Parametric Synthesis of a Robust Controller on a Base of Interval Characteristic Polynomial Coefficients

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Abstract: The paper is dedicated to deriving sufficient conditions, connecting root quality indexes of the control system with interval coefficients of its characteristic polynomial, on the base of interval expansion of the coefficient method. With the help of these conditions, a method of synthesizing a controller, providing an aperiodic transient process and acceptable stability degree, was developed. The method is applied to a problem of synthesizing a controller of an autonomous underwater vehicle submerging control system with interval parameters.

1 INTRODUCTION

Modern level of industrial automation development allows raising a quality of technological objects control with the help of automated control systems. Parameters of control object in such systems may vary slowly or rapidly in some intervals of values randomly or accordingly to known mathematical laws. In both cases such parameters can be considered and interval-uncertain parameters with deterministic uncertainty.

Nowadays, there are two common approaches to manipulating objects with parametric uncertainty: adaptive control, robust control and their combinations.

Adaptive control is based on a use of parametric identifiers or ideal model of control process and requires real-time tuning of adaptation channel [1, 2]. The main restrictions of adaptive approach application are implementation difficulties and lack of adaptation channel operating speed.

Robust control has no disadvantages, mentioned above [3, 4]. It enables the system to operate with desired control quality despite parametric uncertainties, unmodeled dynamics, inaccuracy of parametric identification, external disturbances, etc. Robust controllers also are known for simple implementation as their parameters are constant. It should be noticed, that there is a vast variety of robust controller parametric synthesis methods, providing a desired control quality in interval systems [5] – [9].

One of the most important characteristics of a control systems is a type of a transient process. In most of control systems an aperiodic transient process or transient process of similar type is required. In the figure 1, variants of aperiodic-shaped transient processes are shown. All of these transient processes must have no oscillation around the steady-state value.

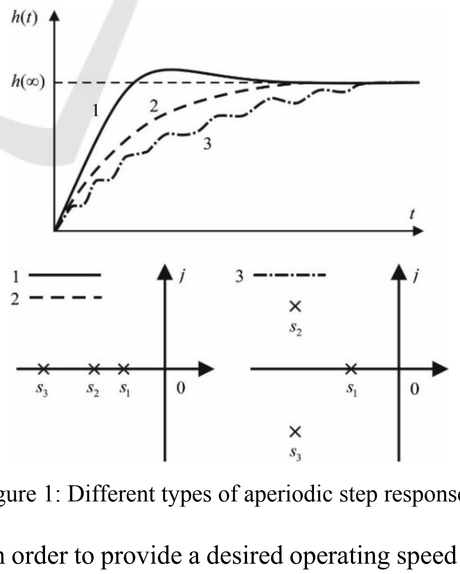

Figure 1: Different types of aperiodic step responses.

In order to provide a desired operating speed for a control system with interval parameters, it is proposed to use typical linear controllers. To provide an aperiodic transient process with the help of linear controller, it is necessary to place a real pole of the

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system nearby imaginary axis and to make it dominant to other poles allocation areas. The problem of synthesizing an interval control system by a criteria of the desired system poles allocation is relatively new. Solutions of this problem can be found in [10, 11].

Another relevant synthesis problem is a parametric synthesis of robust controllers, based on interval expansion of the coefficient method [12, 13]. It directly links root quality indexes of a control system to coefficients of its interval characteristic polynomial, which are linear functions of controller parameters.

In the paper an interval expansion of the coefficient method is applied for synthesizing a robust PID-control, manipulating an autonomous underwater vehicle (AUV) submerging. The AUV operates in conditions of interval parametric uncertainty: hydrodynamic characteristics, angles of attack and drift, etc. Considering this, a problem of synthesizing a PID-controller for an AUV motion control system consists in calculating its parameters, providing an aperiodic transient process with desired setting time.

2 COEFFICIENT CONDITIONS FOR ROBUST CONTROLLER PARAMETRIC SYNTHESIS

Let us describe a control object with a transfer function: $W_{co}\left(s\right) = B(s)/A(s) = \sum_{h=0}^{c}[b_h]s^h / \sum_{q=0}^{w}[a_q]s^q$,

where $\underline{a_q} \le a_q \le \overline{a_q}$, $\underline{b_h} \le b_h \le \overline{b_h}$ – interval coefficients of polynomials $A(s)$ и and B(s), s – Laplace operator. The traffic function of an astatic controller can be written as follows: $W_c\left(s,\vec{k}\right) = F(s,\vec{k})/s$, where $\vec{k}$ – is a vector of controller parameters. Considering this, an interval characteristic polynomial (ICP) can be written as follows:

$$D\left(s,\vec{k}\right) = \sum_{i=0}^{n}[d_i(\vec{k})]s^i = [d_n(\vec{k})]s^n + [d_{n-1}(\vec{k})]s^{n-1} \dots + [d_0(\vec{k})] \quad (1)$$

$[d_i(\vec{k})]$ – interval coefficients of $D(s)$.

In the considered paper it is proposed to derive sufficient conditions for controller parameters, providing necessary setting time of aperiodic transient process and based on interval coefficient stability $[\lambda_i]$ and oscillability $[\delta_p]$ indexes. On the base of interval stability index $[\lambda_i]$, sufficient conditions of providing a accepted robust stability degree η_d can be derived.

Statement 1. In order to provide acceptable robust stability degree, which is determined by a line $(-\eta_d, j0)$ on a complex plane, it is enough to set such parameters of controller $\vec{k}$, which comply with following conditions:

$$\begin{cases} \overline{\lambda_i(\vec{k},\eta_d)} < \lambda^*, \ \lambda^* = 0.465, \ \forall i \in \overline{1,n-2}; \\ \underline{F_g}(\vec{k},\eta_d) \ge 0, \ \forall g \in \overline{1,n-1}; \\ \underline{F_0}(\vec{k},\eta_d) \ge 0. \end{cases} \quad (2)$$

where

$$\overline{\lambda_i(\vec{k},\eta_d)} = \frac{\overline{d_{i-1}(\vec{k})}\ \overline{d_{i+2}(\vec{k})}}{\left(\underline{d_i(\vec{k})} - \overline{d_{i+1}(\vec{k})}(n-i-1)\eta_d\right)\left(\underline{d_{i+1}(\vec{k})} - \overline{d_{i+2}(\vec{k})}(n-i-2)\eta_d\right)}$$

$$\underline{F_g}(\vec{k},\eta_d) = \underline{d_g}(\vec{k}) - \overline{d_{g+1}(\vec{k})}(n-g-1)\eta_d;$$

$$\underline{F_0}(\vec{k},\eta_d) = \underline{d_0(\vec{k})} - \overline{d_1(\vec{k})}\eta_d + 2\underline{d_2(\vec{k})}\frac{\eta_d^2}{3}.$$

Proof. Let us rewrite (2) with interval coefficients

$$\begin{cases} [\lambda_i(\vec{k},\eta_d)] < \lambda^*, \forall i \in \overline{1,n-2}; \\ [F_g(\vec{k},\eta_d)] \ge 0, \ \forall g \in \overline{1,n-1}; \\ [F_0(\vec{k},\eta_d)] \ge 0. \end{cases} \quad (3)$$

where

$$[\lambda_i(\vec{k},\eta_d)] = \frac{[d_{i1-1}(\vec{k})]\,[d_{i1+2}(\vec{k})]}{\left([d_i(\vec{k})] - [d_{i+1}(\vec{k})](n-i-1)\eta_d\right)\left([d_{i+1}(\vec{k})] - [d_{i+2}(\vec{k})](n-i-2)\eta_{Д}\right)}$$

$$[F_g(\vec{k},\eta_d)] = [d_g(\vec{k})] - [d_{g+1}(\vec{k})](n-g-1)\eta_d;$$

$$[F_0(\vec{k},\eta_d)] = [d_0(\vec{k})] - [d_1(\vec{k})]\eta_d + 2[d_2(\vec{k})]\frac{\eta_d^2}{3}.$$

It is necessary to find such limit values of ICP coefficients, which will provide the satisfaction of these inequations with every other set of values. Considering this and (3), functions $\lambda_i(\vec{k},\eta_d)$ must have their maximal values $\overline{\lambda_i(\vec{k},\eta_d)}$, functions $F_g(\vec{k},\eta_d)$ and $F_0(\vec{k},\eta_d)$ – minimal values $\underline{F_g(\vec{k},\eta_d)}$ and $\underline{F_0(\vec{k},\eta_d)}$. To calculate these values, ICP coefficients values must be determined according to (3).

On the base of interval oscillability indexes $[\delta_p]$, a condition of having an aperiodic transient process in interval system was formulated.

Statement 2. In order to provide an aperiodic transient process in an interval system, it is enough to set such values of controller parameters $\vec{k}$, which comply the following condition

$$\underline{\delta_p}\left(\vec{k}\right)=\frac{\underline{d_p^2}\left(\vec{k}\right)}{\overline{d_{p-1}}\left(\vec{k}\right)\overline{d_{p+1}}\left(\vec{k}\right)}\ge\delta_d,\delta_d\ge4,\forall p\in\overline{1,n-1}. \quad (4)$$

Proof. According to [13], to provide an aperiodic transient process it is enough to satisfy following conditions: $\delta_p = d_p^2 / d_{p-1} d_{p+1}, \delta_p \ge 4, \forall p \in \overline{1,n-1}$. Let us apply an interval expansion to these conditions and determine a highest oscillability index of an interval control system. It is obvious, that oscillability index value is maximal if

$$\delta_p \to min \quad (5)$$

If $num = d_p^2$ and $den = d_{p-1} d_{p+1}$, then, to satisfy (5) following conditions must be satisfied:

$$num \to \min \quad den \to \max \quad (6)$$

Conditions (6) require following limits of interval coefficients of ICP: $num = \underline{d_p}$, $den = \overline{d_{p-1} d_{p+1}}$. If (6) are satisfied with these values, then they are satisfied with any other value from considered intervals.

Acceptable minimal (robust) merit index $\underline{D_v}$ of the interval control system can be calculated by the following formula:

$$\underline{D_v} = k_0 \underline{b_0} / \overline{a_0} \quad (7)$$

On the base of (2), (4) and (7), conditions of providing a desired robust stability degree, oscillability degree and merit index can be written as follows:

$$\begin{cases} \overline{\lambda_i(\vec{k},\eta_d)}<\lambda^*, \ \forall i\in\overline{1,n-2}; \\ \underline{F_g}(\vec{k},\eta_d)\ge0, \ \forall g\in\overline{1,n-1}; \\ \underline{F_0}(\vec{k},\eta_d)\ge0. \\ \underline{\delta_p}\left(\vec{k}\right)\ge\delta_d,\delta_d\ge4,\forall p=\overline{1,n-1}.; \\ \underline{D_v}=k_0\underline{b_0}/\overline{\mathrm{a}_0}. \end{cases} \quad (8)$$

In order to synthesize a PI- or PID-controller, controller parameters functions of aperiodic stability degree η_d, oscillability degree δ_d, merit index $\underline{D_v}$ and coefficients of systems transfer function can be derived from the first, fifth and sixth conditions of (6). Such functions are derived in [13].

Considered research resulted in the method of synthesizing a PI- and PID-controllers, which includes following steps:

1. Determine initial data: controller type, acceptable robust oscillability degree $\delta_d \ge 4$, acceptable robust stability degree, acceptable merit index, limits of interval coefficients of control object transfer function.
2. Calculate interval coefficients of an ICP of the considered closed-loop system.
3. According to initial data, choose functions of controller parameters from [13].
4. Derive the system of inequations (8).
5. Solve (8) and calculate controller parameters.

3 MATHEMATICAL MODEL OF THE SUBMERGING VELOCITY CONTROL LOOPCONCLUSIONS

Considered AUV is shown in the figure 2.

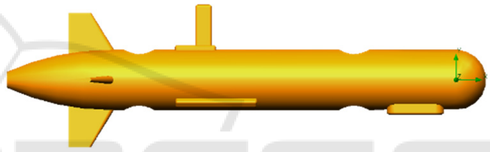


Figure 2: Hull of the considered AUV.

Submerging control is performed with the help of two vertical thrusters. The structure of depth control channel is shown in the figure 3.

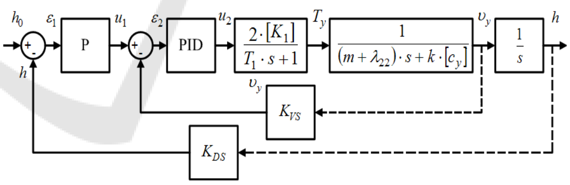


Figure 3: Structure of the submerging control channel.

In the fig. 3 following designations are accepted: h_0 – depth setpoint; h – actual value of depth; ε_1 – a difference between a setpoint and an actual value of the depth; P – P-controller of the outer control loop; u_1 – output of the P-controller; ε_2 – a difference between a P-controller output signal and a submerging velocity signal υ_y; PID – PID-controller of an inner control loop; K_1, T_1 – transfer coefficient and time constant of the thruster; T_y – thrust of vertical steering thrusters; m – AUV mass; λ_{22} – additional mass of water; c_y – hydrodynamic lift force coefficient; k – interval linearization coefficient; υ_y – submerging velocity; K_{VS} – transfer coefficient of the velocity sensor; K_{DS} – transfer coefficient of the depth sensor.

The problem of synthesizing a P-controller of the outer control loop will not be considered in the paper. Let us now consider the mathematical model of the inner control loop of the submerging control channel, which controls submerging velocity.

Considering previous designations, transfer function of the submerging velocity control loop can be written as follows:

$$W(s) = \frac{2 \cdot K_1 \cdot \left(K_D \cdot s^2 + K_P \cdot s + K_I\right)}{a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0}$$

$$a_0 = 2 \cdot K_1 \cdot K_I \cdot K_{VS};$$

$$a_1 = k \cdot c_y + 2 \cdot K_1 \cdot K_P \cdot K_{VS};$$

$$a_2 = m + \lambda_{22} + k \cdot T_1 \cdot c_y + 2 \cdot K_1 \cdot K_D \cdot K_{VS};$$

$$a_3 = T_1 \cdot (m + \lambda_{22}).$$

where K_P, K_I, K_D – proportional, integral and differential coefficients of the PID-controller.

The model considers an interval uncertainty of the AUV hydrodynamic parameters and interval uncertainty of the transfer coefficient of its thrusters.

4 PID-CONTROLLER PARAMETRIC SYNTHESIS

With the help of developed method, let us synthesize a PID-controller for AUV submerging control system. To do this, let determine an acceptable oscillability degree $\delta_d \geq 4$, acceptable degree of robust aperiodic stability $\eta_d = 0.65$ and acceptable merit index $\underline{D_v} = 47$. Then, with the help of expression $K_I = \underline{D_v}\,\overline{p_0} / \underline{b_0}$ let us calculate $K_I = 20$. According to [13], a proportional coefficient can be calculated as follows

$$K_P(K_d) = \frac{(15.1982{+}4.80K_d)^2 - 7.866}{19.25}.$$ Then, a K_d function of η_d can be derived:

$$K_d = \sqrt[3]{-\frac{Q1(\eta_d)}{2} + \sqrt{-\frac{Dis(\eta_d)}{108}}} + \sqrt[3]{-\frac{Q1(\eta_d)}{2} + \sqrt{-\frac{Dis(\eta_d)}{108}}},$$

$$Dis(\eta_d) = -4e(\eta_d)^3 - 27Q1(\eta_d)^2,$$

$$\mathrm{e}(\eta_d) = \frac{\gamma(\eta_d)^3}{3\nu(\eta_d)^2} - \frac{\upsilon(\eta_d)}{\nu(\eta_d)};$$

$$Q1(\eta_d) = \frac{(2\gamma(\eta_d))^3}{27\nu(\eta_d)^3} - \frac{\upsilon(\eta_d)\gamma(\eta_d)}{3\nu(\eta_d)} + \frac{\vartheta(\eta_d)}{\nu(\eta_d)},$$

$$\nu = \text{-}13.355047557214737868;$$

$$\gamma(\eta_d) = -126.2578 - 11.625\eta_d;$$

$$\upsilon(\eta_d) = -388.559 + 72.57\eta_d;$$

$$\vartheta(\eta_d) = -234.833 + 113.261\eta_d.$$

By K_I, $K_P(K_d(\eta_d))$ and $K_d(\eta_d)$ substituting a into (8), an inequations system can be derived:

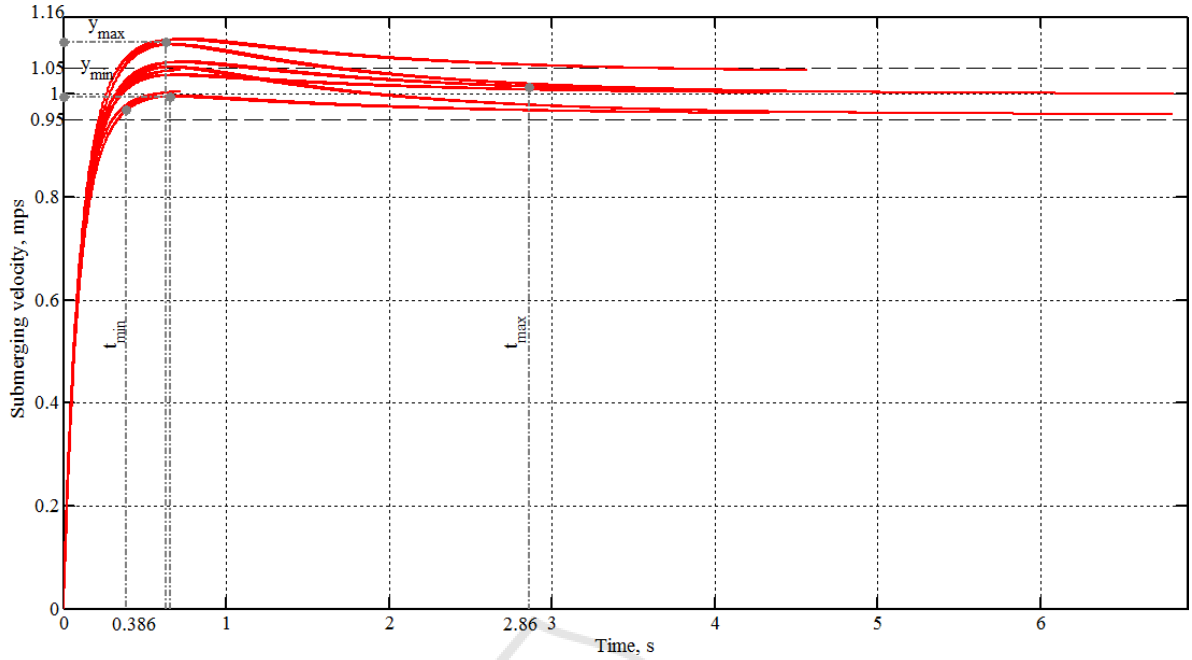
$$\begin{cases} \underline{F_g(K_I, K_P(K_d(\eta_d)), K_d(\eta_d))} = \underline{a_1}(K_P(K_d(\eta_d)), K_d(\eta_d)) - \\ -\overline{d_2(K_d(\eta_d))}(n-i-1)\eta_d \geq 0; \\ \underline{F_0(K_I, K_P(K_d(\eta_d)), K_d(\eta_d))} = \underline{d_0(K_I)} - \overline{d_1(K_P(K_d(\eta_d)), K_d(\eta_d))}\eta_d + \\ +2\underline{d_2(K_d(\eta_d))}\frac{\eta_d^2}{3} \geq 0; \\ \underline{\delta_1(K_I, K_P(K_d(\eta_d)), K_d(\eta_d))} = \dfrac{\underline{\underline{d_1^2(K_P(K_d(\eta_d)), K_d(\eta_d))}}}{\overline{d_0(K_I)}\,\overline{d_2(K_d(\eta_d))}} \geq 4. \end{cases}$$

By solving the system, proportional and integral coefficients can be calculated: $K_d(\eta_d) = 3.402$ and $K_P(K_d(\eta_d)) = 31.859$. Step responses of the synthesized system and allocation of its poles are shown in the figure 4. The figure 4 (a) shows, that setting time of the system varies from 0.386 to 2.86 seconds; overshoot varies from 3.61% to 9.6%. The figure 4 (b) shows, that robust aperiodic stability degree α= 0.68.

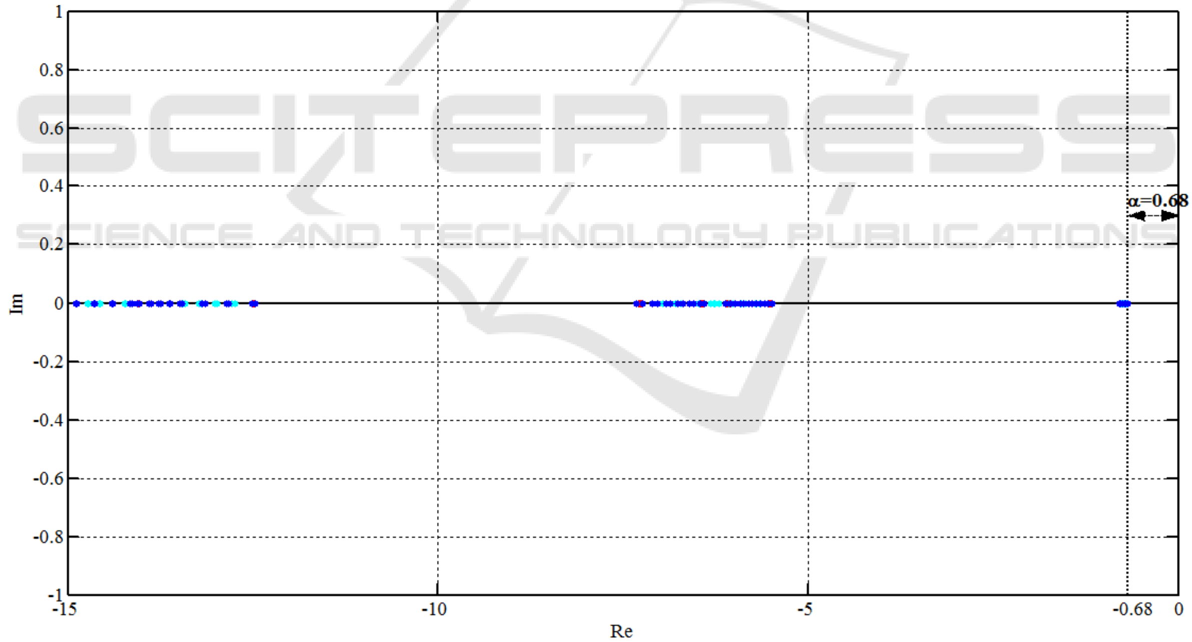
5 CONCLUSIONS

Following results were achieved during the research:

1. Formulas for robust parametric synthesis by coefficient quality indexes, considering minimal accuracy and aperiodic stability of the synthesized system, were derived.
2. General expressions for synthesizing a PI- and PID-controller, providing an aperiodic transient process in interval systems, were derived.
3. The method of interval-parametric synthesis of PI- and PID-controllers of interval systems was developed.
4. Considered method is tested on a problem of synthesizing an AUV submerging control system.



(a)



(b)

Figure 4. (a) Step responses, (b) ICP root allocation areas.

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REFERENCES

J.A. Aseltine, A.R. Mancini, C.W. Sartune. 1958. A survey of adaptive control systems. *IRE Transactions on Automatic Control,* Vol. 3, № 6, P. 102-108.

Egardt B. 1979. Stability of adaptive controllers. *Lecture Notes in Control and Information Sciences*. New York: Springer-Verlag, Vol. 20, 159 p.

S.V. Efimov, M. I. Pushkarev. 2011. Determining direct measures of performance based on the location of zeros and pole of the transfer function. *Optoelectronics, Instrumentation and Data Processing,* Vol. 47, Issue 3, pp. 297-302.

V.A. Zhmud, I.L. Reva, L.V. Dimitrov. 2017. Design of robust systems by means of the numerical optimization with harmonic changing of the model parameters. Journal of Physics: Conference Series, Vol. 803, Issue 1, pp. 1-6 (012185).

C.T. Chen, M.D. Wang.1997. Robust controller design for interval process systems. *Computers and Chemical Engineering*, Vol. 21, P. 707-721.

P. Dorato, L. Fortuna, G. Muscato. 1992. Robust Control for unstructured perturbations – an introduction. *Lecture Notes in Control and Information Sciences*, Vol. 168, 122 p.

P. Apkarian, H.D. Tuan. 2010. Nonsmooth –synthesis. *IEEE Transactions on 11th International Conference on Control Automation Robotics and Vision (ICARCV)*, Singapore, 7-10 December 2010, pp. 917-922.

Y. H. Chang, G. L. Wise. 1998. Robust gamma stability of highly perturbed systems. *IEEE Proceedings of Control Theory Application*, Vol. 142, № 2, pp. 165-175.

H. Chyi, J.H. Hwang, J.F. Leu. 2004. Tuning PID controllers for time-delay processes with maximizing the degree of stability. *Proceedings of the 5th Asian Control Conference*, Melbourne, Victoria, Australia, 20-23 July 2004, Vol. 1, pp. 466-471.

M. V. Meerov, E. I. Jury. 1998. On aperiodicity robustness. *International Journal of Control*, Vol. 70, №. 2, pp. 193–201.

C. B. Soh, C. S. Berger. 1989. Strict aperiodic property of polynomials with perturbed coefficients. *IEEE Transactions on Automatic Control,* Vol. 34, № 5, pp. 546–548.

T. A. Ezangina, S. A. Gayvoronskiy. 2014. The Control of the Descent Submersible in the Confused Sea Environment, Applied Mechanics and Materials, Vols. 536-537, pp. 1200-1206.

Gayvoronskiy S.A., Ezangina T., Khozhaev I. 2014. Bundled software for the desing of interval dynamic systems. *Applied Mechanics and Materials,* Vols. 446-447, pp 1217-1221.