Multi-objective Order Reduction Problem Solving with Restart Meta-heuristic Implementation

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Abstract: An order reduction problem for linear time invariant models brought to the multi-objective optimization problem is considered. Each criterion is multi-extremum and complex, requires an efficient tool for estimating the parameters of the lower order system and characterizes the model adequacy for the unit-step and Dirac function inputs. A common problem definition is to estimate the lower order model coefficients by minimizing the distance between the output of this model and the initial one. We propose an evolution-based multi-objective stochastic optimization algorithm with a restart operator implemented. The algorithm performance was estimated on two order reduction problems for a single input-single output system and a multiple input-multiple output one. The effectiveness of the algorithm increased sufficiently after implementing a meta-heuristic restart operator. It is shown that the proposed approach is comparable to other approaches, but allows a Pareto-front approximation to be found and not just a single solution.

1 INTRODUCTION

The idea of reducing an identification problem to a black-box optimization problem (BBOP) is considered in this study. The initial identification problem is to estimate the parameters of the linear time-invariant (LTI) system of the lower order with the aim of making its behaviour close to the behaviour of the higher order model. In many different studies (Narwal et al., 2016), (Desai et al., 2014) and (Ramesh et al., 2011) the approaches and therefore the models are compared by several criteria, but the model parameters were identified by one of them and so the others are indicative. Commonly, these criteria are based on the sum of the output errors, where the output is a reaction on the unit-step or Dirac function input. Generally, these criteria form a non-dominated set of the identification problem solutions, and that is why the estimation of the lower order parameters leads to a multi-objective (MO) optimization problem. In this case, the proposed problem definition is a generalization of the LTI identification problem.

The BBOP appearing in system identification is a complex multimodal problem. Recent works on the LTI order reduction problem are based on a combination of stochastic nature-inspired optimization algorithms and methods of providing stability, i.e. (Chen et al., 1979), and its first combination with an optimization technique was initially given in (Parmar et al., 2007). Nature-inspired stochastic optimization algorithms are used to solve reduced optimization problems: a genetic algorithm (Ramesh et al., 2011), Big Bang Big Crunch (Desai et al., 2014) and Cuckoo Search Optimization (Narwal et al., 2016). The comparison made in these works proves that heuristic optimization is an efficient tool for solving an extremum seeking problem of this class. Solving the described MO optimization problem also requires an efficient tool, which is used to estimate not only the best solution by each of the criteria, thus dealing with multimodality and complexity, but also the Pareto set.

As the main optimization algorithm, PICEA-g was used. This algorithm was improved by implementing a meta-heuristic, the aim of which is to avoid stagnation areas and improve the search by controlling the initial generation randomization. The main idea of the restart operator is given and developed in studies (Fukunaga, 1998) and (Beligiannis et. al., 2004), but was sufficiently modified in (Ryzhikov and Semenkin, 2017), where it was applied for a single-criterion optimization problem and in the current study it was modified for

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solving MO problems. For this purpose, the main restart criteria were modified and the process of gathering information for the optimization problem is related to other statistics. This data is used to improve the efficiency of the Pareto estimation algorithm and to perform the final Pareto set estimation.

The proposed approach is based on asymptotic equivalence (Ryzhikov et al., 2017), so the lower order model output integral square errors are always convergent. The stability of the dynamical system model is provided by including a penalty function in the criteria. Determining the solution in this way increases the dimension of the search variable space by one for each system output. This approach was compared to other approaches on the same problem set and with the same number of objective function evaluations for solving the LTI order reduction problem for single-input single-output (SISO) systems and multiple-input multiple-output (MIMO) systems.

The rest of the paper is organized as follows: in Section II the order reduction problem is presented. The restart meta-heuristic and MO evolution-based algorithm are introduced in Section III. The experiments conducted and the results obtained are included in Section IV. The conclusions are presented in Section V.

2 ORDER REDUCTION PROBLEM STATEMENT

The SISO LTI system model is determined by the following linear differential equation

$$\sum_{i=0}^{n} a_i \cdot x^{(i)}(t) = \sum_{i=0}^{m} b_i \cdot u^{(i)}(t), \tag{1}$$

where $a_i \in R, i=\overline{1,n}$ and $b_i \in R, i=\overline{1,m}$ are the model parameters, $n : n > m$ is the equation order, $t \in [0, +\infty)$ is the time variable, $x^{(i)}(t)$ is the i-th derivative of the output, $u^{(i)}(t)$ is the i-th derivative of the control input.

In this study we consider the case $x^{(0)}(t)=0$, so after using the Laplace transformation, the model can be represented with a transfer function

$$G(s) = \sum_{j=0}^{m} b_j \cdot s^j \Big/ \sum_{i=0}^{n} a_i \cdot s^i . \tag{2}$$

The MIMO LTI system is determined by the following matrix equations,

$$\begin{aligned} \frac{d}{dt} X(t) &= A \cdot X(t) + B \cdot U(t), \\ Y(t) &= C \cdot X(t) + D \cdot U(t), \end{aligned} \tag{3}$$

where $Y(t) : R^+ \cup \{0\} \to R^{N_o}$ is the output function, N_o is the number of outputs, $U(t) : R^+ \cup \{0\} \to R^{N_c}$ is the input function, N_c is the number of inputs, $X(t) : R^+ \cup \{0\} \to R^{N_s}$ is the space variable, the system matrix $A \in R^{N_s \times N_s}$, the control matrix $B \in R^{N_s \times N_c}$, the output matrix $C \in R^{N_o \times N_s}$ and the feed-forward matrix $D \in R^{N_o \times N_c}$.

In this paper, we consider the MIMO system with two inputs and two outputs, thus, its transient function, which is determined by the equation $W_s(s) = C \cdot (s \cdot I - A)^{-1} \cdot B + D$ can be represented with the expression

$$W_s(s) = \begin{pmatrix} W^{1,1}(s) & W^{1,2}(s) \\ W^{2,1}(s) & W^{2,2}(s) \end{pmatrix}, \tag{4}$$

where $W^{i,j}(s) = D^{i,j}(s) / N^{i,j}(s)$, and $D^{i,j}, N^{i,j}$, $i,j=\overline{1,2}$, are the denominator and nominator, respectively. Factoring out the denominator gives

$$\begin{aligned} W_s(s) &= \frac{1}{D_s(s)} \cdot \begin{pmatrix} N_s^{1,1}(s) & N_s^{1,2}(s) \\ N_s^{2,1}(s) & N_s^{2,2}(s) \end{pmatrix}, \\ D_s(s) &= \prod_{i,j} D^{i,j}(s), \\ N_s^{p,q}(s) &= N^{p,q}(s) \cdot \prod_{\substack{i,j \\ i \neq p \wedge j \neq q}} D^{i,j}(s). \end{aligned} \tag{5}$$

To provide the convergence of integral errors, the asymptotical equivalence approach is used (hidden reference 2), where the higher and lower order model output equivalence is guaranteed by the limit equivalence of the fraction of parameters

$$a^s = \lim_{t \to +\infty} x(t) = \frac{b_0}{a_0}, \tag{6}$$

where the coefficients a_0 and b_0 are given in (1) and known. This means that the first one could calculate the initial model (system) output asymptote a^s, and on the basis of this determinate the parameters of the lower order model using the formula (6).

Since our aim is to approximate the initial model with the lower order model, we need to estimate the parameters of the 2nd order model which is determined by the following transfer function

$$G_m(s,p) = \frac{p_2 \cdot s + a^s \cdot p_1}{s^2 + p_0 \cdot s + p_1}, \quad (7)$$

for the SISO systems and

$$G_m(s,p) = \frac{1}{D_m} \cdot \begin{pmatrix} N_m^{1,1} & N_m^{1,2} \\ N_m^{2,1} & N_m^{2,2} \end{pmatrix}, \quad D_m(s,p) = s^2 + p_0 \cdot s + p_1, \quad N_m^{i,j}(s,p) = p_{1+2\cdot(i-1)+j} \cdot s + a^s_{2\cdot(i-1)+j} \cdot p_1, \quad (8)$$

for the MIMO systems.

Now to provide the 2nd order model stability we require the following condition

$$-p_0 < 0 \Rightarrow p_0 > 0, \quad (9)$$

where the parameter comes from (7) or (8).

We want the model with the reduced order to be an adequate estimation of the LTI system, so its response needs to be close to the response of the model with the higher order on the same control input $u(t)$. The response is a function on a time domain and for both models it can be found by solving the Cauchy problem for (2) and (7) or (4) and (8). Since we consider the unit-step and the Dirac functions as inputs, the output can be expressed via the inverse Laplace transformation:

$$x_u(t,p) = L^{-1}\left(G_m(s,p) \cdot L(u(t))\right), \quad \hat{x}_u(t) = L^{-1}\left(G(s) \cdot L(u(t))\right), \quad (10)$$

for the SISO LTI systems and

$$x_u^{i,j}(t,p) = L^{-1}\left(\frac{N_m^{i,j}(s,p) \cdot L(u(t))}{D_m(s,p)}\right), \quad \hat{x}_u^{i,j}(t) = L^{-1}\left(\frac{N_s^{i,j}(s) \cdot L(u(t))}{D_s(s)}\right), \quad (11)$$

for the MIMO LTI systems. Using expressions (10) and (11) to calculate the responses of the models on different input functions, one can identify parameters as the solution of the extremum problem

$$C_{siso}^u(p) = \sum_{i=0}^{N}\left(\hat{x}_u(t_i) - x_u(t_i,p)\right)^2, \quad C_{siso}^u(p) \to \min_{p \in R^3}. \quad (12)$$

for the SISO system or

$$C_{mimo}^u(p) = \sum_{i,j}\sum_{k=0}^{N}\left(\hat{x}_u^{i,j}(t_k) - x_u^{i,j}(t_k,p)\right)^2, \quad (13)$$

$$C_{mimo}^u(p) \to \min_{p \in R^{2+N_s}},$$

for the MIMO system. In criteria (12) and (13) the values $t_i = T \cdot i/N, i = \overline{1,N}$ are the time points, T is the final time and N is the number of points.

In this study, a penalty function is used to implement the stability condition (9) into the criteria (12) and (13). The modified criteria are as follows,

$$\tilde{C}_{siso}^u(p) = C_{siso}^u + c \cdot P(p_0) \to \min_{p \in R^3}, \quad (14)$$

$$\tilde{C}_{mimo}^u(p) = C_{mimo}^u(p) + c \cdot P(p_0) \to \min_{p \in R^{2+N_s}}, \quad (15)$$

where $P(\cdot): R \to R^+ \cup \{0\}$ is a static penalty function $P(x) = \begin{cases} 0, x > 0 \\ \|x\|, x \le 0 \end{cases}$ and $c > 0$ is a coefficient.

To analyse the solution adequacy on the whole time domain three more criteria are used. These criteria are involved in comparing the efficiency of the approaches. Let $x_\eta(t) = x_\eta(t, p^*), \hat{x}_\eta(t)$ be the solutions of (10) or (11), the input is the unit-step function $u(t) = \eta(t)$ and $p^* = \arg\min \tilde{C}_{siso}^\eta(p)$ or $p^* = \arg\min \tilde{C}_{mimo}^\eta(p)$, depending on the problem. The first criterion we want to calculate is the integral square error

$$I_1 = \int_0^{+\infty}\left(x_\eta(t) - \hat{x}_\eta(t)\right)^2 dt, \quad (16)$$

Its estimation was used to identify the parameters via solving problems (14) or (15). The integral (16) is divergent if $\lim_{t\to+\infty} x_\eta(t) \ne \lim_{t\to+\infty} \hat{x}_\eta(t)$, and for this reason the function (6) is implemented and the stability condition is required.

The next criteria concern the relative integral square error; they are given in (Parmar and Prasad, 2007) and are proposed in order to check the accuracy of the model. Both criteria are expressed with by fraction:

$$I_2 = \frac{\int_0^{+\infty}\left(x_\eta(t) - \hat{x}_\eta(t)\right)^2 dt}{\int_0^{+\infty}\left(x_\eta(t) - x_\eta(+\infty)\right)^2 dt}, \quad (17)$$

and the second is for the input $u(t) = \delta(t)$,

$$I_3 = \frac{\int_0^{+\infty} \left(x_\delta(t) - \hat{x}_\delta(t) \right)^2 dt}{\int_0^{+\infty} \left(x_\delta(t) \right)^2 dt}. \tag{18}$$

The result of the inverse Laplace transformation (10) and (11) can be found symbolically for the current problems, where the initial and reduced order models are linear.

3 RESTART META-HEURISTIC AND PICEA-G

The optimization problem considered in this study can be represented in a following way:

$$C(a): A \to C_A \subset R^m, \dim(A) = n, \\ C(a) = \left(C_1(a) \ \ \dots \ \ C_m(a) \right) \to \underset{a \in A}{extrem}, \tag{19}$$

where A is a space of alternatives with dimension n , C_A is a subspace of some Euclidean vector space R^m , $i = \overline{1,m} : C_i(\cdot) : A \to C_A^i \subset R, \prod_j C_j(A) = C_A$ are the unknown mappings. After the problem formulation and the determination of the identification parameters, we can use a bijection between alternatives and binary strings, so every alternative can be determined with a real value vector and thus a binary string. Generally, the criteria (19) are computable functions or mappings with unknown properties and unknown symbolic form.

For solving MO BBOP we propose using the PICEA-G algorithm, which is population-based. Each population is a set of different solutions – a set of alternatives and our aim is to approximate the Pareto front. In this case, there is a contradiction between the need for an in depth search to improve current solutions and for a search in breadth to approximate the whole front.

To resolve this contradiction we put forward a hypothesis that restarting the Pareto front estimation algorithm improves the population-based optimization algorithm efficiency. This is why an independent restarting operator meta-heuristic was designed and implemented. The proposed meta-heuristic estimates if the stagnation condition is met and evaluates the parameters for the randomized performing of the initial generation. The stagnation estimation is based on the distances between the Pareto front estimations, which are taken at the current generation and the previous one and consist only of non-dominated individuals. If the distance does not change for a given number of generations, the MO optimization algorithm restarts. A more detailed explanation is given below.

Let the population in the i -th generation be noted as P_i. For each algorithm generation a set $S_i = \left\{ a_j \in A : \nexists k < i, j(k) \le |S_k| : a_j \overset{C}{\prec} a_{j(k)}, a_{j(k)} \in S_k \right\}$ and a set $F_i = \left\{ C(a_j), a_j \in S_i \right\}$ are formed. These sets are the Pareto set and front estimations at the i -th generation, respectively. It is easy to see that $\forall i \ S_i \subset S_{i-1} \bigcup P_i$, so the distance $\rho(F_i, F_{i-1})$ between two different sets F_i and F_{i-1} is calculated for the non-dominated solutions found in the current generation. Let F be a set of any limited cardinality $F = \left\{ f_i \in R^m, i = \overline{1,|F|} \right\}$, then

$$\rho(F_a, F_b) : F \times F \to R^+ \bigcup \{0\}, \\ \rho(F_a, F_b) = \frac{1}{|F_a|} \cdot \sum_{i=1}^{|F_a|} \min_{j \le |F_b|} \left(\left\| (F_a)_i - (F_b)_j \right\|_{R^m} \right), \tag{20}$$

where $\|\cdot\|_{R^m} : R^m \to R^+ \bigcup \{0\}$ is a norm on the R^m vector space.

The decision of whether to perform a restart or not is made on the basis of the specific variable value. This variable is the diameter of a set, which is a queue that consists of the metric values of the previous iterations. Let the number of iterations be noted as l_{tail} , then the set is determined in the following way

$$Tail_i(l_{tail}) = \left\{ \rho(F_j, F_{j-1}) : i - l_{tail} < j \le i \right\}, \tag{21}$$

and the meta-heuristic performs the restart if the following condition

$$\max_{j < l_{tail}} \left\{ Tail_i(j) \right\} - \min_{j < l_{tail}} \left\{ Tail_i(j) \right\} \le \delta_{tail}, \tag{22}$$

is met. As can be seen from equations (21) and (22), two different operator settings are used: the tail length l_{tail} controls the size of the observation period and δ_{tail} is a threshold level.

Now, if the restart takes place, we collect the current algorithm run data and put it into the sets to gather information about the MO optimization problem and algorithm's behaviour to provide its control with the meta-heuristic. In this case, we need the estimations of the Pareto front and Pareto set, the last generation population and its criteria values. These sets are used for performing the final solution and initial generation population of the next algorithm run:

$$Memory_S = Memory_S \bigcup \{S_i\},$$
$$Memory_F = Memory_F \bigcup \{F_i\},$$
$$Memory_P = Memory_P \bigcup \{P_i\},$$
$$Memory_C = Memory_C \bigcup \{\tilde{C}_i\},$$

where $\tilde{C}_i = \left\{F\left(c_j\right) : c_j = \left(P_i\right)_j,\ j = \overline{1,|P_i|}\right\}$.

The generation of the initial population is an important feature of the meta-heuristic and it directly influences the algorithm's performance. This generation is controlled by two parameters: the probability of each individual in this initial population being randomly generated - α, and the probability of each gene of the individual being changed to the opposite - β, in the case of the individual not being randomly generated. Each j-th individual can be generated by one of the proposed schemes and it means that its k-th gene in the initial population is generated in one of the following ways:

$$\left(\left(P_0\right)_j\right)_k = r_{j,k},\ P\left(r_{j,k} = 0\right) = P\left(r_{j,k} = 1\right), \tag{23}$$

with the probability α and with the probability $1-\alpha$:

$$\left(\left(P_0\right)_j\right)_k = f_c\left(\left(\left(Memory_S\right)_{r_j^1}\right)_{r_j^2}, r_{j,k}^3\right), \tag{24}$$

where k is the index of a gene, $f_c\left(v,p\right) = \begin{cases} v, & p = 0 \\ \neg v, & p = 1 \end{cases}$ is a special function and r_j^1, r_j^2, $r_{j,k}^3$ are the random values:

$$P\left(r_j^1 = 1\right) = \ldots = P\left(r_j^1 = \left|Memory_S\right|\right),$$
$$P\left(r_j^2 = 1\right) = \ldots = P\left(r_j^2 = \left(Memory_S\right)_{r_j^1}\right),$$
$$P\left(r_{j,k}^3 = 0\right) = 1 - P\left(r_{j,k}^3 = 1\right) = \beta .$$

By varying parameters α and β we control the initial population generation. If we want the initial population to be completely randomized, we set α to 1, and if we want it to be in a some sense near to some previously estimated Pareto set solutions, we set it closer to 0 and β closer to 0 too, where β represents the closeness of the new individual to a found one.

In our study, the restart meta-heuristic is incorporated into the Preference-inspired Co-evolutionary Algorithm using goal vectors (PICEA-g) proposed by Wang in 2013 (Wang, 2013). This algorithm relates to a class of preference-inspired co-evolutionary algorithms (PICEAs) which are based on the concept of co-evolving the population with decision-maker preferences.

PICEA-g includes the following steps:

1. Generate an initial population and evaluate objective values for individuals. Find non-dominated candidate solutions in the population and copy them into the *archive*. Determine the set of *goal vectors* as a number of targets randomly generated within the goal vector bounds.
2. Produce the offspring solutions with *genetic operators*: selection, crossover and mutation. Evaluate objective values for new generated individuals.
3. Pool together parents and children; compile the common set of objective values.
4. Append to the set of goal vectors the additional targets generated within the determined bounds.
5. Assign fitness values for goal vectors and for individuals in the united population.
6. Form the new population and the set of goal vectors based on their fitness.
7. Update the archive with new non-dominated solutions.
8. Check the stopping criterion: if it is satisfied then finish the search with the archive set, otherwise proceed with the second step.

In Steps 1 and 4 decision-maker preferences are incorporated into the algorithm by using goal vectors. They represent points generated in the criteria search space within bounds determined according to the rule:

$$\begin{aligned} g_i^{\min} &= \min(BestF_i), \\ \Delta F_i &= \max(BestF_i) - \min(BestF_i), \\ g_i^{\max} &= \min(BestF_i) + \alpha \times \Delta F_i, \end{aligned} \tag{25}$$

where $g_i^{\min}$ is the lower bound and $g_i^{\max}$ is the upper one for the i-th goal vector component, $BestF_i$ is the best value of the i-th objective function amid solutions in the archive, $i = \overline{1,M}$, M is the number of criteria. The recommended value of the α parameter is 1.2.

4 PERFORMANCE INVESTIGATION

Since we propose the multi-objective optimization problem, and using the proposed approach results in receiving an estimation of a Pareto set and not a single solution, for each problem we should present the best solution by the first criterion and the best one by the second criterion. As in similar investigations, we assign a limit to the maximum number of fitness function evaluation as being equal to 2500. To tune the restart meta-heuristic parameters we performed additional experiments for the same MO optimization problems, where the values of the restart operator parameters were varied. According to the results of these experiments the following parameters were chosen: α=0.9, β=0.7, l_{tail}=5, δ_{tail}=0.0005.

The first problem we consider is the SISO system, which is determined by the equation

$$G(s)=\frac{s^3+7\cdot s^2+24\cdot s+24}{s^4+10\cdot s^3+35\cdot s^2+50\cdot s+24}, \qquad (26)$$

and for which we received models: the best one by the first criterion value $G^*_{\eta(t)}(s)=\frac{0.7696275\cdot s+1.621897}{s^2+2.522232\cdot s+1.621897}$ and the best one by the second criterion value $G^*_{\delta(t)}(s)=\frac{0.86107\cdot s+0.679314}{s^2+1.568184\cdot s+0.679314}$, after 25 independent launches of the proposed PICEA-g with the restart meta-heuristic.

The initial model and reduced model outputs are given in Figure 1, where the dotted line is the initial model output and the solid line is the output of the reduced model. The numeric adequacy estimation is given in Table 1, where the results of the proposed approach are compared with the results received in different studies using other approaches and optimization tools, including the PICEA-g algorithm without the restart meta-heuristic. Knowing the model parameters makes it possible to calculate criteria and compare approaches. Here we use the following notation: with “the proposed approach” we mean the solutions found by PICEA-g with the restart meta-heuristic, 1 – is the same approach, but without restarting, 2 – COBRA optimization tool and asymptotical equivalence (Ryzhikov et al., 2017), 3 – (Desai, Prasad, 2013), 4 – (Parmar el. al., 2007) and 5 – (Narwal, Prasad, 2016).

Table 1: SISO problem (26): performance of approaches.

	Criterion		
Approach	I_1	I_2	I_3
Proposed η	$7.485\cdot10^{-5}$	$1.313\cdot10^{-4}$	$6.515\cdot10^{-3}$
Proposed δ	$4.205\cdot10^{-4}$	$7.373\cdot10^{-4}$	$6.047\cdot10^{-3}$
1, η	$7.564\cdot10^{-5}$	$1.326\cdot10^{-4}$	$6.550\cdot10^{-3}$
1, δ	$1.134\cdot10^{-3}$	$1.989\cdot10^{-3}$	$6.198\cdot10^{-3}$
2	$7.458\cdot10^{-5}$	$1.308\cdot10^{-4}$	$6.901\cdot10^{-3}$
3	$2.841\cdot10^{-4}$	$4.982\cdot10^{-4}$	$5.236\cdot10^{-3}$
4	$2.394\cdot10^{-4}$	$4.197\cdot10^{-4}$	0.018
5	$1.986\cdot10^{-3}$	$3.483\cdot10^{-3}$	$7.612\cdot10^{-3}$

The approximations of the Pareto front, which were made during every algorithm launch and the randomly chosen single Pareto front estimation, are given in Figure 2, where the criteria are represented with a mapping $1/(1+C)$, where C is a criterion, and this mapping was maximized by the searching algorithm. As can be seen, there is not such a solution that would bring the maximum of two of these criteria representations at the same time. This is why it is necessary to solve the multi-objective optimization problem if the model must satisfy more than one criterion.

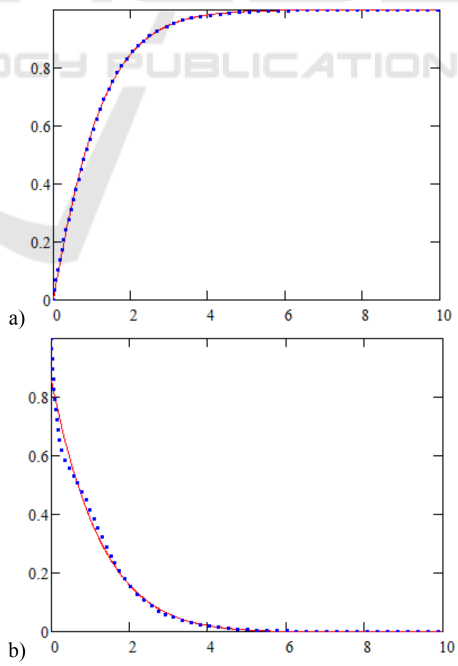

Figure 1: Initial model (dotted line) and lower order model (solid line) outputs for the – a) - unit-step input function and – b) – Dirac input function.

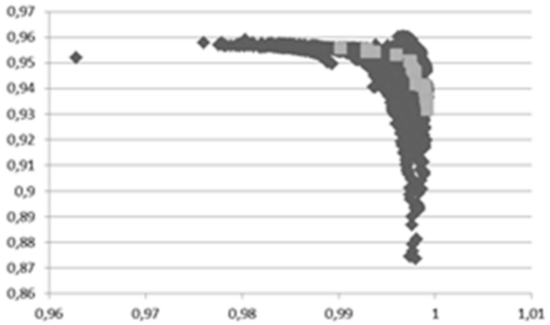

Figure 2: Pareto front estimation in all of the runs (black) and a single front estimation (grey).

Although the PICEA-g with the restart meta-heuristic is a multi-objective optimization tool and it is efficient in solving the problem with two criteria, solutions with maximum criterion values outperform most of the solutions obtained by the optimization algorithms solving a single criterion problem.

A similar problem was considered for the MIMO system order reduction problem

$$H(s)=\begin{pmatrix} \frac{2\cdot(s+5)}{(s+1)\cdot(s+10)} & \frac{s+4}{(s+2)\cdot(s+5)} \\ \frac{s+10}{(s+1)\cdot(s+20)} & \frac{s+6}{(s+2)\cdot(s+3)} \end{pmatrix} \quad (27)$$

for the same computational resources and algorithm runs we received the set of models with the highest criteria values given in Table 2.

Table 2: MIMO problem (27): solution found.

$G^*_{\eta(t)}(s)$
$D^*_m(s)=s^2+3.145035\cdot s+2.168462,$
$N^{*1,1}_m(s)=1.206913\cdot s+2.168462,$
$N^{*1,2}_m(s)=0.927334\cdot s+0.867384,$
$N^{*2,1}_m(s)=0.515576\cdot s+1.084231,$
$N^{*2,2}_m(s)=1.581389\cdot s+2.168462,$
$G^*_{\delta(t)}(s)$
$D^*_m(s)=s^2+4.989368\cdot s+4.344733,$
$N^{*1,1}_m(s)=1.7814044\cdot s+4.344733,$
$N^{*1,2}_m(s)=1.028391\cdot s+1.737893,$
$N^{*2,1}_m(s)=0.792901\cdot s+2.172366,$
$N^{*2,2}_m(s)=1.088212\cdot s+4.344733,$

As for the SISO problem, the outputs for unit-step and Dirac function inputs are given in Figures 3 and 4, respectively. In these figures a) represents the outputs of (1,1) model components, b) represents the outputs of (1,2) components, c) represents the outputs of (2,1) components, and d) represents the outputs of (2,2).

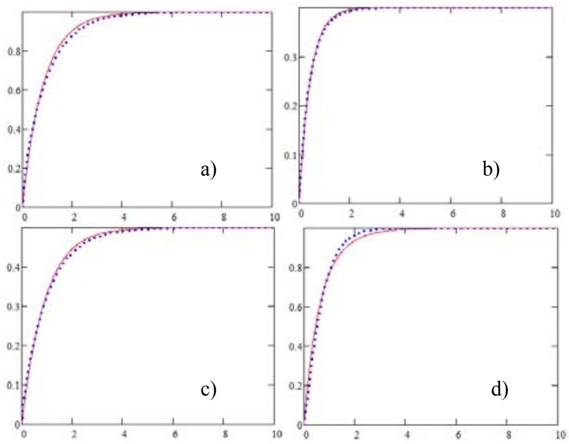

Figure 3: Initial model (dotted line) and lower order model (solid line) outputs for the unit-step function.

Similar experimental results are compared in Table 3, but there criteria are summarized by all the model components.

Also, the Pareto front estimations are given in Figure 5.

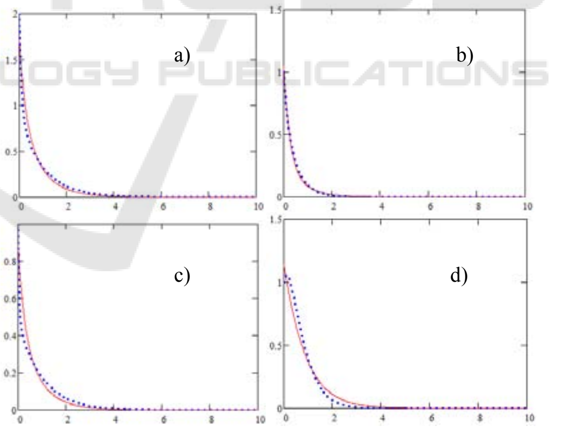

Figure 4: Initial model (dotted line) and lower order model (solid line) outputs for the Dirac function.

Here we use the following notation: "the proposed approach" is PICEA with the restart meta-heuristic, 1 – is the same, but without the restart, 2 – COBRA optimization tool and asymptotical equivalence (Ryzhikov et al., 2017), 3 – (Desai, Prasad, 2013) and 4 – (Narwal, Prasad, 2016).

Table 3: MIMO problem (27): performance of approaches.

	Criterion		
Approach	$\sum I_1$	$\sum I_2$	$\sum I_3$
Proposed η	$5.004 \cdot 10^{-3}$	0.022	0.128
Proposed δ	0.028	0.103	0.095
1 η	$6.670 \cdot 10^{-3}$	0.022	0.140
1 δ	0.025	0.970	0.102
2	$3.323 \cdot 10^{-3}$	0.027	0.136
3	0.02	0.325	0.218
4	0.045	0.372	0.409

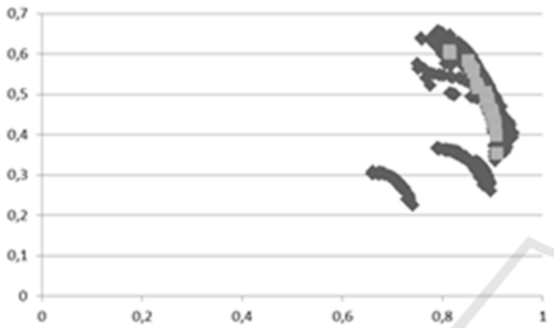

Figure 5: Pareto front estimation in all of the runs (black) and a single front estimation (grey).

To summarize, all the figures and examination results prove that the proposed approach and the optimization algorithm are a reliable combination of techniques for solving the order reduction problems.

5 CONCLUSIONS

It is widely known that solving the order reduction problem for LTI systems requires a powerful and reliable global optimization tool for black-box problems. Many researchers, according to other studies on this topic, are using heuristic optimization techniques, which allow them to achieve satisfying results. However, for some problems there is an aim not just to identify the parameters by some criterion, but to identify the parameters which would fit two or more criteria.

In order to solve the multi-objective problem, it is necessary to use the MO optimization algorithm because the Pareto front is not just a single point in a vector space and, generally, it cannot be determined with additive or multiplicative combination of the criteria. Figures 3 and 5 prove this hypothesis for the considered problems. It can be seen that the Pareto front is a curve, so the best solution for the unit-step function would not prove that this model is the best for another input. Results received in a single run, which are marked in these figures in grey, prove that we receive an acceptable approximation of the Pareto front. As was shown in this study, a meta-heuristic can be used to sufficiently improve the multi-objective optimization algorithm performance with the same computational resources.

This is one more class of optimization problem for which the algorithm efficiency and performance improve after implementing the proposed restart operator. The results of this work demonstrate that this algorithm is not only good at estimating the Pareto front, but can also find good solutions, which are close or even outperform the best solutions found by the single criterion optimization tools using the same resources.

Further work is related to improving the quality of the estimation of the Pareto front in the case of a higher criterion number as well as to developing a meta-heuristic to improve the proposed restart operator and the performance of different multi-objective algorithms. The other aspect of further work is related to using a modified optimization tool to solve MIMO order reduction problems in which each output is characterized by its own criteria.

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