# A New Procedure of Two Stage Data Envelopment Analysis Model under Strict Positivity Restriction

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Abstract: Data Envelopment Analysis (DEA) is a mathematical non-parametric approach for measuring relative efficiency of homogenous decision making units (DMUs) performing. This approach will evaluates the efficiency score of entities. The efficiency is defined as the maximum of the ratio of the sum of its weight output to the sum of its weight inputs. The objective value is subject to the conditions that are corresponding to ratios for each DMU be less than or equal to one. Strict positivity of the weights in the theoretical and the computational result is an important condition to identify whether the $DMU_s$ is efficient or not. One method that can be used to achieve this condition was considering a positive lower bound on its weights, known as a non-Archimedean infinitesimal, $\varepsilon$. In fact, it is very hard to find a set of positive weights among all the alternative solutions of multiplier model. This paper show that a new procedure two-stage approach can solve the decision-making problems that are modelled on the DEA-CCR model under strict positivity restriction.



## 1 INTRODUCTION

In determining the performance of an organization and increasing productivity, the efficiency level must be measured. In general, efficiency is expressed in the form of a comparison between input (input) and output (output). But in a company there may be different input and output entities, in aspects of resources, activities, environmental factors. So in general measurement of efficiency is difficult to use. So to be able to measure the level of efficiency with different input and output entities can be done using Data Envelopment Analysis (DEA) (Charnes et al., 1978).

Charnes et al (1979) proposed the model as a fractional programming problem. After that, the model was transformed as a simple linear programming problem with a objective function and some criteria. DEA's main objective is to determine efficient conditions based on existing problem scenarios. In this case the efficiency can be interpreted as the maximum ratio of the weighted output to the weighted input with the constraints corresponding to each DMU.

Based on the basic concept of the CCR model found by Charnes et al., (1978), known as the DEA CCR, that the unit shows performance the best is with one efficiency score. This shows that the score it is part of the production boundary that cannot be compared to the boundary area. Further techniques that combine principles the basic DEA is known as "Super Efficiency Analysis" introduced by Andersen and Peterson (1993).

In his paper, Thompson et al (1993) discussed several ways to eliminate zero weight in the DEA problem. Various methods have been carried out, including modifying the DEA model as carried out by Charnes et al (1997). In his paper, Charnes et al (1979) added a positivity requirement, using the parameter $\varepsilon$. This method is the right way to do it, but this method has complex limitations and complexities because we don't know the right value for $\varepsilon$. By this situation, Yao (2003) and Amin & Toloo (2004) conducted related research and found the right number for $\varepsilon$.

Cooper et al (2001) in his paper discuss about a method that solved zero weight problem in DEA. proposed two-stage method. This procedure is for selecting non zero weights from the alternative optimal solution of the multiplier model in a DEA. Saen (2010) said that it is very hard to find a set of positive weight among all the alternative solutions of multiplier models.



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## 2 BASIC DEA-CCR MODEL

In this study the author uses the DEA-CCR model as the basis for the model that will later be developed. The basic DEA-CCR model in (1) is formed for evaluating the efficiency of $DMU_s$ (Charnes et al, 1978). Suppose there are n $DMU_s$, $DMU_j$, $(j = 1,2,\dots,n)$, that will be evaluate the efficiency values. Each of DMU consumes the amounts $x_j = [x_{ij}]$of $m$ inputs $(i = 1,2,\dots,m)$, and will produce the amounts $y_j = [y_{rj}]$ of $s$ outputs $(r = 1,2,\dots,s)$ $x_j \geq 0_m, x_j \neq 0_m, y_j \geq 0_s, y_j \neq 0_s$. Then the DEA-CCR model is defined as follows

$$\max \theta = \sum_{r=0}^{s} u_r\, y_{ro}$$

$$\begin{aligned} s.t.\ & \sum_{i=1}^{m} v_i\, x_{io} = 1 && (1) \\ & \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=i}^{m} v_i\, x_{ij} \leq 0 \quad j = 1,2,\dots,n \\ & v_i \geq 0 \qquad i = 1,2,\dots,m \\ & u_r \geq 0 \qquad r = 1,2,\dots,s \end{aligned}$$

Where $x_j = [x_{ij}]$ and $y_j = [x_{rj}]$ are inputs and output respectively. Meanwhile the weights of $i$-th input and $r$-th output are indicated by $v_i$ and $u_r$ respectively.

Completion of the model (1) will get the optimal value for multipliers. Therefore, the model (1) is often referred to as the multipliers form of the CCR problem.

## 3 AN IMPROVED DEA-CCR MODEL

The issue of strict positivity is important in the DEA. Although there are many alternative optimal solutions, it is still difficult to determine the level of efficiency of each DMU. Therefore, Charnes et al (1979) modifies the model (1) as follows:

$$\max \theta = \sum_{r=0}^{s} u_r\, y_{ro}$$

$$\begin{aligned} s.t.\ & \sum_{i=1}^{m} v_i\, x_{io} = 1 && (2) \\ & \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=i}^{m} v_i\, x_{ij} \leq 0 \quad j = 1,2,\dots,n \\ & v_i \geq \varepsilon \qquad i = 1,2,\dots,m \\ & u_r \geq \varepsilon \qquad r = 1,2,\dots,s \end{aligned}$$

where $\varepsilon > 0$ is an infinitecimal element that smaller than any positive real number.

## 4 AN IMPROVED FORMULA OF TWO STAGE DEA

The first step we must take to develop the DEA-CCR model is to complete the model (1) in the first stage. If the value $\theta_o^* < 1$, then $DMU_o$ is said to be CCR-inefficient. If the model (1) has obtained its efficiency value, then the next step is to solving the following model (3) in the second stage.

$$\begin{aligned} \max\quad & \delta \\ s.t.\quad & \sum_{i=1}^{m} v_i\, x_{io} = 1 && (3) \\ & \sum_{r=1}^{s} u_r\, y_{ro} - \sum_{i=i}^{m} v_i\, x_{io} = 0 \\ & \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=i}^{m} v_i\, x_{ij} \leq 0 \qquad j \neq o \\ & v_i - \delta \geq 0 \qquad \forall i \\ & u_r - \delta \geq 0 \qquad \forall r \\ & v_i, u_r, \delta \geq 0 \qquad \forall i, r \end{aligned}$$

After solving the model (3) in the second stage, the next step is to check the optimal solution. If the value of $\delta^* > 0$, then we get $(u^*, v^*) > 0_{s+m}$. If that so, the $DMU_o$ is called to be efficient.

In model (1) and (3), we replace the constrain $\sum_{i=1}^{m} v_i x_{io} = 1$ with $\sum_{i=1}^{m} v_i x_{io} = K$, $K$ is an arbitrary nonnegative number to improve the recent procedure of two stage DEA. Therefore, we rewrite the models (1) and (3) respectively as follows:

$$\max \Theta_o = \sum_{r=0}^{s} u_r\, y_{ro}$$

$$\begin{aligned} s.t.\ & \sum_{i=1}^{m} v_i\, x_{io} = K && (4) \\ & \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=i}^{m} v_i\, x_{ij} \leq 0 \quad j = 1,2,\dots,n \\ & v_i \geq 0 \qquad i = 1,2,\dots,m \\ & u_r \geq 0 \qquad r = 1,2,\dots,s \end{aligned}$$

$$\begin{aligned} \max \Delta \\ s.t.\quad & \sum_{i=1}^{m} v_i\, x_{io} = 1 && (5) \\ & \sum_{r=1}^{s} U_r\, y_{ro} - \sum_{i=i}^{m} V_i\, x_{io} = 0 \end{aligned}$$

$$\sum_{r=1}^{s} U_r\, y_{rj} - \sum_{i=i}^{m} V_i\, x_{ij} \leq 0 \qquad j \neq o$$
$$v_i - \Delta \geq 0 \qquad \forall i$$
$$u_r - \Delta \geq 0 \qquad \forall r$$
$$V_i, U_r, \delta \geq 0 \qquad \forall i, r$$

In this paper a simple example will be given to seeing the proposed DEA model application. In addition, there will also be case example from bank performance. As a tool, we use LINDO for solving and making analysis of the models.

## 5 NUMERICAL EXAMPLE

As a simple numerical example, we evaluate 7 DMU with two inputs and two outputs as shown in Table 1. First, applying stage I to evaluate each DMUs. We have four DMU which efficiency score is 1 as shown in Table 2.

Table 1: Input and output of 7 bank

| DMU | $Input_1$ | $Input_2$ | $Output_1$ | $Output_2$ |
|---|---|---|---|---|
| $DMU_1$ | 19 | 3 | 5 | 4 |
| $DMU_2$ | 6 | 4 | 2 | 4 |
| $DMU_3$ | 7 | 2 | 2 | 2 |
| $DMU_4$ | 3 | 3 | 5 | 5 |
| $DMU_5$ | 1 | 5 | 3 | 3 |
| $DMU_6$ | 9 | 2 | 2 | 6 |
| $DMU_7$ | 3 | 4 | 2 | 3 |

We evaluate the data on Table 1 using LINDO. By means of $\delta^*$ of the $DMU_4$ and $DMU_6$ is greater than zero, it means that both of them are efficient.

Table 2: The result of numerical example using LINDO

| DMU | Stage I | | | | | | Stage II | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\theta_o^*$ | $v_1^*$ | $v_2^*$ | $u_1^*$ | $u_2^*$ | | $\delta^*$ | $v_1^*$ | $v_2^*$ | $u_1^*$ | $u_2^*$ |
| $DMU_1$ | 1.0000 | 0.0000 | 0.5000 | 0.2500 | 0.0000 | | 0.0000 | 0.0000 | 0.5000 | 0.2500 | 0.0000 |
| $DMU_2$ | 0.4710 | 0.0588 | 0.1961 | 0.0000 | 0.1569 | | | | | | |
| $DMU_3$ | 0.5000 | 0.0000 | 1.0000 | 0.3750 | 0.1250 | | | | | | |
| $DMU_4$ | 1.0000 | 0.2500 | 0.0000 | 0.2500 | 0.0000 | | 0.1250 | 0.1875 | 0.1250 | 0.1250 | 0.1250 |
| $DMU_5$ | 1.0000 | 0.5000 | 0.0000 | 0.0000 | 0.5000 | | 0.0000 | 0.5000 | 0.0000 | 0.5000 | 0.0000 |
| $DMU_6$ | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 0.2000 | | 0.0625 | 0.2500 | 0.3750 | 0.6250 | 0.1875 |
| $DMU_7$ | 0.5000 | 0.2500 | 0.0000 | 0.0000 | 0.2500 | | | | | | |

Table 2 shows the results of stages I and II obtained using LINDO. From Table 2 it can be seen that the efficient DMUs are $DMU_4$ and $DMU_6$. This is due to the optimal value of $DMU_4$ and $DMU_6$ which are $\delta^* = 0.1250$ ; $\delta^* = 0.0625$.

As another example, data from 50 banks was provided. There are three inputs and 3 outputs. This problem is solved by an improved two-stage DEA. The optimal values of the first stage and the objective function of the second stage are showed by the last two columns of Table 3. There are eight $DMU_s$ whose optimal value $\delta^* > 0$.

Table 3: Input and output of the 50 $DMU_s$

| $DMU_s$ | Inputs | | | | Outputs | | | | Stage 1 | | Stage II |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Empl. | Cost | Debt. | | Deposits | Income | Loan | | $\theta_o^*$ | | $\delta^*$ |
| $DMU_1$ | 32 | 161 | 446,869 | | 551,768 | 2,068 | 1,209,876 | | 1.0000 | | 0.000005448 |
| $DMU_2$ | 19 | 2,026 | 22,345 | | 87,365 | 2,848 | 103,573 | | 1.0000 | | 0.000017493 |
| $DMU_3$ | 14 | 1,456 | 12,830 | | 50,206 | 2,755 | 208,456 | | 0.8943 | | |
| $DMU_4$ | 5 | 4,566 | 145 | | 77,436 | 1,554 | 12,789 | | 1.0000 | | 0.000096789 |
| $DMU_5$ | 18 | 1,324 | 21,567 | | 24,794 | 1,638 | 45,790 | | 0.6360 | | |
| $DMU_6$ | 18 | 1,562 | 25,689 | | 25,894 | 1,448 | 44,567 | | 0.7862 | | |
| $DMU_7$ | 16 | 1,468 | 54,243 | | 95,804 | 1,578 | 80,942 | | 0.6453 | | |
| $DMU_8$ | 17 | 1,884 | 39,453 | | 25,266 | 1,895 | 35,790 | | 0.8543 | | |
| $DMU_9$ | 9 | 1,636 | 12,456 | | 28,885 | 1,572 | 55,782 | | 1.0000 | | 0.000036918 |
| $DMU_{10}$ | 13 | 1,993 | 7,623 | | 34,226 | 1,277 | 209,765 | | 0.8764 | | |
| $DMU_{11}$ | 8 | 1,934 | 34,562 | | 87,990 | 1,445 | 45,674 | | 1.0000 | | 0.000032445 |
| $DMU_{12}$ | 11 | 1,279 | 2,487 | | 77,567 | 2,051 | 77,833 | | 0.4325 | | |
| $DMU_{13}$ | 17 | 2,426 | 11,453 | | 45,698 | 2,745 | 50,975 | | 0.5543 | | |
| $DMU_{14}$ | 14 | 1,236 | 10,934 | | 78,965 | 2,774 | 120,987 | | 0.5547 | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $DMU_{15}$ | 14 | 2,011 | 22,176 | | 88,784 | 2,341 | 35,678 | | 0.5722 | | |
| $DMU_{16}$ | 7 | 2,894 | 26,832 | | 33,489 | 1,090 | 58,542 | | 0.3974 | | |
| $DMU_{17}$ | 12 | 1,500 | 8,643 | | 56,779 | 1,462 | 556,709 | | 0.3894 | | |
| $DMU_{18}$ | 9 | 1,475 | 3,411 | | 69,055 | 1,572 | 450,097 | | 0.4490 | | |
| $DMU_{19}$ | 5 | 1,290 | 1,421 | | 67,784 | 1,635 | 169,005 | | 0.5768 | | |
| $DMU_{20}$ | 6 | 2,094 | 3,744 | | 92,675 | 1,725 | 33,789 | | 0.5947 | | |
| $DMU_{21}$ | 6 | 2,068 | 5,321 | | 38,000 | 1,613 | 87,734 | | 0.3462 | | |
| $DMU_{22}$ | 8 | 2,848 | 31,589 | | 65,470 | 2,025 | 56,733 | | 0.5231 | | |
| $DMU_{23}$ | 9 | 2,755 | 4,215 | | 34,226 | 1,486 | 34,098 | | 0.5279 | | |
| $DMU_{24}$ | 8 | 1,554 | 65,782 | | 87,990 | 3,566 | 66,990 | | 0.4469 | | |
| $DMU_{25}$ | 7 | 1,638 | 20,021 | | 77,567 | 2,324 | 59,032 | | 0.5103 | | |
| $DMU_{26}$ | 9 | 1,448 | 25,072 | | 95,804 | 4,572 | 133,456 | | 0.3974 | | |
| $DMU_{27}$ | 7 | 1,578 | 14,081 | | 25,266 | 1,498 | 12,500 | | 0.5478 | | |
| $DMU_{28}$ | 7 | 1,895 | 16,702 | | 28,885 | 1,874 | 31,567 | | 0.4580 | | |
| $DMU_{29}$ | 7 | 1,572 | 6,574 | | 34,226 | 1,536 | 51,578 | | 0.4356 | | |
| $DMU_{30}$ | 6 | 1,277 | 5,432 | | 87,990 | 1,984 | 76,890 | | 0.5569 | | |
| $DMU_{31}$ | 7 | 1,445 | 7,331 | | 77,567 | 1,935 | 34,590 | | 0.5021 | | 0.000034526 |
| $DMU_{32}$ | 7 | 2,051 | 2,361 | | 45,698 | 1,289 | 98,004 | | 0.4592 | | |
| $DMU_{33}$ | 8 | 2,745 | 2,093 | | 78,965 | 2,426 | 95,709 | | 0.3678 | | |
| $DMU_{34}$ | 9 | 2,774 | 2,100 | | 88,784 | 1,236 | 39,056 | | 0.5946 | | 0.000033468 |
| $DMU_{35}$ | 5 | 2,341 | 1,946 | | 33,489 | 2,011 | 34,781 | | 0.5793 | | |
| $DMU_{36}$ | 7 | 1,090 | 1,421 | | 56,779 | 2,894 | 72,890 | | 0.5198 | | |
| $DMU_{37}$ | 8 | 1,462 | 3,744 | | 37,586 | 1,500 | 39,357 | | 0.5356 | | 0.000005549 |
| $DMU_{38}$ | 6 | 1,572 | 5,321 | | 77,895 | 1,475 | 55,490 | | 0.5782 | | |
| $DMU_{39}$ | 5 | 1,635 | 31,589 | | 76,880 | 1,290 | 33,789 | | 0.5583 | | |
| $DMU_{40}$ | 9 | 1,725 | 4,215 | | 34,556 | 2,094 | 87,734 | | 0.5932 | | |
| $DMU_{41}$ | 5 | 1,545 | 65,782 | | 67,032 | 1,678 | 56,733 | | 0.5435 | | |
| $DMU_{42}$ | 6 | 1,792 | 25,689 | | 87,004 | 1,568 | 65,470 | | 0.5321 | | |
| $DMU_{43}$ | 6 | 1,227 | 54,243 | | 79,034 | 3,899 | 34,226 | | 0.5271 | | |
| $DMU_{44}$ | 7 | 1,967 | 39,453 | | 66,503 | 1,257 | 87,990 | | 0.4367 | | |
| $DMU_{45}$ | 5 | 1,215 | 12,456 | | 80,933 | 1,065 | 77,567 | | 0.3561 | | |
| $DMU_{46}$ | 5 | 1,157 | 7,623 | | 79,335 | 1,803 | 95,804 | | 0.5519 | | |
| $DMU_{47}$ | 6 | 1,592 | 34,562 | | 44,897 | 2,560 | 56,903 | | 1.0000 | | |
| $DMU_{48}$ | 5 | 1,278 | 2,487 | | 76,449 | 1,774 | 103,466 | | 0.4706 | | |
| $DMU_{49}$ | 6 | 1,373 | 11,453 | | 77,803 | 1,356 | 12,890 | | 0.5335 | | |
| $DMU_{50}$ | 4 | 1,298 | 10,934 | | 69,067 | 2,508 | 33,390 | | 1.0000 | | |

Table 4: The strictly positive weights of the efficient $DMU_s$

| $DMU_s$ | Inputs | | | | Outputs | | | | Stage II |
|---|---|---|---|---|---|---|---|---|---|
| | $v_1^*$ | $v_2^*$ | $v_3^*$ | | $u_1^*$ | $u_2^*$ | $u_3^*$ | | $\delta^*$ |
| $DMU_1$ | 0.00000544 | 0.00007144 | 0.00002824 | | 0.0000544 | 0.0000544 | 0.0000544 | | 0.0000544 |
| $DMU_2$ | 0.00001749 | 0.00037749 | 0.00001049 | | 0.0001749 | 0.0003669 | 0.0001749 | | 0.0001749 |
| $DMU_4$ | 0.00009678 | 0.00009678 | 0.00078578 | | 0.0002578 | 0.0009678 | 0.0009678 | | 0.0009678 |
| $DMU_9$ | 0.00003691 | 0.00003691 | 0.00014991 | | 0.0003691 | 0.0003691 | 0.0003691 | | 0.0003691 |
| $DMU_{11}$ | 0.00003244 | 0.00003244 | 0.00093744 | | 0.0003244 | 0.0003244 | 0.0003244 | | 0.0003244 |
| $DMU_{31}$ | 0.00003452 | 0.00003452 | 0.00067452 | | 0.0003452 | 0.0003452 | 0.0003452 | | 0.0003452 |
| $DMU_{34}$ | 0.000033468 | 0.004733468 | 0.000031569 | | 0.00033468 | 0.00033468 | 0.00033468 | | 0.0003346 |
| $DMU_{37}$ | 0.000005549 | 0.000391549 | 0.000101549 | | 0.00005549 | 0.00005549 | 0.00005549 | | 0.0000554 |

DMU with optimal value will then determine its strictly positive weight value as shown in table 4. From table 3 it can be seen that there are 8 DMUs that have optimal values in stage 2, they are $DMU_1$, $DMU_2$, $DMU_4$, $DMU_9$, $DMU_{11}$, $DMU_{31}$, $DMU_{34}$ and $DMU_{37}$. The eight efficient DMUs will then be determined its strictly positive weight as shown by Table 4.

## 6 CONCLUSIONS

In this paper, in order to achieving strictly positive of multipliers, we have to eliminate the role of non-Archimedean (ε), in the DEA models. The model used in this study is the multiplier form of the DEA-CCR model. By considering that all weights on its constraints are non-negative number.

In the first stage, we solved a new CCR model to specifying the CCR-efficient DMUs using LINDO. At this stage we get an efficient DMUs. In the second stage we will evaluate the efficient DMU that we get in the first stage to get the strictly positive value for their inputs and outputs.

On the other hand, from the computational test result using LINDO, we have to pay attention to gain the accuracy of computations result. This method is able to provide better efficiency results for cases of positive strictly constraints. This will help decision makers in making decisions on issues with scenarios that correspond to the proposed model.

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