Computing Path Bundles in Bipartite Networks

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Keywords: Path Bundling, Optimization, Bipartite Networks.

Abstract: Path bundling, a class of path planning problem, consists of compounding multiple routes to minimize a global distance metric. Naturally, a tree-like structure is obtained as a result wherein roots play the role of coordinating the joint transport of information, goods, and people. In this paper we tackle the path bundling problem in bipartite networks by using gradient-free optimization and a convex representation. Then, by using 7,500 computational experiments in diverse scenarios with and without obstacles, implying 7.5 billion shortest path computations, show the feasibility and efficiency of the mesh adaptive search.

1 INTRODUCTION

Path bundling is the problem which consists on compounding multiple paths and finding anchoring points at intermediate joints in order to minimize a global distance metric. Naturally, by using coordinate nodes, the aim of computing path bundles is to coordinate the transport and communication of goods, information and people. Path bundling is mainly relevant in scenarios where (1) the resources for transport are scarce, and (2) the environment is hard to navigate due to narrow space or limited navigability. Thus it becomes necessary to join single paths into compounded ones to ensure efficient transport/communication. For example consider the optimization of a the location of coordinating nodes in a ZigBee network (and its IoT applications), or consider the problem of building optimal wire harness of the electrical system of a vehicle (or any complex mechanical system), or consider the decentralized communication of multiagent robotic systems over a large area (where the location of the coordinating agents is to be optimized for efficient communication).

Research on path bundling has its origins in the well-known developments of the shortest-path problem: how to search for the shortest route path over polygonal domains? (Dijkstra, 1959; P.E. Hart, 1968; Kallmann, 2005). Here, the main goal is to find the most optimal path between single origin-destination pairs; and the widely-known algorithms are Dijsktra (Dijkstra, 1959) and A* (P.E. Hart, 1968), and their extensions are well-studied.

In practical domains, yet with a different scope, the research of path bundling has attracted the attention of the following fields: optimization of sensor and wireless networks (Falud, 2014; Wightman and Labardor, 2011; Torkestani, 2013; Panigrahi and Khilar, 2015; Szurley et al., 2015; Singh and Sharma, 2015; Parque et al., 2015), and network visualization (Cui et al., 2008; Selassie et al., 2011; Ersoy et al., 2011; Gansner et al., 2011; Holten and van Wijk, 2009; R. Osada and Dobki, 2002; Parque et al., 2014b). The closest developments to path bundling regard the edge bundling problem in network visualization. Here, the conventional works have focused on the geometry-based clustering of edges(Cui et al., 2008; Parque et al., 2014b), the force-based edge bundling where edges are able to attract to each other(Cui et al., 2008; Selassie et al., 2011), the clustering and attraction to the skeleton of adjacent edges (Ersoy et al., 2011), and the kd-tree based optimization of the centroid points of close edges(Gansner et al., 2011). The above solutions for route bundling render compounded networks which are aesthetically pleasing, topologically compact and locally optimal. However, the study of route bundling under global optimization, in the sense of minimizing a global distance metric, has been elusive.

In this paper, in order to fill the above gap, we further advance our previous work(Parque et al., 2017) by designing globally optimal path bundles through sample-based global optimization algorithms over a convex representation of polygonal domains and bipartite networks. The unique point of our approach

Parque, V., Miura, S. and Miyashita, T.
Computing Path Bundles in Bipartite Networks.
DOI: 10.5220/0006480604220427
In *Proceedings of the 7th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2017)*, pages 422-427
ISBN: 978-989-758-265-3

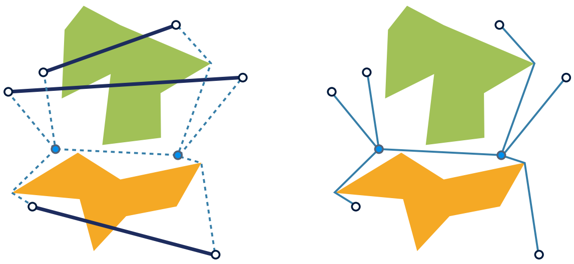


Figure 1: Basic idea of path bundling. Given a polygonal map and edges representing desirable origin-destination pairs, the goal is to find optimal anchoring points minimizing the global distance metric of the bundled path.

is to explicitly avoid the computations of point inside polygons while sampling for globally optimal path bundles in both convex and non-convex polygonal domains. Computational experiments in scenarios with a relevant set of polygonal domains and relevant global optimization algorithms show the feasibility and efficiency of our approach.

2 COMPUTING PATH BUNDLES

2.1 Basic Idea

The basic concept of path bundling is depicted by Fig. 1 wherein the output is a tree structure with compounded paths avoiding obstacle collision, wherein roots of the tree denote coordinating points to their leaves. A-priori knowledge of the following is necessary: (1) a bipartite graph $G = (V,E)$ wherein every edge $e \in E$ represents the communication/transportation needs between origin-destination pairs, and (2) obstacle geometry which denote unfeasible areas for navigation/transportation.

2.2 Representation of Bundled Paths

In our approach, any *feasible* point is represented by the 3-element tuple:

$$P = (i, r_1, r_2) \quad (1)$$

where $i \in [n]$ and $r_1, r_2 \in [0,1]$. In the above encoding, i is the index of i-th triangle $t_i \in T$ of the triangulation of the free-space, and r_1, r_2 are real numbers in the interval $[0,1]$. The unique feature of the above *convex* representation lies in the ability to encode arbitrary points which guarantee to be inside the navigable space by using the tuple (i, r_1, r_2) for $i \in [n]$, for $P \in \mathbb{N}^{[n]} \times \mathbb{R}^{[0,1]} \times \mathbb{R}^{[0,1]}$. Furthermore, the equivalent 2-dimensional cartesian coordinates can be computed as follows (R. Osada and Dobki, 2002):

$$(P_x, P_y) = (1-r_1)A_i + \sqrt{r_1}(1-r_2)B_i + \sqrt{r_1}r_2C_i \quad (2)$$

where A_i, B_i, C_i are the 2-dimensional coordinates of the vertices of the i-th triangle $t_i \in T$. Then, by using the encoding in Eq. 1, the bundled path can be represented by the 6-element tuple:

$$x = (i^P, r_1^P, r_2^P, i^Q, r_1^Q, r_2^Q) \quad (3)$$

where i^P, i^Q are natural numbers in the interval $[n]$, and $r_1^P, r_2^P, r_1^Q, r_2^Q \in [0,1]$. For simplicity and without loss of generality, we denote the search space $x \in \mathbf{T}$:

$$\mathbf{T} = \mathbb{N}^{[n]} \times \mathbb{R}^{[0,1]} \times \mathbb{R}^{[0,1]} \times \mathbb{N}^{[n]} \times \mathbb{R}^{[0,1]} \times \mathbb{R}^{[0,1]} \quad (4)$$

2.3 Optimization Problem

We solve the following equation:

$$\begin{aligned} \underset{x}{\textbf{Minimize}} \quad & F(x) \\ \text{subject to} \quad & x \in \mathbf{T} \end{aligned} \quad (5)$$

where, x is the encoding (representation) of the bundled path, $F(x)$ is the global distance metric to evaluate the quality of the bundled paths, and $\mathbf{T}$ is the search space of feasible bundled paths. The main rationale of the above is as follows: once the search space $x \in \mathbf{T}$ is constructed by the procedures described in the previous subsection, our goal is to find anchoring points P and Q which minimize a distance metric. For simplicity, we use the following function:

$$F(x) = \sum_{e \in E} d(e_o, P) + d(P,Q) + \sum_{e \in E} d(Q, e_d) \quad (6)$$

where, $d(a,b)$ is the Euclidean obstacle-free shortest distance metric between points a and b, e_o (e_d) is the coordinate of the *origin* (*destination*) node of the edge $e \in E$, and P and Q are anchoring points being closer to the *origin* e_o and *destination* e_d, respectively. The 2-dimensional coordinates of P and Q can be computed by combining Eq. (1)-(3). Solving Eq. 5 is realized by:

- DE: Differential Evolution with Successful Parent Selection/Best1(S-M Guo, 2015).
- NPSO: Particle Swarm Optimization with Niching Properties(B. Y. Qu and Suganthan, 2012).
- RBDE, Real-Binary Differential Evolution (RBDE)(Sutton et al., 2007).
- SHADE, Success History Parameter Adaptation for Differential Evolution(R. Tanabe, 2013).

- DIRECT, Direct Global Optimization Algorithm(Jones, 1999).

The main reason/motivation of using the above algorithmic set is to rigorously tackle path bundling problem by using a representative class of gradient-free optimization algorithms. The above algorithms are relevant in the literature due to the fact of considering multimodality, parameter adaptation, search memory, selection pressure, search over neighbourhood concepts, and mesh partitioning. Parameters for each algorithm are default and described in the references. Fine tuning the respective parameters is out of the scope of the paper.

3 COMPUTATIONAL EXPERIMENTS

In order to evaluate the performance of our approach, we used diverse polygonal domains with convex and non-convex obstacles, as well as different configurations of bipartite networks. This section describes our experimental conditions, results and insights obtained.

3.1 Experimental Settings

The computing environment used is Intel i7-4930K @ 3.4GHz with Windows 8.1, and computational experiments were performed using Matlab 2016a. In order to enable a meaningful evaluation of our proposed approach, we consider the following environmental settings:

- No. of edges in the input bipartite graph, $|E| = \{5, 10, 15, 20, 25\}$,
- Number of Polygonal Obstacles: $\{1, 2, 3, 4, 5\}$,
- No. Sides in each Polygonal Obstacle: $\{5, 10\}$.
- For each combination of the above, 30 independent experiments were performed to solve Eq. 5,
- For each independent experiment, the maximum number of functions evaluations is set as 10^4.
- In each independent experiment, the initial solutions of route bundles $x_o \in \mathbf{T}$ are initialized randomly and independently.

In order to show the kind of environments and bipartite networks used in our experiments, Fig. 2 shows the rendering of the polygonal domains and bipartite networks with obstacles, each of which has (a) 5 sides and (b) 10 sides. In this figure, we show a matrix-like configuration, where the x-axis denote the number of edges in the bipartite network, and the y-axis denote the number of obstacles in the environment. The configuration of edges in the bipartite network (origin and destination pairs) are arbitrarily generated to allow exhaustive evaluation of the optimization algorithms.

Furthermore, the main reason of using values of the number of edges $|E|$ up to 25 is due to our interest in evaluating the performance close to the number of transport needs in indoor environments, where the complexity of the environment is controlled by

- the number of obstacles in the polygonal map, and
- the number of sides for each obstacle.

In the above, complex polygonal environments induce in large number of triangles, thus representing a challenging search space for any search algorithm. Our future work aims at using configurations considering large scenarios and being close to outdoor environments.

The use of 30 independent runs in each experimental setting allows to evaluate the gradient-free optimization algorithms under arbitrary initialization conditions, thus avoiding random luckiness. Also, the key rationale of using 10^4 function evaluations as upper bound of computational budget is due to our interest of evaluating the effectiveness and efficiency of the heuristics under restrictive computational resources. Note that the use of function evaluations as a surrogate metric for efficiency is relevant to avoid bias in hardware or algorithmic implementation.

In line of the above, as a result, 7500 experimental conditions were evaluated[1], and 7.5×10^9 functions evaluations were computed[2], assuming a single optimization using population size $|\P| = 100$.

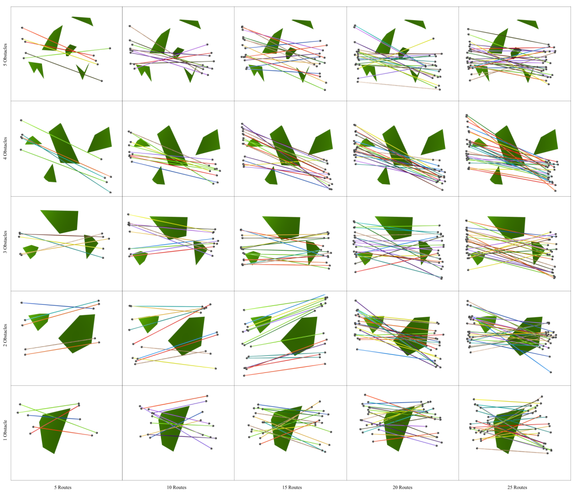
3.2 Results and Discussion

In order to show the kind of tree structures obtained, as well as to evaluate the efficiency in path bundling, Fig. 3 shows the optimized path bundles; and Fig. 4 - 8 show the convergence behaviour. Note that our results follows the same organization of Fig. 2, that is x-axis show the number of edges in the bipartite network, while y-axis show the number of obstacles in the environment.

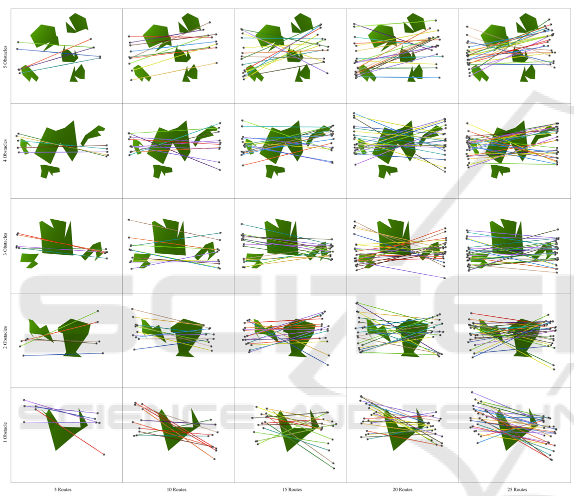
In regards to the obtained path bundles, by observing Fig. 3 we can confirm the following facts:

- Regardless of polygonal domain, location of origin-destination pairs in the bipartite network, and evaluated optimization algorithm, it is possible to generate tree structures representing the

[1] $5 \times 5 \times 5 \times 2 \times 30$
[2] $5 \times 1500 \times 10^4 \times |\P|$

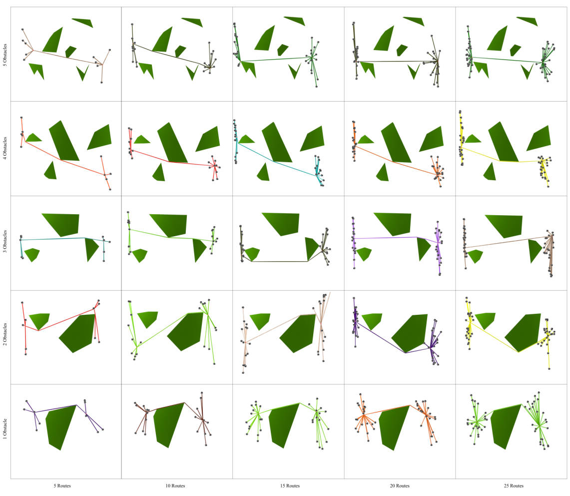


(a) 5 sides

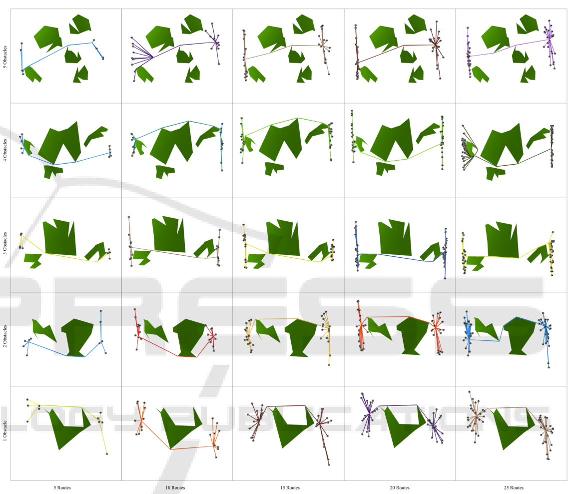


(b) 10 sides

Figure 2: Bipartite networks in a polygonal domain with obstacles of **(a) 5 sides and (b) 10 sides**.



(a) 5 sides



(b) 10 sides

Figure 3: Path Bundles in a polygonal domain with obstacles of **(a) 5 sides and (b) 10 sides**.

bundled paths which aim at minimizing the global distance metric.

- The location of the anchoring points of the bundled paths are close to, but not necessarily at, the center of the origin and destination pairs of the bipartite graph.

Regarding convergence behaviour, we confirmed the following facts (representative examples in Fig. 4 and Fig. 5):

- Regardless of polygonal domain, location of origin-destination pairs in the bipartite network, and evaluated optimization algorithm, it is possible to converge to the bundled paths minimizing the global distance metric within 2000 function evaluations.
- DIRECT is the most efficient algorithm achieving convergence to the path bundles with minimum global distance within 100 function evaluations, in 90% of the experimental cases. Out of 50 experimental cases, there exists 5 experimental cases wherein DIRECT achieves convergence in more than 2000 function evaluations.
- Over independent runs, all population-based algorithms show variance in the rate of convergence. This result is due to the fact of randomness in the initialization process and the sampling behaviour of the algorithms, whereas DIRECT is a deterministic algorithm using the DIviding RECTangles concept, which samples solutions vectors at the center of hypercubes, and then subdivides potentially optimal hypercubes recursively.
- Convergence to local optima in observed in all population-based algorithms, except RBDE. We believe this result is due to RBDE uses a select-

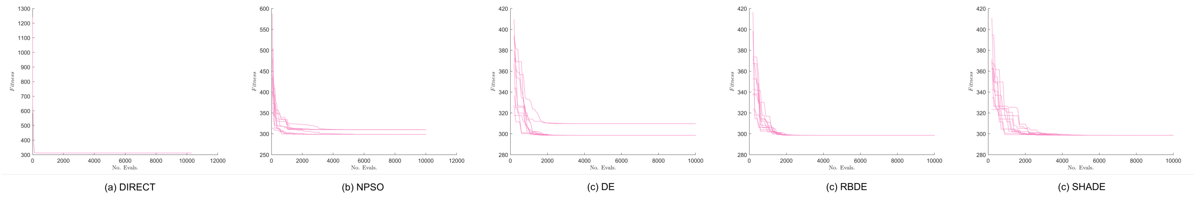


Figure 4: Convergence in polygonal domain with obstacles having **5 sides** and a bipartite network with **25 edges**.

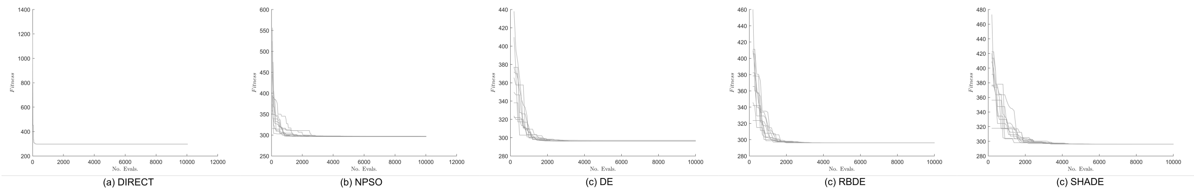


Figure 5: Convergence in polygonal domain with obstacles having **10 sides** and a bipartite network with **25 edges**.

ing mechanism which is more greedy compared to other population-based heuristics. Thus, due to the single-optima nature of the path bundling problem, RBDE focuses more on exploitation, rather than exploration of the search space.

- Increasing the number of edges in the bipartite network has a direct effect on increasing the distance metric by some small factor smaller than 1.

We believe the above observations has important implications to design effective algorithms that solve the path bundling problem effectively and efficiently. In line of the above, we provide the following propositions:

- Instead of using arbitrary initial solutions in the optimization algorithm, it may be possible to compute the initial solutions of x of path bundles which are close to the center/centroid of the origin and destination pairs of the bipartite networks,
- Whenever the number of edges in the bipartite network is expected to change (as a result of increasing/decreasing the number of agents and both origin-destination pairs), it may be possible to use pre-computed paths as initial solutions x of path bundles, since the new paths are expected to be structurally similar and close distance metric.
- Instead of sampling vectors close to potential solution vectors, it may be possible to sample at equally and locally distributed partition of the search space. Furthermore, a convex search space (as the one proposed in this paper), may be key for effective and efficient performance of partitioning the search space.

The above results imply the feasibility and efficiency to obtain optimal path bundles in polygonal maps with both convex and non-convex obstacles.

Further work remains on the agenda. Key limitations of our approach lie in our environments and networks: generalization to dynamic environments, non-bipartite networks, and networks having very large number of nodes is still unclear. Further computational experiments using large number of edges and diverse obstacle configurations reminiscent of outdoor environments are in our agenda.

4 CONCLUSION

In this paper, we have proposed an approach for designing optimal path bundles based on the idea of sampling over a convex search space to optimize a global distance metric. The unique point of our proposed approach is to compute feasible path bundles efficiently since the convex search space ensures the avoidance of overlapping (computation of point inside polygon is explicitly avoided) while sampling for optimal solutions.

Exhaustive computational experiments using a diverse and representative class of polygonal domains, bipartite networks and gradient-free optimization algorithms, show that (1) it is possible to obtain bundled paths with an optimized global distance metric via a reasonable number of sample evaluations (100 in the best case), and (2) the convergence is most efficient with the DIviding RECTangles concept.

We provided relevant insights to develop gradient-free algorithms for the bundling problem which regard (1) the use of initialization close to center/centroid of origin/destination pairs in the bipartite networks, (2) the use of pre-computed paths to approximate optimal bundles whenever the bipartite network varies, and the use of partitioning combined with a convex representation of the search space.

In future work, we aim at exploiting our insights in polygonal environments reminiscent of outdoor configurations. Also, we aim at exploring the generalization ability in dynamic and unknown environments. Furthermore, we aim at extending our approach to tackle the bundling of networks with different topologies, e.g. it may be possible to use the DIviding RECTangles concept with a number-based representation of undirected networks (Parque et al., 2014a) and directed networks (Parque and Miyashita, 2017), where the partition is realized in number-space (rather than a high-dimensional matrix-space).

We believe our approach opens new possibilities to develop compounded and global path planning algorithms via gradient-free sampled-based learning and convex representations of the search space.

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