Optimizing Storage Capacity of Retailers in Stochastic Periodic Inventory Routing Problem

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Abstract: A challenging question in Stochastic Periodic Inventory Routing Problem (SPIRP) is how to deal with stochastic demand rates, while minimizing the costs (transportation, inventory, and storage) and finding the best routing system. In this paper, we reformulate the SPIRP model to a safety stock-based SPIRP where the inventory storage capacity at the retailers are considered as variables and retailer's demand rate is stochastic. The supply chain planner needs to find the best routing system to replenish the retailers with the most optimum level of inventory, while the service level is satisfied in a long term planning horizon. Four different policies for storage capacity optimization are presented, evaluated, and compared in an illustrative example. The impact of storage capacity limitation is considered based on the defined policies to measure their compatibility for different situations.

1 INTRODUCTION

Inventory-Routing Problem (IRP) integrates inventory management and vehicle routing decisions over several periods and has received increased attention in recent years (Aghezzaf, 2007, Bertazzi et al., 2013, Yadollahi et al., 2017, Federgruen and Zipkin, 1984, Bell et al., 1983). Bell et al., (1983) are one of the first researchers who used VRP and inventory management together to deal with the case where only transportation costs are included, demand is stochastic, and customer inventory levels must be met. Demand stochasticity means that shortages may occur since the supplier only knows a probabilistic distribution of demand for the retailer. To avoid having stock-outs, a penalty is imposed whenever a retailer runs out of stock, and this penalty is usually paid with the unsatisfied demand (negative inventory). Unsatisfied demand is either considered as lost-sale or backlogged. More explanation about IRP and SPIRP can be found in (Coelho et al., 2014a, Coelho et al., 2014b).

Variability of service, uncertainty in demand, and delay are the well-known characteristics of SPIRP. The trade-off between costs (transportation and inventory) and products' availability makes SPIRP a hard problem to solve. Even though there is a noticeable body of literature about IRP and SPIRP, only few studies have involved capacity limitation as constraints. Stacey et al., (2007) are one of the pioneers in specifying the significance of storage capacity on both the routing and inventory decisions in the context of inbound transportation. They have evaluated the benefits of applying storage constraints at different levels by developing two new heuristics that sequentially take into account the inventory level and routing decisions.

Pujawan et al., (2015) have proposed a new method to integrate operational and strategic decision parameters, namely shipment planning and storage capacity decision under uncertainty. Their objective is to provide a close to optimal solution to find the best balance for logistics cost and product availability. The authors develop a simulation model to investigate the effects of various indicators on costs and service levels in a distribution system. The model mimics the transportation and distribution problems of bulk cement, consisting of a silo at the port of origin, two silos at two ports of destination, and a number of ships that transport the bulk cement. The outcome of their model clarifies the significant effect of the number of ships deployed, silo capacity,

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working hours of ports, and the dispatching rules of ships on both total logistics costs and service level.

Finding the appropriate storage capacity is one of the main objectives of SPIRP that desires more investigation. The comparison between small and big storage capacity can be assessed from several different aspects such as costs, service level, silo availability, product's perishability, etc. In addition, different options for storage capacity at the retailers with different costs makes it more challenging for the supply chain decision maker to find the most optimum solution.

The idea of having the capacity optimized is a new concept in SPIRP and has not been treated completely in the literature. In this paper by involving storage capacity constraints into SPIRP, we develop the solutions to deal with stochasticity in demand rates and costs minimizations while service level is satisfied. Four different policies for storage capacity allocation are considered in this paper. The strategies are evaluated and compared by implementing them on an illustrative example based on two indicators namely costs and computation time. The outcome of these solutions are discussed in details for the short and long term planning horizon in order to have a better insight of their influence on the whole system.

The rest of the paper is organized as follows; section 2 presents the Safety Stock-based SPIRP together with the different approaches for the capacity optimization. In section 3, we explain all the approaches and discuss the advantages and dis-advantages.

2 SAFETY STOCK-BASED SPIRP MODEL WITH STORAGE CAPACITY LIMITATION POLICIES

The inventory routing system studied in this paper consists of a single depot and a set of geographically scattered retailers. The retailers are indexed by j, $j \in \{1, 2, .., m\}$ (where m is the total number of retailers) and the depot is indexed by r. Each retailer j has a stochastic independent demand rate of d_{jt} per unit of time, that is assumed to be approximately based on Gamma distribution $\Gamma(\alpha, \beta)$. Let S be the set of retailers indexed by i and j; and $S^+ = S \cup \{r\}$.

Let $H = \{1, 2, .., T\}$ be the planning horizon indexed by t, and $H^+ = H \cup \{0\}$ be the planning horizon that includes period $t = 0$. Let τ_t be the size in time units of each period t, for example eight working hours per day. For the deliveries, a fleet of vehicles V, $v \in \{1,2, .., k\}$ each with a capacity of κ is available. The supplier and each retailer j agree to a service level (SL_j) based on a predetermined inventory violation rate of θ_j during each period and retailer, and $SL_j = (1 - \theta_j)$. Let $SG = \{1,2, \dots, G\}$ be the number of available silos for each retailer j.

Additional parameters of the model are as follows:

ϕ_{jt} : the fixed handling cost (in euros) per delivery at location $j \in S^+$ (retailers and depot) in period $t \in H$.

h_{jt} : the per unit holding cost of the product at location $j \in S$ (in euros per ton) in period $t \in H$;

ψ_v: the fixed operating cost of vehicle $v \in V$ (in euros per vehicle per use);

δ_v: travel cost of vehicle $v \in V$ (in euros per km);

η_v: average speed of vehicle $v \in V$ (in km per hour);

Δ_{ij} : duration of a direct trip from retailer $i \in S^+$ to retailer $j \in S^+$ (in hours);

I_{j0}: the initial inventory levels at each retailer $j \in S$ in period zero;

CI_{jt}: The cost of using a silo for each retailer $j \in S$, in period $t \in H$

KI_{jg}: maximum capacity of each silo $g \in SG$, for retailer $j \in S$

The variables of the model are defined as follows:

Q_{vijt} : the quantity of product remaining in vehicle $v \in V$ when it travels directly to location $j \in S^+$ from location $i \in S^+$ in period $t \in H$. This quantity equals zero when the trip (i, j) is not on any tour of the route travelled by vehicle $v \in V$ in period t;

q_{jt} : the quantity delivered to location $j \in S$ in period $t \in H$;

I_{jt}: the inventory level at location $j \in S$ by the end of period $t \in H$;

x_{vijt}: a binary variable set to 1 if location $j \in S^+$ is visited immediately after location $i \in S^+$ by vehicle $v \in V$ in period $t \in H$, and 0 otherwise;

y_{vt}: a binary variable set to 1 if vehicle $v \in V$ is being used in period t, and 0 otherwise;

IS_{jtg}: a binary variable set to 1 if silo $g \in SG$ is being used for retailer $j \in S$ in period t, and 0 otherwise;

The minimization objective function is:

$$CV = \sum_{t\in H}\sum_{v\in V}\left[\psi_v y_{vt} + \sum_{i\in S^+}\sum_{j\in S^+}(\delta_v\,\eta_v\Delta_{ij} + \phi_{jt})x_{vijt}\right] + \sum_{t\in H}\sum_{j\in S} h_{jt}I_{jt} + \sum_{j\in S}\sum_{g\in SG}(KI_{jg}\,CI_{jt})IS_{jtg}\,T \tag{1}$$

Subject to:

$$\sum_{v\in V}\sum_{i\in S^+} x_{vijt} \leq 1, \qquad \forall\, j \in S,\, t \in H \tag{2}$$

$$\sum_{i\in S^+} x_{vijt} - \sum_{k\in S^+} x_{vjkt} = 0, \qquad \forall\, j \in S^+, t \in H, v \in V \tag{3}$$

$$\sum_{i\in S^+}\sum_{j\in S^+} \Delta_{ij}x_{vijt} \leq \tau_t, \qquad \forall\, t \in H, v \in V \tag{4}$$

$$\sum_{v\in V}\sum_{i\in S^+} Q_{vijt} - \sum_{v\in V}\sum_{k\in S^+} Q_{vjkt} = q_{jt}, \qquad \forall\, j \in S, t \in H \tag{5}$$

$$Q_{vijt} \leq \kappa\, x_{vijt}, \qquad \forall\, i \in S^+, j \in S^+, t \in H, v \in V \tag{6}$$

$$I_{j0} + \sum_{l=1}^{t} q_{jl} = \sum_{l=1}^{t} E(d_{jl}) + SS_{jt} + I_{jt}, \qquad \forall\, j \in S, t \in H^+ \tag{7}$$

$$I_{j0} \leq I_{jT}, \qquad \forall\, j \in S, t \in H \tag{8}$$

$$I_{jt} \leq \sum_{g\in SG} IS_{jtg}\, KI_{jg}, \qquad \forall\, j \in S, t \in H \tag{9}$$

$$x_{vrjt} \leq y_{vt}, \qquad \forall\, j \in S^+, t \in H, v \in V \tag{10}$$

$$x_{vijt}, y_{vt}, IS_{jtg} \in \{0,1\}, I_{jt} \geq 0, Q_{vijt} \geq 0, q_{jt} \geq 0, \qquad \forall\, j \in S^+, t \in H, v \in V$$

The objective function (1) shows the variables to minimize the level of costs in this replenishment system. It includes five cost components, namely, total fixed operating cost of using the vehicle(s), total transportation cost, total delivery handling cost, total inventory holding cost at the end of each period, and total cost of renting silos at the retailers.

Constraints (2) assure that each retailer is visited at most once during each period. Constraints (3) guarantee that a vehicle moves to the next retailer/depot after serving the current one. Constraints (4) prevent that the time required to complete each tour does not exceed the duration of the period. The quantities to be delivered to each retailer are determined by constraints (5). These constraints also avoid sub-tour from occurring. Constraints (6) are capacity constraints induced by the vehicles capacities. Constraints (7) determine the delivered number of products from period 1 to t together with the initial inventory to be equal to the expected demand's values from period 1 to t, safety stock, and remaining inventory at the end of period t for each retailer j. Constraints (8) insure that the level of inventory at the end of last period is equal or larger than initial inventory. Constraints (9) determine the optimum number of required silos for each retailer during each period. Finally, constraints (11) specify a vehicle cannot be assigned to serve retailers unless the related fixed cost is payed.

Eq. (11) presents the safety stock calculation model to be used in constraints (7).

As is specified by equation (11), safety stock is a function of service level parameter (z_{θ_j}), number of time periods (t), and standard deviation of demand (σ_{jt}) for each retailer (j). The parameter z_{θ_j} is the service factor determined by retailer's requested service level (SL_j%) gained by the level of θ_j. It is used as a multiplier with the standard deviation and number of time periods to calculate a specific quantity (as safety stock) to meet the pre-set service level.

$$SS_{jt} = z_{\theta_j}\sqrt{\sum_{l=1}^{t}\sigma_{jl}^2} \tag{11}$$

3 DIFFERENT APPROACHES FOR STORAGE CAPACITY ALLOCATION

We propose 4 different policies in this study. These policies are suggested based on the requirements in short/long term planning horizons and high variability in demand rates to evaluate their applicability in distribution systems. Different industries have different preferences in renting a silo. Therefore, presenting various strategies for silo allocation could help the decision maker to decide wisely. In the reminder four proposed policies for silo allocation are modelled and described.

3.1 Fixed Number of Silos

This is the basic policy that allocates a certain number of silos to the retailers during the whole planning horizon. Equations (1)-(10) formulate the Safety Stock-based SPIRP for this policy. Number of silos are fixed from period 1 to the last period. It means the maximum required silos need to be rented in the beginning of the planning horizon based on the expected level of inventory from the optimization model.

In some distribution centres where the variability of demand rates is high, and high level of service is promised to the customers, it is better to rent a certain number of silos for the whole planning horizon. Therefore, there is less risk of having limited space for the inventory during the planning horizon. The calculated number of silos is based on the maximum expected level of inventory, meaning there are some periods that some silos are not full, but the rent must be paid. The allocation of the silos to the retailers are based on the rental fee, and the trade-off between inventory/silo costs and transportation costs.

3.2 Fixed Cumulative

Fixed-cumulative approach optimizes the silo allocation mechanism, in order to use the maximum capacity of rented silos during the periods with low inventory level at the retailers. In other words, the cumulative level of inventory from the beginning to period t is taken into account instead of the level of inventory for period t. To have this strategy applied in the Safety Stock-based SPIRP, constraints (9) needs to be replaced by constraints (13). In constraints (13) the inventory level for all the periods from 1 to t need to be smaller or equal to storage capacity in one period multiplied by t. Retailers with higher variability in demand rates and/or long term planning horizon are more convenient to have this strategy for renting the silos, since for those retailer the risk of having excess inventory/demand in long term is compensated by other periods with lower demand rate.

3.3 Flexible Number of Silos

In this policy the retailers are allowed to have different number of silos for each period. It means the number of silos are different during the planning horizon, but 1the decision for each period is made only based on the inventory for that period. It makes the inventory costs as low as possible since there is no need to pay the rent when the silo is not used. Equation (12) involves this flexibility in the objective function by summing up the silo fee costs for each period. Therefore, the model selects the number of silos for each period differently based on the maximum inventory level on that period. Equations (2-10) and (12), present the Safety Stock-based SPIRP model with flexible storage capacity.

All these decisions are made before the planning horizon, therefore this mechanism may cause risks for the retailers in terms of stock-out occurrence. Generally, the retailers with lower coefficient of variation with short term planning horizon are more preferred to apply this policy.

3.4 Flexible Cumulative

This mechanism is similar to Fixed-cumulative, with this difference that in this policy the retailer does not

$$CV = \sum_{t\in H}\sum_{v\in V}\left[\psi_v y_{vt} + \sum_{i\in S^+}\sum_{j\in S^+}(\delta_v \eta_v \Delta_{ij} + \phi_{jt})x_{vijt}\right] + \sum_{t\in H}\sum_{j\in S} h_{jt} I_{jt} + \sum_{t\in H}\sum_{j\in S}\sum_{g\in SG}(KI_{jg}\, CI_{jt}) IS_{jtg} \tag{12}$$

$$\sum_{s=1}^{t} I_{js} \leq \sum_{g\in SG} IS_{jtg}\ KI_{jg}\, t, \qquad \forall\, j \in S, t \in H \tag{13}$$

need to keep a certain number of silos for the whole planning horizon. The idea is to be flexible in renting the silos as well as involving the variability in inventory level among the periods to minimize the costs. Equations (2-8), (10), (12), and (13) present the Safety Stock-based SPIRP model with flexible-cumulative approach for silo allocation.

4 ILLUSTRATIVE EXAMPLE

We consider a distribution centre with 8 retailers. There is a fleet of vehicles with 2 available vehicles, each one with the capacity of 40 tons. The vehicles work 8 hours per day with an average speed of 50 km/h. Fix and variable costs of the vehicles are presented in table 1. The retailers are scattered randomly around the warehouse. Distances between retailers themselves and warehouse are shown in table 3.

Table 1: Some elements.

Notation	Parameter	Cost
ϕ_{it}	Delivery costs	25
η_{jt}	Inventory holding costs per unit per period	0.5
δ_v	Travel costs for vehicle in Euro per KM	1
ψ_{vt}	Fix operating cost of vehicle	30
ν_v	Average speed of vehicle	50

The demand rate for each retailer is considered stochastic and follows Gamma distribution and all the stock-outs are fully backlogged. Table 2 presents the demand rates for 8 hours (1 period) and standard deviations as well as their coefficient of variations.

The rest of the parameters of this example are provided in table 1. We use CPLEX 12.5.1 for solving all models. All the computations are performed on a 3.60 GHz Intel® Xeon® CPU.

Table 2: Demand rate parameters per period.

Retailers	Average demand $E(d_{jt})$ (ton/day)	Standard deviation σ_{jt}(ton/day)	CV	(α)	(β)
1	1.507	1.228	0.81	1.507	1
2	0.979	0.989	1.01	0.979	1
3	0.498	0.706	1.41	0.498	1
4	3.455	1.859	0.53	3.455	1
5	11.596	3.405	0.29	11.59	1
6	0.497	0.705	1.41	0.497	1
7	3.278	1.811	0.55	3.278	1
8	5.747	2.397	0.41	5.747	1

5 RESULTS AND DISCUSSION

The two indicators considered in this study are cost level and computation time. Both indicators have been measured and evaluated for the defined policies in this example to clarify the differences. Figure 1 shows the expected costs for each policy during the whole planning horizon. As mentioned in equations (1) and (2), these costs are fixed and variable costs of transportations, silos, and inventory. Figure 11 clearly indicates the low level of cost for flexible cumulative strategy while fixed strategy is the highest. Flexible cumulative strategy has saved 40% of the expected costs in this model, while flexible strategy reduces the costs by almost 30%. The cumulative approach shows a big improvement compared to periodic approach, by allocating the silos and trucks properly as well as minimizing the inventory level at the retailers among the periods.

Table 3: Duration of a trip from retailer $i \in S^+$to retailer $j \in S^+$ (in hour).

	warehouse	c1	c2	c3	c4	c5	c6	c7	c8
warehouse	0	0.5001	0.9621	0.5155	0.3106	0.4601	0.5536	0.986	0.8472
c1	0.5001	0	1.3152	0.676	0.7533	0.2581	0.9052	1.4541	1.3465
c2	0.9621	1.3152	0	1.436	0.6586	1.0973	1.3068	1.4446	0.7826
c3	0.5155	0.676	1.436	0	0.7823	0.82	0.3225	0.9552	1.0698
c4	0.3106	0.7533	0.6586	0.7823	0	0.6161	0.7071	0.9266	0.644
c5	0.4601	0.2581	1.0973	0.82	0.6161	0	0.9775	1.4446	1.2525
c6	0.5536	0.9052	1.3068	0.3225	0.7071	0.9775	0	0.6329	0.7992
c7	0.986	1.4541	1.4446	0.9552	0.9266	1.4446	0.6329	0	0.4826
c8	0.8472	1.3465	0.7826	1.0698	0.644	1.2525	0.7992	0.4826	0

We also consider computation time for each policy in order to verify the applicability of the strategy, particularly for larger models. Table 4 presents the computation time per policy for the whole distribution system. Fixed and flexible silo allocation models need the minimum time among the other strategies, while when the model is cumulative in storage capacity allocation, the required time becomes larger. Fixed cumulative approach needs 87 seconds to achieve the optimized solution, while it is even more with Flexible Cumulative approach with 106 seconds. Higher computational time specifies the model complexity level and computation difficulty that results in lower interest to apply the complex solutions for large systems.

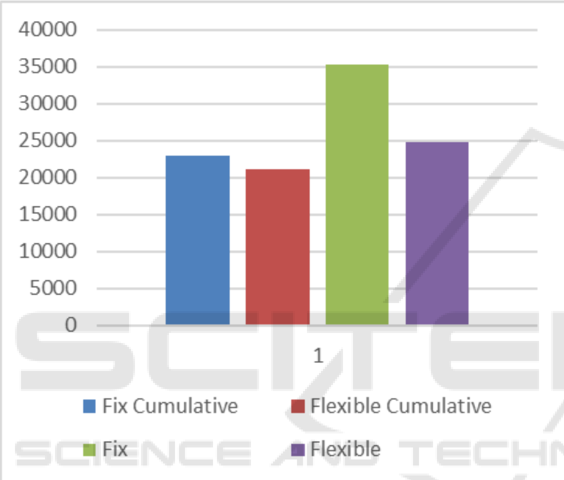

Figure 1: Overall costs for each policy.

Table 4: Computation time per policy.

Policies	Time (seconds)
Fixed	20
Fixed cumulative	87
Flexible	23
Flexible Cumulative	106

According to the results of the illustrative example, flexible approach has got the most reasonable results for both computation time and cost reduction. But if the model is small in size, the fix cumulative approach seems more reasonable, since it is more logical to rent a silo for the whole planning horizon.

6 CONCLUSIONS

In this paper we considered Stochastic Periodic Inventory Routing Problem with storage capacity limitation. The proposed safety stock-based SPIRP model involved storage capacity as a constraint in the model to optimize it with regard to cost minimization. Four different policies are proposed to deal with storage capacity limitation at retailers. The advantages and disadvantages of these approaches have been discussed in this paper. Finding the balance between transportation and inventory costs together with the storage costs (silo rent) is the most important factor in SPIRP model. Definitely it depends on the value of product itself, silo fee, promised service level, demand variability rate at the retailers, length of the planning horizon, etc., to allocate silos to the retailers. The illustrative example presented in this paper has revealed the advantages of flexible model among other policies. In addition for smaller distribution centres, fixed cumulative approach seems to be an appropriate strategy to optimize the storage capacity. As for future research, the applicability of these approaches will be evaluated in some experimental cases with design of various experiments based on the variables. In addition, their impact on service level, inventory and transportation costs, and computational time will be measured and discussed.

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