change according to his incurred demand rate, redistributed quantities from the previous period and possible deliveries from the supplier.

The desired service level is expressed by the *fill rate* (i.e., the fraction of demand filled from items available in inventory). For each time period, three inventory levels are determined for each retailer, the *critical inventory level* $CI_{i,t}$, the *threshold inventory level* $TI_{i,t}$ and the *optimal inventory level* $OI_{i,t}$. The *critical inventory level* $CI_{i,t}$ of a retailer is the inventory level below which he wants to receive goods from other retailers. So, a retailer is a receiver in time period t if $EI_{i,t} < CI_{i,t}$. The *threshold inventory level* $TI_{i,t}$ is the inventory level above which he is willing to share goods with other retailers. Hence, a retailer is a contributor in time period t if $EI_{i,t} > TI_{i,t}$. The *optimal inventory level* $OI_{i,t}$ of a retailer is the inventory level up to which he wants to receive items from the other retailers when his inventory has fallen below his critical inventory level. If the inventory of retailer i in time period t lies in the interval $[CI_{i,t};TI_{i,t}]$, he will not participate in the redistribution in time period t. The fill rates determining the critical inventory level and the threshold inventory level of a retailer can change over the time periods, depending on how many time periods there are left before he receives his next order from the supplier (i.e., the lead time L).

Based on the *critical inventory level*, the *optimal inventory level* and the *threshold inventory level* of a retailer, the number of goods he wants to contribute ($c_{i,t}$) or receive ($r_{i,t}$) in a time period t can be determined.

$$c_{i,t} = EI_{i,t} - TI_{i,t} \quad (5)$$

$$r_{i,t} = OI_{i,t} - EI_{i,t} \quad (6)$$

When the contributor amount ($\sum_i c_{i,t}$) equals the receiver amount ($\sum_i r_{i,t}$), they can be optimally redistributed. However, these amounts will often be different. When the contributor amount is smaller than the receiver amount ($\sum_i c_{i,t} < \sum_i r_{i,t}$), the receivers will not all be able to reach their optimal inventory level, so an appropriate decision rule is required to divide the contributed items over the receivers. Conversely, when $\sum_i c_{i,t} > \sum_i r_{i,t}$, we need to decide what contributors will contribute which quantities.

These decisions are also taken based on the fill rates of the receivers and the contributors. When the contributors want to contribute more items than receivers want to receive ($\sum_i c_{i,t} > \sum_i r_{i,t}$), the contributed items are collected from the contributors in such a way that they end up with a new inventory level that results in the same fill rate for the contributors (which is still above the required fill rate). When the receivers want to receive more items than the contributors want to contribute, the contributed items are allocated over the receivers in such a way they all end up with a new inventory level with the same fill rate (which is still below the required fill rate). The number of items a retailer i actually receives or contributes in period t are denoted as $R_{i,t}$ and $C_{i,t}$.

When the received and contributed number of items for all retailers are calculated at the end of time period t, the start inventory level of the retailers in the next period $SI_{i,t+1}$ can be calculated. This is done by adding the quantity delivered by the supplier in that time period $D_{i,t+1}$ (which will be zero when no delivery occurs in that time period), subtracting the number of contributed items $C_{i,t}$ and adding the number of received items $R_{i,t}$ from the end inventory level $EI_{i,t}$ in period t. The end inventory level $EI_{i,t}$ in all time periods is determined by subtracting the incurred demand $a_{i,t}$ from the start inventory level $SI_{i,t}$. Note that this end inventory level can become negative. When it is negative, it means that retailer i could not meet the demand and lost sales occur ($LS_{i,t}$). In periods in which the end inventory of a retailer is higher than or equal to zero, no lost sales occur (i.e. $LS_{i,t} = 0$). When the end inventory is lower than zero, lost sales amount to $-EI_{i,t}$. Note that the formula for the start inventory $SI_{i,t}$ takes into account lost sales from the previous period.

$$D_{i,t} = \begin{cases} 0 & \text{if there is no delivery in t} \\ d_i T_r & \text{if there is a delivery in t} \end{cases} \quad (7)$$

$$SI_{i,t} = max(0, EI_{i,t-1}) + D_{i,t} + R_{i,t-1} - C_{i,t-1} \quad (8)$$

$$EI_{i,t} = SI_{i,t} - a_{i,t} \quad (9)$$

$$LS_{i,t} = -min(0, EI_{i,t}) \quad (10)$$

Besides the cost of distribution and redistribution, the cost of lost sales and the inventory holding cost at the retailers are also considered to assess the collaborative supply chain with lateral transshipments. The lost sales cost rate of a retailer LSC_i is calculated by taking his average number of lost sales per time period and multiplying it with the cost of one item that could not be sold (λ_i) (cfr. formula 11). To calculate the inventory holding cost rate of a retailer H_i, his average inventory in each time period is summed over the planning horizon and multiplied with his inventory holding cost per item per time period (η_i) divided by the number of time periods in the planning horizon p (cfr. formula 12).

$$LSC_i = \frac{\lambda_i}{P} \sum_{t \in P} LS_{i,t} \quad (11)$$

$$H_i = \frac{\eta_i}{P} \sum_{t \in P} \frac{SI_{i,t} + EI_{i,t}}{2} \quad (12)$$

Note that this new inventory holding cost at the retailers replaces the inventory holding cost calculated in the CIRP solution algorithm ($T_r \sum_{i \in I_r} \frac{\eta_i d_i}{2}$, cfr. formula 1), since it is the inventory holding cost that the retailers will actually incur.

To determine the daily redistribution cost, a TSP is solved each day. The nodes in the TSP on a specific day are the receivers and the contributors. We assume there is no limitation on the capacity of the third party courier, since the redistributed volumes are small. The model takes into account a travel cost per km (θ). The redistribution cost rate is calculated

Table 1: CIRP routes supplier illustrative example.

r	TC_r	$\frac{F_r}{T_r}$	$\sum_{i \in r} \frac{\eta_i d_i}{2}$	T_r	Sequence
1	139	64	75	4	$S \to 8 \to 4 \to 5 \to 2 \to S$
2	172.66	99.16	73.5	5	$S \to 10 \to 1 \to 11 \to 7 \to 13 \to 12 \to S$
3	224	149.3	74.7	3	$S \to 6 \to 14 \to 3 \to 15 \to 9 \to S$
Total	535.66	312.46	223.2		

by multiplying the distances resulting from the TSPs with the travel cost per km and taking the average over the planning horizon P.

$$RC = \frac{\sum_{t \in P} TSP_t.\theta}{P} \tag{13}$$

The total cost comprises the distribution cost rate of the supplier TC_s (minus the inventory holding cost at the retailers incorporated in this cost rate ($\sum_{i \in I_r} T_r \frac{\eta_i d_i}{2}$)), the redistribution cost rate RC, the sum of the inventory holding cost over all retailers ($\sum_{i \in I} H_i$) and the sum of the lost sales cost over all retailers ($\sum_{i \in I} LSPC_i$). This total cost is computed for the baseline system without lateral transshipments and for the cooperative system with lateral transshipments. Both cases are compared to identify the benefits of the lateral transshipments.

5 ILLUSTRATIVE EXAMPLE

The suggested solution approach is illustrated with an illustrative example. The example comprises one supplier replenishing 15 retailers with one type of product. One time period is assumed to be one day. The average daily demand rates of the retailers d_i are generated randomly between 0.2 and 10 items per day and their maximum storage capacity between 10 and 100 items. The inventory holding cost and the lost sales cost of the retailers are assumed to be equal for all retailers and are respectively 1.5/item/day and 100/item. The retailers' locations are generated randomly in such a way that they are uniformly distributed in a circle around the supplier

The routes resulting from the CIRP algorithm are shown in Table 1. The total transportation cost rate of the supplier TC_S equals 535.66 (= 139 + 172.66 + 224). The inventory holding costs of the retailers have to be subtracted from the routes' cost rates. These inventory holding cost rates amount to 75, 73.5 and 74.7. So, the transportation cost rates that are taken into account for the comparison of the baseline system without lateral transshipments and the collaborative system with lateral transshipments are 64 (= 139 - 75), 99.16 (=172.66 - 73.5) and 149.3 (= 224 - 74.7), summing to a total transportation cost rate of 312.46.

The daily demand rates of the retailers are assumed to be normally distributed with mean d_i and standard deviation $0.1\ d_i$. Given that the cycle times of the routes of the suppliers are 4, 5 and 3, actual daily demand rates are generated for all retailers over a planning horizon of 60 days (= P). Table 2 shows the actual daily demand rates for retailer 2 over 12 days. These demand rates are generated based on his average demand rate of 4.6 and standard deviation 0.46. The supplier receives a delivery of 18.4 items (= 4 . 4.6) on days 1, 5 and 9 since the cycle time of his route is 4. Based on the actual demand rates, the start and end inventory levels of the retailers in the baseline system without lateral transshipments can be calculated using formulas 8 and 9. The negative end inventory levels on days 4, 8 and 12 (resp. -0.3, -0.9 and -1.0 items) indicate that a stockout (and consequently lost sales) occurs on these days due to the higher than average demand over the previous days. Note that the lost sales cannot be backlogged, so the start inventory on day 5 equals the delivery retailer 2 receives from the supplier and does not take into account the lost sales of 0.3 from day 4.

The cost of lost sales and the inventory holding cost are calculated for all retailers (cfr. formulas 11 and 12). Table 3 shows the overall lost sales cost rate and inventory holding cost rate in the baseline system without lateral transshipments over all retailers. The total cost rate in the baseline system without redistribution equals 618.11 (= 312.46 + 254.15 + 51.5).

When redistribution is introduced, the threshold inventory level, the optimal inventory level and the critical inventory level are calculated for all retailers on all days. In our example, we assume a fill rate of 99.5% for the threshold inventory level, 99% for the optimal inventory level and 98.5% for the critical inventory level. Table 4 shows the three inventory levels for retailer 2 over 12 days. Note that on day 1 the inventory levels are calculated using a lead time of 3 since it is the inventory level at the end of the day that is compared to the threshold inventory level, the optimal inventory level and the critical inventory level. At the end of day 1, retailer 2 expects his next delivery in 3 days. Hence, the lead time of 3 days.

Based on these inventory levels and the end inventory level of the retailers, the receiver and contributor

Table 2: Retailer 2 days 1 to 12 illustrative example baseline system.

t	1	2	3	4	5	6	7	8	9	10	11	12
$D_{2,t}$	18.4	0	0	0	18.4	0	0	0	18.4	0	0	0
$a_{2,t}$	4.3	4.2	4.9	5.3	5.4	4.6	5	4.3	4.9	4.9	4.4	5.2
$SI_{2,t}$	18.4	14.1	9.9	5.0	18.4	13.0	8.4	3.4	18.4	13.5	8.6	4.2
$EI_{2,t}$	14.1	9.9	5.0	-0.3	13.0	8.4	3.4	-0.9	13.5	8.6	4.2	-1.0
$LS_{2,t}$	0	0	0	0.3	0	0	0	0.9	0	0	0	1

Table 3: Lost sales, inventory holding, distribution and redistribution cost rates.

	Baseline	Collaborative
$\sum_i LSC_i$	51.50	23.50
$\sum_i H_i$	254.15	244.00
TC_S	312.46	312.46
RC	0.00	31.84
Total cost rate	618.11	611.80

amounts are determined. On day 2 for example, the end inventory of retailer 2 is higher than his threshold inventory (9.9>9.7), so he is willing to hand over 0.2 items to the other retailers. Table 4 also shows the total receiver and contributor amount (resp. $\sum_i r_{i,t}$ and $\sum_i c_{i,t}$) over all retailers over the 12 days. On day 2 there are more receiver items requested than there are items contributed (1.2<3.1). Hence, supplier 2 will contribute the total 0.2 items. The receiver and contributor quantities are redistributed 'overnight'. So, the start inventory of retailer 2 on day 3 is updated for these 0.2 contributed items (9.9 - 0.2 = 9.7). On day 5 on the other hand, the end inventory of retailer 2 falls below his critical inventory (13.2<13.9). So, retailer 2 is a receiver on that day. The total contributor amount is now higher than the total receiver amount (4.4>0.8), so retailer 2 will receive his full number of requested items (0.7). Again, the start inventory of retailer 2 on day 6 takes into account this number of received items (13.2 + 0.7 = 13.9).

The new lost sales of retailer 2 in the 12-day period are also included in table 4. These are determined in the same way as in the baseline system (i.e., as the negative end inventory when the end inventory falls below zero). Note that despite the redistribution of inventory among the retailers, retailer 2 incurs lost sales on day 4. Overall, his lost sales are lower than in the baseline sytem though (0.3 + 0.9 + 1).

The number of lost sales, the average inventory level and the corresponding lost sales cost rate and inventory holding cost rate is calculated over all retailers in the planning horizon in the collaborative system with lateral transshipments. The results are also shown in Table 3. The lost sales cost rate has dropped (-54.3%) compared to the baseline system, as has the inventory holding cost rate (-3.9%). However, the cost of redistribution has to be taken into account. A TSP is solved for each day within the 60-days planning horizon. On day 7 for example, retailers 2 and 13 want to receive items (resp. 0.1 and 0.9 items). They receive these items from retailers 4, 9 and 10. So, on day 7 a TSP is solved among retailers 2, 4, 9, 10 and 13. The total redistribution cost rate resulting from the TSPs over the planning horizon equals 31.84.

The collaborative system with lateral transshipments among the retailers results in our example only in a small decrease in the total cost rate (-1,02%). However, lost sales have decreased considerably (from 30.9 items to 14.1 items over the 60-days planning horizon), so the customer service level has improved too.

6 CONCLUSIONS

We studied the benefits of lateral transshipments of inventory among retailers in a two-stage supply chain. The supplier replenishes the retailers in a VMI setting and his/her distribution routes and distribution cost are determined using a CIRP solution heuristic. Replenishments by the supplier are periodic. However, since the replenishment frequencies are determined in the CIRP heuristic, cycle times of the retailers can differ from one to another. Redistribution of inventory is possible in all time periods. The redistributed quantities are determined based on the inventory level of the retailers and their desired fill rate. The benefits of the lateral transshipments are savings created through reductions in lost sales and inventory holding costs and increased customers service levels. The cost reductions have to be balanced with the increase in transportation cost caused by the redistribution of inventory. The cost of redistribution is determined by solving TSP problems among the retailers that want to participate in the redistribution.

Preliminary results show that savings in inventory costs and lost sales can occur through lateral transshipments in the VMI setting. Furthermore, the customer service levels increase. However, the savings are highly dependent on the problem parameters, like the cost of lost sales, the inventory holding cost at the

Table 4: Retailer 2 days 1 to 12 illustrative example collaborative system.

t	1	2	3	4	5	6	7	8	9	10	11	12
$D_{2,t}$	18.4	0	0	0	18.4	0	0	0	18.4	0	0	0
L	3	2	1	4	3	2	1	4	3	2	1	4
$TI_{2,t}$	14.5	9.7	4.8	19.2	14.5	9.7	4.8	19.2	14.5	9.7	4.8	19.2
$OI_{2,t}$	14.1	9.4	4.6	18.9	14.1	9.4	4.6	18.9	14.1	9.4	4.6	18.9
$CI_{2,t}$	13.9	9.2	4.4	18.6	13.9	9.2	4.4	18.6	13.9	9.2	4.4	18.6
$a_{2,t}$	4.3	4.2	4.9	4.3	5.4	4.6	5	4.3	4.9	4.9	4.4	5.2
$SI_{2,t}$	18.4	14.1	9.7	4.8	18.6	13.9	9.3	4.4	18.6	13.9	9.2	4.8
$EI_{2,t}$	14.1	9.9	4.8	-0.5	13.2	9.3	4.3	0.1	13.7	9	4.8	-0.4
$C_{2,t}$	0	0.2	0	0	0	0	0	0	0	0	0	0
$R_{2,t}$	0	0	0	0.2	0.7	0	0.1	0.1	0.2	0.2	0	0.2
$\sum_i c_{i,t}$	0.1	1.2	0.6	4.7	4.4	3.1	7.2	7.1	9.5	10.6	11.3	10.5
$\sum_i r_{i,t}$	3.3	3.1	2.3	0.6	0.8	1.5	1	0.2	0.5	0.2	0.5	0.7
$LS_{2,t}$	0	0	0	0.5	0	0	0	0	0	0	0	0.4

retailers and the location of the retailers (and consequently the cost of distribution and redistribution).

More in-depth research into larger datasets is necessary. A design of experiments should be performed to calibrate the redistribution policy parameters across a wide range of instances. The chosen levels of the fill rate to determine the threshold, critical and optimal inventory levels appear to have a large impact on the quantities all retailers want to contribute or receive to the redistribution, and consequently on the redistribution costs. Also the way in which we determine which retailers actually contribute items when the contributor amount is higher than the receiver amount (or vice versa, when the amount of receiver items is higher than the contributor amount) must be investigated more in detail. The preliminary analysis showed for example that it can be beneficial to exclude some retailers from the redistribution in certain time periods to prevent unnecessary movements of small quantities and to keep the redistribution cost resulting from the TSP under control. These redistribution policy parameters become certainly important in more realistic datasets with a high number of retailers.

Furthermore, when determining which retailers should contribute and receive what quantities in the redistribution, we must keep in mind that all retailers want to receive benefits when entering the collaboration. Hence, the benefits should be shared over all participants in a fair way, otherwise some participants will be inclined to leave the collaboration. Further research can be performed on how all retailers can receive a fair share of the redistribution benefits.

Additionally, taking into account demand variability into the CIRP to reduce the need for redistribution should also be investigated.

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