

# Dynamic Programming Resolution and Database Knowledge for Online Predictive Energy Management of Hybrid Vehicles

Rustem Abdrakhmanov and Lounis Adouane

*Institut Pascal/IMobS3, UCA/SIGMA - UMR CNRS 6602, Clermont-Ferrand, France*

**Keywords:** Hybrid Electric Vehicle, Energy Management Strategy, Dynamic Programming, Online Sub-optimal Optimization, SOC Prediction, Multi-dimensional Database Knowledge.

**Abstract:** This paper presents a sub-optimal strategy, based on Dynamic Programming (DP) approach, for online energy (electric battery and fuel) optimization of a Hybrid Electric Vehicle (HEV). An optimal offline optimization is first proposed in this work, permitting to have simultaneous speed profile optimization and optimal power split strategy of a series-parallel hybrid bus. The aim of this optimization is mainly to reduce the fuel and electrical energy consumption of the studied HEV while maintaining smooth bus navigation to ensure the passengers' comfort. It is assumed in this first proposal that current road profile (slope, geometry, etc.) and the overall bus trip (time at the stations) are known in advance. Afterward, the basis of the offline optimal strategy has been adapted in order to deal online with the current road profile and driver velocity demand. The proposed sub-optimal online strategy uses mainly an appropriate speed profile and power-split database, obtained offline with DP, in order to cope with the current bus situations, and this is carried out by using a multi-dimensional interpolation method. The present work is conducted on a dedicated high-fidelity model of the hybrid bus that was developed on MATLAB/TruckMaker software.

## 1 INTRODUCTION

The problem of reducing the environment pollution in order to save the planet became one of the most important challenges in the world. Besides, the worldwide crisis of the fossil fuel resources, which diminish at high rate, aggravates it. These two global aspects made the big industrial companies and the state governments invest increasingly into the alternative energy sources. The hybrid electric vehicles (HEV) promise a relevant solution with regard to the objectives of reducing the fuel consumption, as well as the decrease of the exhaust gases emission (Murphey, 2008).

Concerning the energy optimization in the HEV and pure electric vehicles (EV), the researches mainly deal with two kind of problems: 1) energy power management for a given velocity profile (Rousseau, 2008) (Chen et al., 2014) (Kitayama et al., 2015); 2) velocity profile optimization for EV or conventional vehicles (Ozatay et al., 2014) (Tokekar et al., 2014) (Dib et al., 2014). The application of the optimal control theory to power management on HEV has been the most popular approach, which includes linear programming, optimal control and espe-

cially Dynamic Programming (DP) (Rousseau, 2008) (Chen et al., 2014) (Pei and Leamy, 2013) (Song et al., 2015) (Kamal et al., 2017) (Abdrakhmanov and Adouane, 2017). These techniques have been widely studied and applied to a broad range of vehicles.

The authors in (Pei and Leamy, 2013) propose an approach for determining the State Of the Charge (SOC)-dependent equivalent cost factor in HEV supervisory control problems using DP. (Song et al., 2015) use the DP approach to deal with the global optimization problem for deriving the best configuration for the drivetrain components sizes and energy split strategies of a hybrid energy storage system, including a battery and a supercapacitor, for an electric city bus. (Zhang et al., 2014) proposed a DP-rule based (DP-RB) algorithm to solve the global energy optimization problem in a real time controller of a plug-in hybrid electric bus (PHEB). A control grid (a set of deterministic rules) is built for a typical city route according to the station locations and discrete SOC levels. An offline DP with historical running information of the driving cycle is used to deduce optimal control parameters of RB on all points of the control grid. For a RB control strategy, control parameters are selected according to the current position and battery SOC of

PHEB. The trajectory is divided into  $N$  segments and the control parameters between station  $st_n$  and station  $st_{n+1}$  are selected as a  $Rule_n$ , which is determined by the  $SOC S^n$  at station  $st_n$ .

Other authors as (Tokekar et al., 2014) studied the problem of the velocity profiles search for a car-like robots in order to minimize the energy consumed while traveling along a given path, whereas (Dib et al., 2014) tackle an energy management problem for an electric vehicle compliant with online requirements for “eco-driving application”. The main difference between two last papers cited above is that the robot is fully autonomous, and the electric vehicle is controlled by a driver, but the driver receives the velocity profile proposed by an eco-driving system. (Ozatay et al., 2014) propose an optimization of the speed trajectory to minimize the fuel consumption and communicate it to the driver. In their approach the driver sends the information of the intended travel destination to the server. The server generates a route, collects the associated traffic and geographical information, and solves the optimization problem by a spatial domain DP algorithm that utilizes accurate vehicle and fuel consumption models to determine the optimal speed trajectory along the route. (Kim et al., 2009) use model predictive control for the velocity and power split optimization in HEVs. A given velocity profile is optimized by setting the constraints on the velocity and the acceleration of the vehicle. This allows to smooth the current velocity profile without generating a new one. The authors (Van Keulen et al., 2010a) proposed a method that solves the velocity optimization problem for HEVs, based upon information from Global Navigation Satellite-based Systems, assuming that the velocity trajectory has a predefined shape. Although this method is used for HEVs, the authors do not deal with the energy management optimization aspect. An optimal control problem was proposed to determine the potential of a HEV operational strategy controlling gear shift, torque split, and velocity at the same time (Heppeler et al., 2014). It is proposed in this paper a discrete dynamic programming approach to find a global optimal solution for fuel efficiency potential analysis, optimizing torque split, gear shifting and velocity trajectory.

In their latter works (Heppeler et al., 2016), as well as (Shen et al., 2015) and (Sun et al., 2015), deal with the problem of prediction of the battery  $SOC$ . These articles use the offline global optimal control to generate the desired  $SOC$  trajectory, later this values are used as an input in Model Predictive Control (MPC). It is proved that prediction of the future trajectories, based upon either past or predicted vehicle velocity and road grade trajectories, could help in ob-

taining a solution close to the optimal (Van Keulen et al., 2010b).

Unlike the previous publications, the present paper proposes an optimal offline optimization based on DP permitting to have simultaneous speed profile optimization and optimal power split strategy of a series-parallel hybrid bus, aiming to reduce the fuel and electrical energy consumption. For an urban bus the route is normally known in advance, so the optimization is performed for given road profile. One of the most important aspects of the public transportation is the passengers comfort. For that purpose the maximal permitted acceleration and deceleration are taken into account. As the hybrid bus' electric motor and engine have different dynamic characteristics (the power supplied, the response time, etc.), the dynamic constraints linked to the dynamic motors are taken into account. An online sub-optimal speed profile and related power split generation using a multi-dimensional interpolation is developed to deal online with the current road profile and driver velocity demand. This is carried out using the Optimal Profile Database based on DP (OPD-DP). The battery  $SOC$  is estimated and predicted, using Kalman Filter estimation, in order to guarantee that at the end of operational cycle (in the end of a course of a day) the electric battery charge is not below its permitted minimum level.

The rest of the paper is organized as follows. In section 2, the studied bus powertrain and its dynamical model are presented. Section 3 presents the proposed offline DP algorithm and its constraint set. Section 4 describes DP modeling of offline strategy, as well as the use of the obtained results for online implementation of the sub-optimal strategy and  $SOC$  prediction. In section 5, several simulation results are presented showing the efficiency of the proposed velocity profile optimization and energy management strategies. Finally, conclusions and some prospects are given in the last section.

## 2 MODELING OF THE HYBRID BUS

The aim of this section is to illustrate the architecture and the mathematical model of the studied system, i.e., BUSINOVA hybrid bus, developed by SAFRA company (cf. Figure 1)<sup>1</sup>. This bus is composed of an electric motor, a hydraulic motor, an internal combustion engine and battery as the propulsion powertrain system of the vehicle.

<sup>1</sup><http://www.businova.com>



Figure 1: BUSINOVA hybrid bus.

## 2.1 Hybrid Bus Powertrain Architecture

The model of the studied hybrid bus is based on a series-parallel power-split hybrid architecture (Bayindir et al., 2011). A simple block diagram of the power flows in the bus is shown in Figure 2.

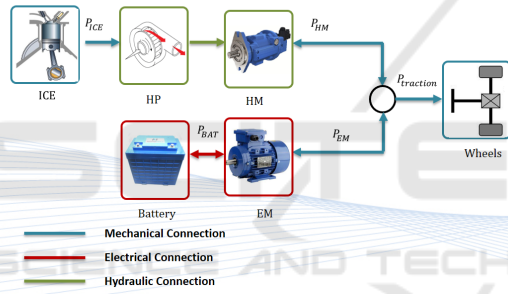


Figure 2: Block diagram of the powertrain power flows. (ICE: internal combustion engine, HP: hydraulic pump, HM: Hydraulic motor, EM: electric motor).

The electric (EM) and hydraulic (HM) motors are both directly connected to the transmission and can ensure simultaneously or independently the traction of the bus. On the other hand, the internal combustion engine (ICE) is coupled to a hydraulic pump (HP) for driving the HM, and therefore allowing the ICE load shifting.

The rotational speeds of the HM and the EM are imposed by the wheels speed in proportion to the reduction ratios of HM and EM respectively. Moreover, the rotational speed  $\omega_{HM}$  and the torque  $T_{HM}$  are expressed as follows:

$$\begin{cases} \omega_{HM}(T_{ICE}, D_{HM}) = \frac{D_{HP} \cdot \eta_{VHM} \cdot \omega_{ICE}}{D_{HM} \cdot \eta_{VHP}} \\ T_{HM}(T_{ICE}, D_{HM}) = \frac{D_{HM} \cdot \eta_{mHM} \cdot T_{ICE}}{D_{HP} \cdot \eta_{mHP}} \end{cases} \quad (1)$$

where  $\omega_{ICE}$ ,  $T_{ICE}$  are respectively rotational speed and torque of the ICE, and  $D_{HM}$ ,  $D_{HP}$ ,  $\eta_{mHM}$ ,  $\eta_{mHP}$ ,  $\eta_{VHM}$ ,  $\eta_{VHP}$  are respectively displacement, mechanical

efficiency and volumetric efficiency of the HM and the HP.

The BUSINOVA can operate according to the modes described below:

1. the propulsion is fully supplied by the electric motor (mode I),
2. the bus is actuated by the hydraulic motor via the ICE (mode II),
3. the mode III implies the hybrid operation of the EM and the HM via ICE,
4. the regenerative braking (mode IV) - the part of the kinetic energy during braking phase is recuperated to charge the electric battery.

## 2.2 Dynamical Model

This part is dedicated to the dynamical equations describing the bus. The purpose of the dynamical model is to have a realistic global behavior of the bus in order to validate the proposed energy management techniques. To describe it in a generic manner, assume that the bus is moving up the slope of  $\theta$  degree (cf. Figure 3). The origin of the coordinates is situated in the Center of Mass (CoM). It is supposed that CoM of the bus is in its geometric center. The dynamical equation of the bus is given as follows:

$$\vec{F}_{tr} + \vec{F}_{rr} + \vec{F}_{ad} + \vec{F}_g + \vec{F}_{brake} = (M + M_{eq})\vec{a} \quad (2)$$

where  $\vec{F}_{tr}$  traction force,  $\vec{F}_{rr}$  rolling resistance,  $\vec{F}_{ad}$  aerodynamic force,  $\vec{F}_g$  gravity force,  $\vec{F}_{brake}$  mechanical brake force,  $M$  bus weight,  $M_{eq}$  equivalent mass of rotating parts,  $\vec{a}$  bus acceleration.

To produce the bus acceleration, it is necessary to take into account the moments of inertia of the rotating components (e.g., rotor of the EM, crankshaft of the ICE, driving axle, etc.). It is done by introducing the equivalent mass  $M_{eq}$  of the rotating components:

$$M_{eq} = \frac{i_g \eta_{pt} J_{rot}}{r^2} \quad (3)$$

where  $i_g$  gear ratio,  $\eta_{pt}$  powertrain efficiency,  $J_{rot}$  total inertia of the rotating components in the transmission, and  $r$  the wheel radius (Cheng et al., 2007).

The traction force  $F_{tr}$  is linked to the torque produced by the powertrain  $T_{pt}$  via gear ratio  $i_g$ , powertrain efficiency  $\eta_{pt}$ . Expanding the dynamical equation (2), the following relation is obtained:

$$a = \frac{dv}{dt} = \frac{1}{M + M_{eq}} H \quad (4)$$

with

$$H = \frac{i_g \eta_{pt} T_{pt}}{r} - \mu_{rr} F_N \text{sign}(v) - \frac{1}{2} \rho A C_d (v + v_{wind})^2 - Mg \sin(\theta) - \frac{T_{brake}}{r} \quad (5)$$

where:

- $T_{pt}$ : output powertrain torque from the gearbox,
- $\mu_{rr}$ : rolling resistance coefficient,  $F_N = Mg \cos(\theta)$  normal force,  $g$  gravity acceleration,  $\theta$  slope angle,  $v$  bus speed,
- $\rho$ : the air density,  $A$  the frontal area of the bus,  $C_d$  drag coefficient,  $v_{wind}$  wind speed,
- $T_{brake}$ : the brake torque provided by the bus mechanical brake system.

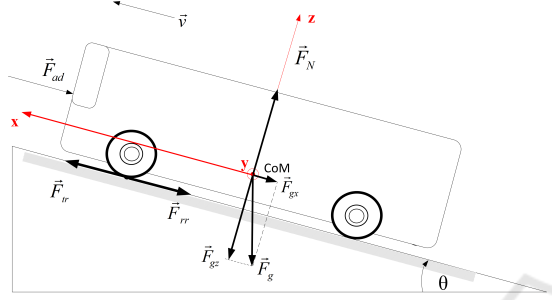


Figure 3: Forces acting on the bus.

### 3 OVERALL MULTI-CRITERIA OPTIMIZATION FORMULATION

The objective of the optimal control problem is to find the optimal bus velocity profile and energy split between the actuators for a given trajectory  $D$ . The optimization is performed in the spatial domain by means of the following transformation (conversion from time domain into spatial domain) (Ozatay et al., 2014):

$$a = \frac{dv}{dt} = \frac{dv}{dD} \frac{dD}{dt} = \frac{dv}{dD} v \quad (6)$$

In that case the dynamics of the BUSINOVA given in equation (4) could be re-written with the following equation:

$$\frac{dv}{dD} = \frac{1}{(M + M_{eq})v} H \quad (7)$$

As any problem of optimization, it is important to define optimization criteria. In our case, the criteria is defined in order to optimize the fuel mass consumed  $\dot{m}_{fuel}$  (Zeng, 2009) and the electric power  $P_{EM}$  consumed by the electric motor during the trip  $D$ :

$$J_D = \alpha J_1 + (1 - \alpha) J_2 + \beta J_3 \quad (8)$$

where:

$$J_1 = \int_0^{D_f} \frac{\dot{m}_{fuel}(P_{ICE}(D), P_{HM}(D), D_{HM})}{v(D)} dD \quad (9)$$

with  $\dot{m}_{fuel}$  fuel mass consumed is a function of ICE power  $P_{ICE}$ , as well as of HM power  $P_{HM}$  and HM displacement  $D_{HM}$  (Zeng, 2009);

$$J_2 = \int_0^{D_f} \frac{P_{EM}(D)}{v(D)} dD \quad (10)$$

with  $P_{EM}(D)$  EM power.

$$J_3 = \int_0^{D_f} \frac{|a|}{v(D)} dD \quad (11)$$

which is introduced to avoid abrupt velocity changes and make the speed profile smoother with  $a$  vehicle acceleration,

$D_f$  is the total traveled distance;

$v$  the bus speed;

$\alpha$  is a constant weight coefficient such as  $\alpha \in [0, 1]$  and  $\beta$  is a scale factor.

In order to have adequate results, each part of the cost function (8) is normalized. The fuel consumption rate  $\dot{m}_{fuel}$  is transformed into equivalent consumed engine power  $P_{engine}$ :

$$P_{engine} = \dot{m}_{fuel} Q_{LHV} \quad (12)$$

where  $Q_{LHV}$  is lower heating value of a used fuel. For diesel  $Q_{LHV} = 43 \text{ MJ/kg}$ .

The minimization of the cost function (8) is the subject to the bus dynamical model (7), as well as to the constraints imposed on the control input and state during the performed optimization.

#### 3.1 Boundary Conditions

To set the boundary conditions for optimization procedure, we take into consideration a normal operation for an urban bus, traveling from one stop to another with the appropriate velocity profile  $v(D)$  (cf. Figure 4), which should be obtained according to the proposed optimization algorithm based on DP (cf. section 4).

To simplify the notation, initial  $v(0)$  and final  $v(D_f)$  speeds are called  $v_0$  and  $v_f$ . Generally for the developed DP-based algorithm (cf. section 4), they

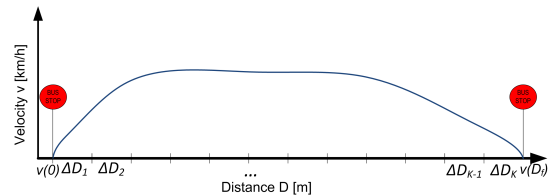


Figure 4: An example of a bus trip from one bus stop to another.



can have any non-negative value. But in practice, the bus trip to run from one bus stop to another implies that:

$$v_0 = v_f = 0 \quad (13)$$

The trip duration  $T$  to arrive to the next bus stop is imposed by the schedule that must be respected. As the optimization is computed with floating-point numbers, a constant  $\tau$  is introduced (cf. equation (14)) to calculate the range of the permitted final time  $t_f$  flexibility. But introduction of  $\tau$  is also justified in real systems, as the trip duration  $T$  cannot be absolutely precise during the trip from  $D_0$  to  $D_f$  due to the traffic jams, traffic lights, pedestrian crossing, etc.

$$t_f = t_0 + (1 \pm \tau)T \quad (14)$$

where  $T$  is the pre-set trip duration,  $t_0$  is the initial time,  $\tau$  is a very small value ( $\tau \rightarrow 0$ ).

## 4 OFFLINE AND ONLINE OPTIMIZATION STRATEGIES BASED ON DP ALGORITHMS

This section is dedicated to the development of a simultaneous speed profile optimization and an energy management strategy in offline and online mode. The procedure of the decision tree construction is described in details in section 4.1. The developed offline algorithm permits to move to the online implementation by means of the Optimal Profiles Database (cf. section ??), which generates the speed profile and its energy management strategy, depending on the road profile, the bus current bus state and velocity setpoint. The battery *SOC* estimation and prediction procedure is described in section 4.1.2.

### 4.1 Offline Optimization Strategy

#### DP Formulation

The optimization method used to solve the given optimal control problem is based on DP (Bellman and Dreyfus, 2015) (Bertsekas, 1995), which provides the global optimal solution over a given trip. The algorithm proceeds from 0 to  $K$  steps in order to minimize the following cost function:

$$J_k(v_k) = \min_{u_k \in \mathbb{U}_D} \{g_k(v_k, u_k, \Delta D_k) + J_{k-1}(f(v_k, u_k))\} \quad (15)$$

where

- $J_k(v_k)$  is the cost-to-go function from step 0 to step  $K$  starting from  $v_0$  with initial cost  $J_0(v_0) = g_0(v_0) = 0$ ;

- $g_k(v_k, u_k, \Delta D_k)$  is the cost-to-go from state  $i$  to state  $j$ .
- $J_{k-1}(f(v_k, u_k))$  is the total cost starting from the initial state to the state  $i$ .
- $u_k \in \mathbb{U}_D$  is the control input determining the velocity to go from the state  $i$  to the next state  $j$ .

The given optimization is aimed to solve two main problems simultaneously: (i) Find the optimal speed profile from  $D_0$  to  $D_f$  in time  $t_f$ , minimizing the electric energy and fuel consumption; (ii) find the optimal power split strategy to the speed profile in order to provide the optimal functioning mode (cf. section 2) and the percentage of the contribution of each motor (EM and HM) in order to move the hybrid bus.

The paragraphs below detail the resolution of the nonlinear optimization control problem formulated in the spatial domain by using DP algorithm. A set of points defines the route. Namely, the route consists of the points  $P = [p_0, p_1, p_2, \dots, p_K]$ . Every point  $p_k \in P$  for  $k = 0, 1, 2, \dots, K$  has its own characteristics:  $p_k = [x_k, y_k, \theta_k]$ , where  $x_k$  longitudinal position,  $y_k$  lateral position,  $\theta_k$  road slope angle. The given route of the length  $D$  is divided into  $K$  segments of the sample length  $\Delta D$ . Depending on the acceleration/deceleration limits and on the  $\Delta D$  segment length (cf. Figure 4), the velocity  $v(D_k)$  for a given segment  $D_k$  can increase or decrease with a fixed step  $\Delta v$ . The maximum number of  $\Delta v$  is equal to  $2N_v + 1$ :

$$\mathbb{V} = \{-N_v \Delta v, \dots, 0, \Delta v, \dots, N_v \Delta v\} \quad (16)$$

where  $N_v \in \mathbb{N}$ .

#### DP-SEO Algorithm Flowchart

Figure 5 shows the proposed DP based Speed and Energy Optimization (DP-SEO) algorithm. The proposed DP-SEO has the following main steps.

*Action 1.* It initializes the road profile  $P$ , distance discretization step  $\Delta D$  (cf. Figure 4),  $K$  number of  $\Delta D$ , a set of maximum permissible values of speed increment  $\Delta v$ . To perform a simultaneous bus speed profile and its power split optimization, the specific vector  $\Lambda = \{0, 0.1, 0.2, \dots, 1\}$  is assigned with  $Card(\Lambda) = m$  cardinal number of the set  $\Lambda$ , which corresponds to the contribution of the EM and HM in traction. Each state  $v_j \equiv [P_k, v_j, t_j, Parent(v_j)]$ , with  $v_j$  speed,  $t_j$  time,  $Parent(v_j)$  previous state.

*Action 2.* It attributes an acceleration  $\Delta v$ .

*Action 3.* It calculates the required torque  $T_{setpoint}$ .

*Decision 1.* If  $T_{setpoint}$  is negative, the braking is applied (cf. *Action 7*).

*Decision 2.* If  $T_{setpoint}$  less than the maximum torque that can be produced by EM and HM, then go *Action 4*.

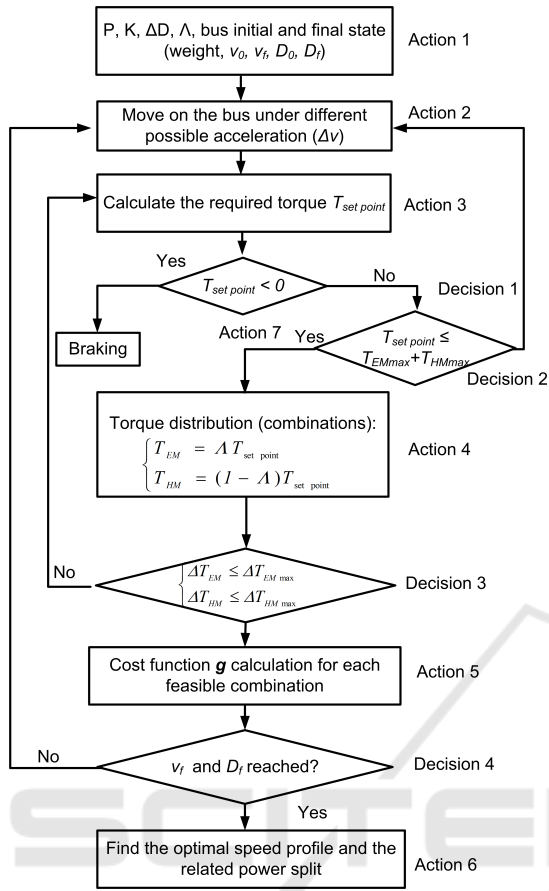


Figure 5: Block diagram of the Dynamic Programming based Speed and Energy Optimization (DP-SEO).

**Action 4.** It calculates the possible torque ratio combinations according to  $\Lambda$ .

**Decision 3.** The EM and the engine, which drives the HM, have different dynamic characteristics. EM is capable of delivering a high starting torque. Unlike EM, engine cannot provide torque at zero speed and it produces maximum power at a certain speed (Pisu and Rizzoni, 2007). The efficiency of the engine is very much dependent on the operating point in the engine's performance map. Thus, to go from one state to another not all the  $\Delta T$  are feasible. To take this constraint into account, the maximum values  $\Delta T_{EMmax}$  and  $\Delta T_{HMmax}$  are introduced. If the demanded torque is higher than these values, then the development of the decision tree in that direction is stopped. Otherwise, go to **Action 5**.

**Action 5.** For the feasible combinations the cost-to-go function  $g_k(v_k, u_k, \Delta D_k)$  is calculated (cf. equation (15)).

**Decision 4.** If the final conditions are reached, then go to **Action 6**. Otherwise, go to **Action 2**.

**Action 6.** The minimum cost function is calculated according to equation (15). Optimal speed profile and related power split are calculated. In this work, DP is used to obtain the minimal cost leading the hybrid bus from its initial state to the final one.

**Action 7.** A part of the kinetic energy is regenerated to charge the battery, and the rest is dissipated via mechanical brake. If the vehicle speed is high enough then the kinetic energy can be regenerated (limited to the maximum torque  $T_{regenmax}$  produced by EM and depending on the current vehicle speed). No regeneration is possible under  $v < 10$  km/h. In future work, the regenerative aspect and dynamic will be studied in more details.

## DP-SEO Results Analysis

In order to validate the proposed offline optimization, it is performed several DP-SEO in different elementary conditions of slope and desired velocity progress (constant speed ( $v_0 = v_f$ ), acceleration ( $v_0 < v_f$ ) and deceleration ( $v_0 > v_f$ ) phases) for a fixed distance  $D$ ,  $K$  number of  $\Delta D$  and  $\Lambda$ . Figures 6-7 show the average energy consumption for different slopes  $\theta$  for aforementioned phases. It can be seen that the energy consumption increases with a bigger  $\theta$ . A bigger bus weight also increases the fuel consumption. To keep the cruise speed on the downhill slope (negative  $\theta$ ), the acceleration produced by the bus' own dynamics is enough (sometimes even excessive and it is necessary to apply a negative torque). So on the downhill slope the EM part is 100% for deceleration and cruise speed phases. When accelerating, the EM torque goes to its peak with the increase of the uphill slope, and it is complemented by the HM (via ICE), so we can see that HM ratio increases.

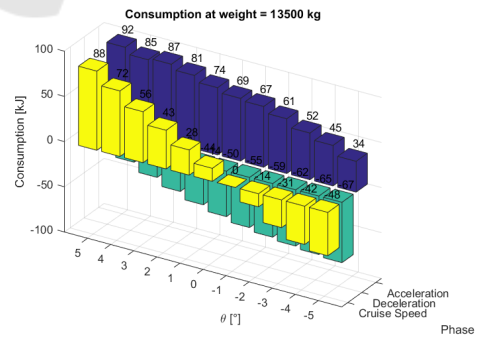


Figure 6: Average energy consumption for different slopes.

Even if the proposed strategy of the DP-SEO gives good optimal results (all the possible states are explored and the global minimum is found), nevertheless it is too time consuming. However, from the results of the optimization presented above, the op-

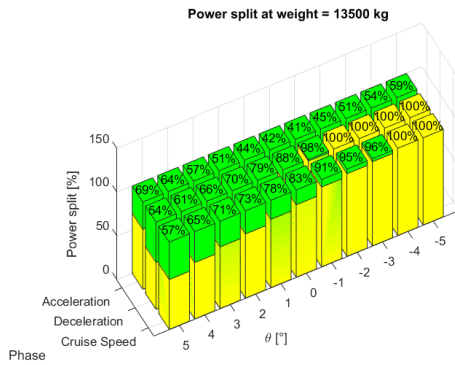


Figure 7: Average power split for different slopes.

timal offline solutions can be used online in order to generate the sub-optimal solutions. For that purpose, the optimal profiles were generated offline for the distance of  $\Delta d = 21$  m (chosen prediction horizon), with different  $\Delta v$  for several masses and slopes. Different scenarios (for acceleration, deceleration, maintaining the same speed) were simulated and for each simulation configuration an optimal speed profile and related power split are obtained.

The Optimal Profiles Database based on DP (OPD-DP) was built for all the possible combination of velocities from 0 to 15 m/s with  $\Delta v = 1$  m/s, the bus weight varies from 13 to 15 tonnes with  $\Delta M = 0.5$  tonne, and the slope angle value ranges from  $-5^\circ$  to  $5^\circ$ . Based on the offline optimal solutions, OPD-DP will be used to generate the speed profiles and the related power split in a real-time system.

The multi-dimensional OPD-DP is used for online speed profile and its power split generation. The scheme of the online control implementation is presented in Figure 8. The inputs of the multi-dimensional OPD-DP are the road profile ( $x, y, \theta$ ), the bus weight, the current speed and the driver's reference (desired) speed, and estimated  $SOC$  value (this block is detailed later). Each 21 meters this block generates the sub-optimal speed profile reference and torque split depending on the current state of the vehicle and the road profile. The bus weight is considered constant to travel from one bus station to another, thus the weight is susceptible to change only after each bus station. The driver's command must be followed as precise as possible due to the safety measures, ensuring the collision avoidance.

The OPD-DP generates the sub-optimal profile references, as well as  $T_{EM}$  and  $T_{HM}$  set points. The set point torques sent to EM and HM local controllers, which generate the control input to the motors ( $I_{EM}$  EM current,  $D_{HM}$  HM displacement).

Knowing that the multidimensional OPD-DP has only finite set of values for the parameters (slope  $\theta$ ,

weight  $M$ , etc.), it is very important to have a mean to use this Database even for values not belonging to it. To solve this problem, below it is proposed a Multidimensional interpolation method.

#### 4.1.1 Multidimensional Interpolation

There are several possible ways to approximate the actual optimal profile to follow. In our approach, the Linear Multidimensional (LMD) interpolation method was used to approximate the optimal speed profile and the powersplit vector (LaValle, 2006). As a desired velocity value  $v$  lies between two values in the OPD-DP, the weighted sum of the lower bound speed  $v_{UB}$  and the upper bound speed  $v_{LB}$  are applied to generate the speed profile:

$$v_{opt} = \zeta_v v_{LB} + (1 - \zeta_v) v_{UB} \quad (17)$$

The weight coefficient  $\zeta_v$  is obtained as follows:

$$\zeta_v = 1 - \frac{v - v_{LB}}{v_{UB} - v_{LB}} \quad (18)$$

The weight coefficients applied to calculate the corresponding power split  $\Lambda_{opt}$ , depend on the current speed  $v$ , the bus weight  $M$ , and the road slope  $\theta$ . This results into three dimensional interpolation function given as follows:

$$\begin{aligned} \Lambda_{opt}(v, M, \theta) = & \zeta_v \zeta_M \zeta_\theta \Lambda(v_{LB}, M_{LB}, \theta_{LB}) + \\ & (1 - \zeta_v) \zeta_M \zeta_\theta \Lambda(v_{UB}, M_{LB}, \theta_{LB}) + \\ & \zeta_v (1 - \zeta_M) \zeta_\theta \Lambda(v_{LB}, M_{UB}, \theta_{LB}) + \\ & \zeta_v \zeta_M (1 - \zeta_\theta) \Lambda(v_{LB}, M_{LB}, \theta_{UB}) + \\ & (1 - \zeta_v) (1 - \zeta_M) \zeta_\theta \Lambda(v_{UB}, M_{UB}, \theta_{LB}) + \\ & (1 - \zeta_v) \zeta_M (1 - \zeta_\theta) \Lambda(v_{UB}, M_{LB}, \theta_{UB}) + \\ & \zeta_v (1 - \zeta_M) (1 - \zeta_\theta) \Lambda(v_{LB}, M_{UB}, \theta_{UB}) + \\ & (1 - \zeta_v) (1 - \zeta_M) (1 - \zeta_\theta) \Lambda(v_{UB}, M_{UB}, \theta_{UB}) \end{aligned} \quad (19)$$

where  $\Lambda_{opt}$  is the power split vector, and weight coefficients  $\zeta_M$  and  $\zeta_\theta$  are calculated as follows:

$$\zeta_M = 1 - \frac{M - M_{LB}}{M_{UB} - M_{LB}} \quad (20)$$

$$\zeta_\theta = 1 - \frac{\theta - \theta_{LB}}{\theta_{UB} - \theta_{LB}} \quad (21)$$

with indexes  $LB$  - lower bound and  $UB$  - upper bound of the corresponding parameter.

To illustrate how this method works in 3D, let us fix speed parameters, and suppose that the powersplit  $\lambda$  change depends only on the variation of the bus weight and the road slope. Figure 9 illustrates the case when for different combinations of weight and slope angle, we have four known values of  $\lambda$  stored in the OPD-DP. Now let us address the case when weight and slope angle values lie in between the known data. This method permits to calculate the corresponding powersplit.

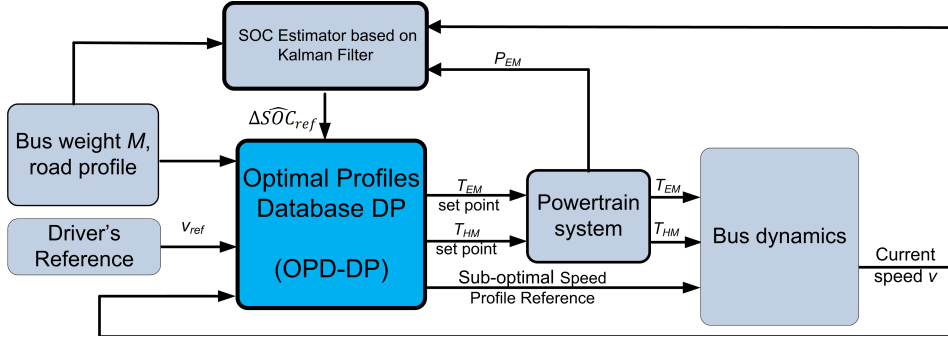


Figure 8: Scheme of the online sub-optimal speed profiles and its power split generation.

#### 4.1.2 SOC Estimator based on Kalman Filter

The BUSINOVA bus is a plug-in hybrid electric vehicle and its standard functioning time is 8 hours a day (so called “course of a day”). Figure 10 illustrates the spatial bounds of a bus running cycle. The bus travels from its starting location to another route terminus, stopping at bus stations (BS) along the route to allow passengers to board and to alight. This movement is called a trip. By the end of a day, the bus reaches its  $SOC_{min}$  value and can be recharged during all the night long to ensure the service the next day.

In this work, the principle idea is to consider that a better usage of the electric energy is such that it is available until the end of the day (during 8 hour operational cycle), and this is considered as an optimal functioning of the bus. The working hypothesis behind this assumption is to use the maximum amount of energy that can be consumed from the battery in one day driving so that the battery energy is spread as uniformly as possible in one working day. This implies the smooth battery discharging rate (C-rate), avoidance of the high or low  $SOC$  and excessive depth of charge, which lead to a high rate of battery capacity loss (Tang et al., 2015) (Choi and Lim, 2002) (Brousely et al., 2005). As Li-ion batteries represent a big

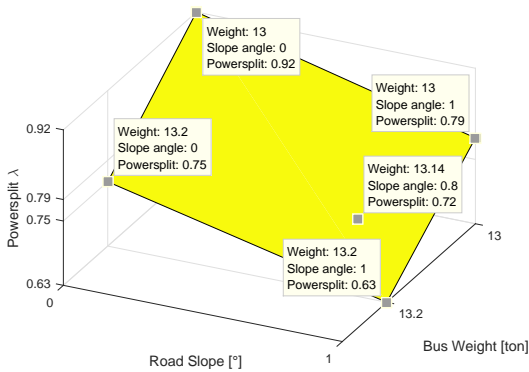


Figure 9: Linear Multi-dimensional Interpolation method illustration.

part of a vehicle cost, the clear interest is to prolongate the battery life. For that purpose a  $SOC$  Estimator based on Kalman Filter (Welch and Bishop, 1995) (Grewal, 2011) is proposed.

It is assumed that the traffic data are measured and collected, so that for each Trip, and BS distance there is an amount of  $\Delta SOC = f(E_{EM})$  that is permitted to consume according to the statistical analysis, depending on an average congestion level, travel distance, average velocity, etc. (Sun et al., 2015) (Van Keulen et al., 2010b). The Kalman filter addresses the general problem of trying to estimate the state  $x$  of a discrete-time controlled process that is governed by the linear stochastic difference equation:

$$x_k = Ax_{k-1} + Bu_k + w_k \quad (22)$$

with measurements  $z$ :

$$z_k = Hx_k + v_k \quad (23)$$

Matrix  $A$  in the difference equation 22 relates the state at the previous step  $k - 1$  to the state at the current step  $k$ . The matrix  $B$  relates the optional control input  $u$  to the state  $x$ . The matrix  $H$  in the measurement equation 23 relates the state  $x$  to the measurement  $z_k$ . The random variables  $w_k$  and  $v_k$  represent the process and measurement noise, respectively. In our case, the state vector  $x = [E_{EM} \ P_{EM}]^T$ , which is a total electric energy consumed during a trip and instantaneous electric motor power, respectively. The control input

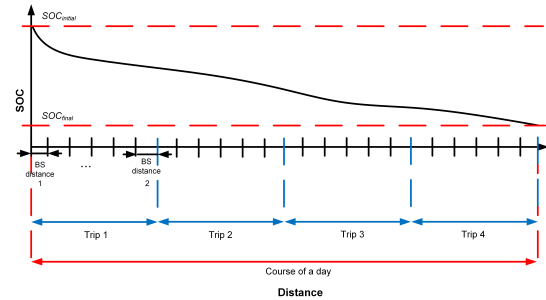


Figure 10: Spatial bounds of a bus running cycle.



$u = T_{EM}\omega_{EM}$  is a product of electric motor torque and its speed.

A predicted state is calculated according to the equation 22:

$$x_{k_p} = Ax_{k-1} + Bu_k + w_k \quad (24)$$

A Predicted Process Covariance Matrix is calculated as follows:

$$P_{k_p} = AP_{k-1}A^T + Q_k \quad (25)$$

where  $P_{k-1}$  is previous step Process Covariance Matrix,  $Q_k$  stands for process Noise Covariance Matrix associated with noisy control inputs.

To update the measurements, the following three steps are carried out:

1. Calculation of the Kalman Gain  $KG$ :

$$KG = \frac{P_{k_p}H^T}{HP_{k_p}H^T + R} \quad (26)$$

where  $R$  is Observation Errors Matrix.

2. Calculation of the current state:

$$x_k = x_{k_p} + KG(z_k - Hx_{k_p}) \quad (27)$$

3. Next step is updating a Process Covariance Matrix:

$$P_k = (I - KG \cdot H)P_{k_p} \quad (28)$$

with  $I$  identity matrix.

In this manner,  $\Delta\hat{SOC}_k = f(E_{EM})$  is estimated at each step  $k$  in order to respect  $SOC_{min}$  at the end of the bus day work. The value of  $\Delta\hat{SOC}_k$  is sent to the OPD-DP block to find the solution which respects the given constraint and choose an appropriate powersplit. The simulation results for a specific scenario are demonstrated in the next section.

## 5 SIMULATION RESULTS

It is presented below several results of the use of the OPD-DP based real time predictive energy optimization. The Kalman Filter is used to predict the  $SOC$  and to constrain the use of the battery. Two scenarios are proposed. For both of them the bus travels a trip consisting of five Bus Station distances.

1. The bus arrives at all the Bus Stations without stopping at the traffic lights and without being stuck in a traffic jam.
2. In this case, at some points it had to stop because of the traffic lights or before the vehicle ahead, which entailed underspecified stops along the trip.

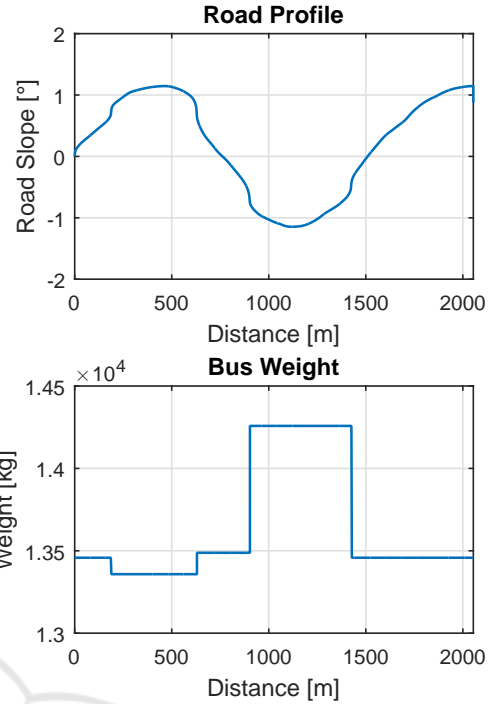


Figure 11: Road slope and bus weight.

Road profile and the weight of the bus change during the trip (cf. Figure 11). However, the weight changes only after each bus stop when people get on or get off the bus. It is considered in this simulation that the road-tire frictional coefficient, which depends on the surface where the bus moves and on the weather conditions, is constant and equal to 1, which corresponds to the dry pavement (Ming, 1997).

The value of the  $SOC_{min}$  in the end of the trip remained the same. Basically, the  $SOC_{initial} = 90\%$ , and in the end of the trip must be around  $SOC_{min} = 87.79\%$ . Total trip distance is 2 km (more precisely 2058m).

Figure 12 represents the results of the first scenario. The first figure shows the obtained speed profile. Red crosses indicate the driver's demanded speed at each  $\Delta d$ . The speed tracking of the driver's command is well followed. The figure below shows battery  $SOC$  trajectory. It can be seen that by the end of the trip we tend to the  $SOC_{min}$ . The last graph shows the power split value  $\lambda$ . The applied notation is as follows:  $\lambda = 1$  corresponds to 100% electric mode,  $\lambda = 0$  corresponds to 100% hydraulic mode (via ICE), and  $\lambda = -0.1$  corresponds to regenerative mode.

Figure 13 represents the second scenario. In this case, we can see that at the distance  $\approx 500m$  there is a non-planned stop, as well as at the distance  $\approx 1200m$ , etc. Although the speed profile changed, the online energy optimization strategy is adapted in such a man-

Table 1: Comparison of the ODP-DP online strategy and the offline DP.

$SOC_{min} = 87.79\%$						
	OPD-DP		DP		DP vs OPD-DP	
	$SOC_{final}, \%$	Fuel, l	$SOC_{final}, \%$	Fuel, l	$SOC_{final}, \%$	Fuel, l
Scenario 1	87.85	0.195	88.02	0.178	-0.02	0.017
Scenario 2	87.84	0.228	87.88	0.189	-0.03	0.039

ner that it will always tend to  $SOC_{min}$  at the end of the trip. We can see this adaptation, according to the change in  $\lambda$ .

The OPD-DP based real time predictive energy optimization is compared to the offline DP. The offline DP based optimization was carried out for the same speed profiles. Table 1 shows that the obtained results are close to the offline DP solution.

## 6 CONCLUSIONS AND PROSPECTS

In this paper, the DP technique is used to simultaneously generate the optimal velocity profile and its power split strategy in order to ensure the electric energy and fuel economy, respecting passengers comfort (by limiting the acceleration/deceleration). As the

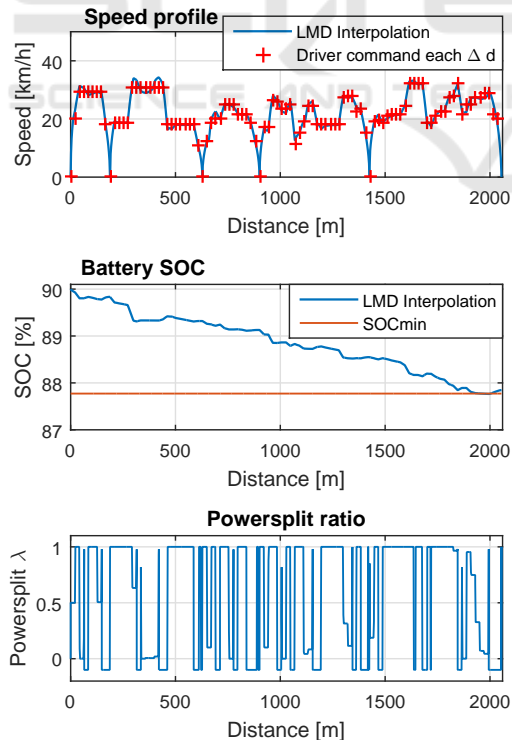


Figure 12: First scenario: 1) speed profile 2) actual  $SOC$  and the desired value in the end of the trip  $SOC_{min}$  3) power split  $\lambda$ .

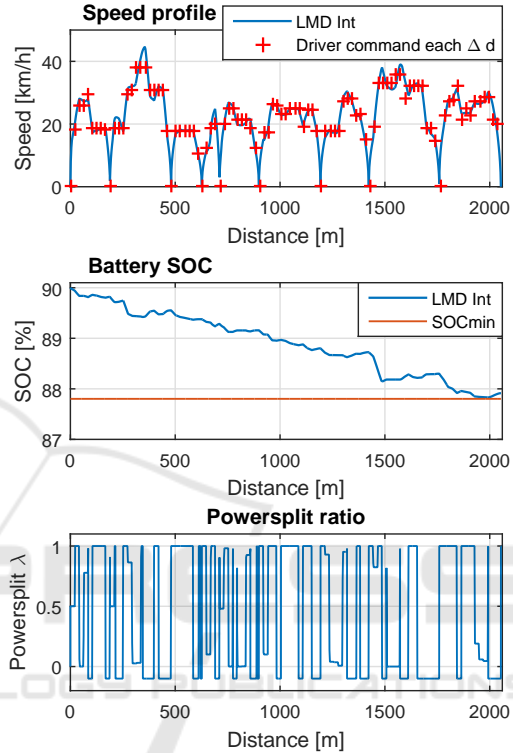


Figure 13: Second scenario: 1) speed profile 2) actual  $SOC$  and the desired value in the end of the trip  $SOC_{min}$  3) power split  $\lambda$ .

dynamics of the electric motor and engine are different, not all the energy management configurations are feasible. This aspect is taken into account in the DP-SEO algorithm.

Thereafter, the offline optimal solutions of the DP based optimization were collected into a multi-dimensional OPD-DP for the online implementation for a priori unknown traffic conditions for instance. To cope with the problem of the finite set of sample points in the database, a Linear Multidimensional interpolation was used to obtain the values at all other points. The  $SOC$  estimator based on Kalman Filter was used to restrict the use of the battery throughout the cycles and allows us to have smooth battery discharge rate, guaranteeing a regular electric energy consumption without falling below the minimum  $SOC$  value. The results obtained by Online Predictive Energy Management strategy were compared to DP

solutions obtained offline, and it was shown that near optimal results were obtained in real-time application. The influence of road-tire frictional coefficient on the online energy management strategy will be studied in near future. Later, the given approach will be implemented in the real bus.

## ACKNOWLEDGEMENTS

This project is supported by the ADEME (Agence De l'Environnement et de la Maitrise de l'Energie) for the National French program "Investissement d'Avenir", through BUSINOVA Evolution project.

## REFERENCES

- Abdrakhmanov, R. and Adouane, L. (2017). Efficient acc with stop&go maneuvers for hybrid vehicle with on-line sub-optimal energy management. In *RoMoCo'17, 11th International Workshop on Robot Motion and Control, Wasowo-Poland, 3-5 July*.
- Bayindir, K. Ç., Gözükcük, M. A., and Teke, A. (2011). A comprehensive overview of hybrid electric vehicle: Powertrain configurations, powertrain control techniques and electronic control units. *Energy Conversion and Management*, 52(2):1305–1313.
- Bellman, R. E. and Dreyfus, S. E. (2015). *Applied dynamic programming*. Princeton university press.
- Bertsekas, D. P. (1995). *Dynamic programming and optimal control*, volume 1. Athena Scientific Belmont, MA.
- Broussely, M., Biensan, P., Bonhomme, F., Blanchard, P., Herreyre, S., Nechev, K., and Staniewicz, R. (2005). Main aging mechanisms in li ion batteries. *Journal of power sources*, 146(1):90–96.
- Chen, B.-C., Wu, Y.-Y., and Tsai, H.-C. (2014). Design and analysis of power management strategy for range extended electric vehicle using dynamic programming. *Applied Energy*, 113:1764–1774.
- Cheng, Y., Lataire, P., et al. (2007). Research and test platform for hybrid electric vehicle with the super capacitor based energy storage. In *Power Electronics and Applications, 2007 European Conference on*, pages 1–10. IEEE.
- Choi, S. S. and Lim, H. S. (2002). Factors that affect cycle-life and possible degradation mechanisms of a li-ion cell based on licoo 2. *Journal of Power Sources*, 111(1):130–136.
- Dib, W., Chasse, A., Moulin, P., Sciarretta, A., and Corde, G. (2014). Optimal energy management for an electric vehicle in eco-driving applications. *Control Engineering Practice*, 29:299–307.
- Grewal, M. S. (2011). *Kalman filtering*. Springer.
- Heppeler, G., Sonntag, M., and Sawodny, O. (2014). Fuel efficiency analysis for simultaneous optimization of the velocity trajectory and the energy management in hybrid electric vehicles. *IFAC Proceedings Volumes*, 47(3):6612–6617.
- Heppeler, G., Sonntag, M., Wohlhaupter, U., and Sawodny, O. (2016). Predictive planning of optimal velocity and state of charge trajectories for hybrid electric vehicles. *Control Engineering Practice*.
- Kamal, E., Adouane, L., Abdrakhmanov, R., and Ouddah, N. (2017). Hierarchical and adaptive neuro-fuzzy control for intelligent energy management in hybrid electric vehicles. In *IFAC World Congress, Toulouse-France*.
- Kim, T. S., Manzie, C., and Sharma, R. (2009). Model predictive control of velocity and torque split in a parallel hybrid vehicle. In *Systems, Man and Cybernetics, 2009. SMC 2009. IEEE International Conference on*, pages 2014–2019. IEEE.
- Kitayama, S., Saikyo, M., Nishio, Y., and Tsutsumi, K. (2015). Torque control strategy and optimization for fuel consumption and emission reduction in parallel hybrid electric vehicles. *Structural and Multidisciplinary Optimization*, 52(3):595–611.
- LaValle, S. M. (2006). *Planning algorithms*. Cambridge university press.
- Ming, Q. (1997). *Sliding mode controller design for abs system*. PhD thesis, Virginia Tech.
- Murphey, Y. L. (2008). Intelligent vehicle power management: An overview. In *Computational Intelligence in Automotive Applications*, pages 169–190. Springer.
- Ozatay, E., Onori, S., Wollaeger, J., Ozguner, U., Rizzoni, G., Filev, D., Michelini, J., and Di Cairano, S. (2014). Cloud-based velocity profile optimization for everyday driving: A dynamic-programming-based solution. *Intelligent Transportation Systems, IEEE Transactions on*, 15(6):2491–2505.
- Pei, D. and Leamy, M. J. (2013). Dynamic programming-informed equivalent cost minimization control strategies for hybrid-electric vehicles. *Journal of Dynamic Systems, Measurement, and Control*, 135(5):051013.
- Pisu, P. and Rizzoni, G. (2007). A comparative study of supervisory control strategies for hybrid electric vehicles. *IEEE Transactions on Control Systems Technology*, 15(3):506–518.
- Rousseau, G. (2008). *Véhicule hybride et commande optimale*. PhD thesis, École Nationale Supérieure des Mines de Paris.
- Shen, D., Bensch, V., and Miiller, S. (2015). Model predictive energy management for a range extender hybrid vehicle using map information. *IFAC-PapersOnLine*, 48(15):263–270.
- Song, Z., Hofmann, H., Li, J., Han, X., and Ouyang, M. (2015). Optimization for a hybrid energy storage system in electric vehicles using dynamic programming approach. *Applied Energy*, 139:151–162.
- Sun, C., Sun, F., Hu, X., Hedrick, J. K., and Moura, S. (2015). Integrating traffic velocity data into predictive energy management of plug-in hybrid electric vehicles. In *American Control Conference (ACC), 2015*, pages 3267–3272. IEEE.
- Tang, L., Rizzoni, G., and Onori, S. (2015). Energy management strategy for hevs including battery life opti-

- mization. *IEEE Transactions on Transportation Electrification*, 1(3):211–222.
- Tokekar, P., Karnad, N., and Isler, V. (2014). Energy-optimal trajectory planning for car-like robots. *Autonomous Robots*, 37(3):279–300.
- Van Keulen, T., de Jager, B., Foster, D., and Steinbuch, M. (2010a). Velocity trajectory optimization in hybrid electric trucks. In *Proceedings of the 2010 American Control Conference*, pages 5074–5079. IEEE.
- Van Keulen, T., de Jager, B., Kessels, J., and Steinbuch, M. (2010b). Energy management in hybrid electric vehicles: Benefit of prediction. *IFAC Proceedings Volumes*, 43(7):264–269.
- Welch, G. and Bishop, G. (1995). An introduction to the kalman filter.
- Zeng, X. (2009). Improving the energy density of hydraulic hybridvehicle (hhvs) and evaluating plug-in hhvs.
- Zhang, Y., Jiao, X., Li, L., Yang, C., Zhang, L., and Song, J. (2014). A hybrid dynamic programming-rule based algorithm for real-time energy optimization of plug-in hybrid electric bus. *Science China Technological Sciences*, 57(12):2542–2550.

