Game-theoretic End-to-end Throughput Optimisation in Wireless Sensor Networks

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Keywords: Transmission Power, Expected Transmission Count (ETX), Throughput, Fictitious Play, Lyapunov, Potential Game.

Abstract: One of the most important problems in the Wireless Sensor Network community is the enhancement of the end-to-end throughput that strengthens the reliability of the network. Transmission power adjustment may play a key role in accomplishing better throughput. Increasing transmission power to make the signal strength better is the intuitive solution; however, this may introduce certain problems such as interference and more energy consumption. However, decreasing the transmission power may result in a weak signal strength that may result in unreliable links, which also affect throughput significantly. One of the most important metrics for link reliability is the Expected Transmission Count (ETX). We take the additive ETX from the basestation to every node and we aim to optimise the route throughput by setting the transmission power accordingly. We address these trade-offs and we propose a game-theoretic solution that aims to maximize the end-to-end throughput between network nodes, while using the optimal transmission power. In this paper, we provide the conditions for the convergence of our algorithm to a pure Nash equilibrium. We show that our algorithm converges to the global optimum and that it is Lyapunov stable. We provide evidence that our algorithm converges to the best response dynamics under the fictitious play learning algorithm.

1 INTRODUCTION

End-to-end throughput (Li et al., 2001) is a major issue in Wireless Sensor Networks (WSN)s. A WSN comprises a set of links formed by nodes that transmit their packets, in order to reach a basestation. However, nodes experience interference during their transmission, thereby making the transmission difficult. Furthermore, the delay of the transmission of the packet increases, since the interfered node has to do an exponential backoff (Committee et al., 1997) and retransmit the packet. At the same time, this node may be receiving packets making its link quality with the sender to increase. Thereafter, the messages enter a queue and need to be serviced with the minimum delay, thus impacting the capacity of the link (Jun and Sichitiu, 2003).

One approach to enhance the link quality between nodes forming links, which expand to the network's reliability of communication is to increase the radio transmission power level, in order to strengthen the signal strength. Link quality is directly related to throughput, as we can see in the novel paper presenting the Expected Transmission Count (ETX) metric (De Couto et al., 2005). However, raising the transmission power may result in packets being lost due to the complexities of the wireless channel. An increase in transmission power might cause an increase in interference and collisions, decreasing the number of packets received; hence, the end-to-end throughput of the network. On the other hand, as we see in (Spyrou and Mitrakos, 2015b), if the distance between the transmitter-receiver and interferer-receiver is difference by approximately a factor of 2, interference does not cause packet loss. This indicates that a node may select a high transmission power level, in order to strengthen its signal, without suffering from packet loss. Moreover, packets using different transmission powers may result in their packets simultaneously transmitted successfully, depending on their distance and transmission power level (Moscibroda et al., 2006).

There is a sweet spot in ETX related to transmission power levels that can keep throughput to a high level, while not using a larger transmission power level than necessary. The transmission power also affects the energy consumption of the node, directly influencing the lifetime of the WSN (Antonopoulos et al., 2009). In order to handle this trade-off, we present a finite strategy distributed game-

Spyrou, E. and Mitrakos, D.
Game-theoretic End-to-end Throughput Optimisation in Wireless Sensor Networks.
DOI: 10.5220/0006399400170026
In *Proceedings of the 14th International Joint Conference on e-Business and Telecommunications (ICETE 2017) - Volume 6: WINSYS*, pages 17-26
ISBN: 978-989-758-261-5

theoretic approach that maximizes each node's end-to-end throughput, while using the optimal transmission power from an optimisation point of view. Specifically, we focus on the trade-offs between energy consumption, and ETX. We use game theory, since it can appropriately describe the behavior of selfish nodes and find an optimal solution in a distributed manner. Modeling systems with selfish algorithms have been shown to provide efficient solutions that improve network performance (Yeung and Kwok, 2006). We consider nodes to be individual players that play selfishly in order to find a best response for their objectives. We propose a game-theoretic model of the end-to-end throughput optimisation algorithm. We call this algorithm Game-theoretic End-to-End Throughput Optimisation Algorithm (GETOA).

The contributions in this paper are the following:

- We aim to solve the end-to-end throughput by utilising the additive ETX value.
- We show the relationship between ETX with capacity and delay.
- We formulate a game theoretic model with finite strategies and show that it is a potential game. This means that it converges to a Nash Equilibrium
- We show that it reaches the global optimum.
- We prove that the Nash Equilibrium is Lyapunov stable.

This paper is structured as follows: Section 2 provides the related work, section 3 gives a brief description of game theory and potential games, section 4 gives the system model, section 5 provides our game-theoretic algorithm, section 6 gives the results of our approach and section 7 presents the conclusions and the future work.

2 RELATED WORK

To the end of solving the end-to-end throughput issue, there has been a plethora of approaches that dealt with the Medium Access Control (MAC) layer (Sun et al., 2015), (Ai et al., 2004),(Rajendran et al., 2006), (wal,). However, in this work we are addressing the problem in the network layer and we are providing the reader with necessary information regarding the relationship of capacity and delay with ETX. There are practical works in the literature that implicitly show enhancement of throughput by adjusting transmission power and relate it to link quality (Son et al., 2004), (Lin et al., 2016), (Hackmann et al., 2008). Hence, in this work we are focusing on an approach that attempts to enhance end-to-end throughput further up the stack from the MAC layer.

Zeng et al. (Zeng et al., 2008) studied opportunistic routing, which may cope with poor link reliability by taking advantage of the broadcast nature and spatial diversity of the wireless medium. The authors target scenarios with multiple rates, interference, candidate selection and prioritisation on the maximum end-to-end throughput or capacity of opportunistic routing. By carefully considering wireless interference, transmitter conflict graphs are composed, in order to introduce concurrent transmitter sets as constraints related to the transmission conflicts or opportunistic routing. Thereafter, the maximisation of the end-to-end throughput is formulated as a maximum-flow linear programming problem subject to the transmission conflict constraints. Moreover, a rate selection method is proposed to perform a comparison of multiple rate scenarios against single rate ones. The results given in the paper provide evidence that end-to-end throughput can be enhanced as well as that the multiple rate scenario improves throughput as well.

Choi et al. (Choi and Lee, 2014) address the multi-hop link property of link selection, where the increase in the rate of link may be the reason for the decrease of another link's rate. The end-to-end throughput in a multi-hop network is restricted by the lowest rate of a link. This suggests that the max-min fair allocation of the link rates constitutes an optimal strategy that maximises end-to-end-throughput. The authors suggest an approach that makes all link rates equal, thus having the max-min fair allocation property, using a transmission power control algorithm. In particular this distributed algorithm operates by a node averaging the link rates close to it and adjusts its transmission power to accomplish the average rate. Thereafter, it repeats this operation until all rates are equalised. The results shown in the paper maximizes end-to-end throughput while enhancing energy efficiency of multi-hop nodes.

Yu et al. (Yu et al., 2015) addressed the problem of network capacity performance in the presence of interference in a multi-hop wireless network, when nodes are competing for the channel medium. In this work, the minimisation of interference power, in order to maximise network capacity is discussed. To this end, the authors propose a consensus power control algorithm that maximises end-to-end throughput. This algorithm adjusts the transmission powers of the nodes to maximize the average end-to-end throughput with a consensus coefficient. Results in this work show that maximum average end-to-end throughput is achieved for all traffic flows and energy efficiency is accomplished. The drawback of this approach lies in the use of the algorithm in dense wireless network deployments.

Durmaz Incel et al. (Incel et al., 2012) studied the information collection in tree based sensor network deployments. Hence, they evaluate methodologies that belong to the family of many-to-one communication scheme, known as convergecast. The authors consider time scheduling on a channel with and without transmission power control settings. The former targets the minimisation of the required time slots to achieve convergecast. The latter employs power control to reduce schedule length using multiple frequencies. The authors provide lower bounds on the schedule length when interference complete diminishes, and suggest approaches that ensure the achievement of such bounds. Furthermore, performance of a number of channel assignment methods suggest that multi-frequency scheduling is enough to eliminate most of the interference. The finding is that data rate is not only dependent on interference; thus, spanning trees are constructed that result in the improvement of scheduling performance using a number of deployment densities.

Chantzis et al. (Chantzis et al., 2014), suggests a scheme that aims to provide information regarding the local network throughput impact on topological properties, such as maintenance of neighbourhoods and load balancing. To this end, the authors propose a protocol that adaptively tunes transmission power with low throughput settings where nodes accomplish a degree of symmetric and coherent links. Furthermore, the network throughput is maximised provided that degree is satisfied. Results show that link quality and symmetry as well as links degree satisfaction can be regulated appropriately by transmission power and adaptive throughput control.

3 GAME THEORY AND POTENTIAL GAMES

Game theory studies mathematical models of conflict and cooperation (Von Neumann et al., 2007), between players. Therefore, our meaning of the term game corresponds to any form of social interaction between two or more nodes. The rationality of a node is satisfied if it pursuits the satisfaction of its preferences through the selection of appropriate strategies. The preferences of a node need to satisfy general rationality axioms, then its behavior can be described by a utility function. Utility functions provide a quantitative description of the node's preferences and the main objective is therefore the maximization of its utility function.

In this work, we focus on strategic non-cooperative games, since we consider nodes to act as selfish players that want to preserve their interests. The intuition behind this is that the nodes will reach an optimal state, without having to pay a price to maximize their payoffs. The Nash equilibrium (Nash Jr, 1950) is the most important equilibrium in non-cooperative strategic form games. It is defined as the point where no node will increase its utility by unilaterally changing its strategy.

In 2008, Daskalakis proved that finding a Nash equilibrium is PPAD-complete (Daskalakis et al., 2009). Polynomial Parity Arguments on Directed graphs (PPAD) is a class of total search problems (Papadimitriou, 1994) for which solutions have been proven to exist, however, finding a specific solution is difficult if not intractable. This development drove the community to 'Potential Games', since they guarantee the convergence to pure Nash equilibria and best response dynamics.

This class of games consists of the exact and ordinal potential games. This work employs exact potential games and refer the reader to (Monderer and Shapley, 1996) for details on the methodology. In order to use exact potential games, it is essential to have a potential function that has the same behavior as the individual utility function, when a player unilaterally deviates.

More formally:

A game $G\langle N, A, u\rangle$, with N players, A strategy profiles and u the payoff function, is an exact potential game if there exists a potential function

$$V : A \rightarrow \mathbb{R} \tag{1}$$

subject to

$$\forall i \in N, \forall \sigma_{-i} \in A_{-i}, \forall \sigma_i, \sigma_i^{'} \in A_i \tag{2}$$

where σ_i is the strategy of player i, $\sigma_i^{'}$ is the deviation of player i, σ_{-i} is the set of strategies followed by all the players except player i and A_{-i} is the set of strategy profiles of all players except i such as

$$V(\sigma_i, \sigma_{-i}) - V(\sigma_i^{'}, \sigma_{-i}) = u_i(\sigma_i, \sigma_{-i}) - u_i(\sigma_i^{'}, \sigma_{-i}) \tag{3}$$

4 SYSTEM MODEL

We consider a wireless network that consists of a number of nodes that transmit their data in a wireless fashion. The network is essentially an undirected graph G which has V number of vertices and E number of edges $G = (V, E)$. Link asymmetry is considered, thereby the network nodes send data and acknowledgment packets to each other.

One of the key issues of the wireless links, is the number of link layer transmissions of a packet. This is a good metric that aims to increase throughput of a link as well as the network, by minimising transmissions and, of course, retransmissions of packets. Thus the metric ETX emerged, which is the average number of transmissions of data and ACK packets. A node calculates ETX by obtaining the frame loss ratio of a wireless link l with each of its neighbouring nodes in the data direction, denoted as PRR_{frwd}. Thereafter it continues by repeating the aforementioned procedure in the opposite direction, denoted as PRR_{bkwd}. ETX is widely known as the inverse of the probability of Packet Success Delivery given as

$$ETX_l = \frac{1}{PRR_{frwd} * PRR_{bkwd}} \tag{4}$$

As it is clear from equation (4) a link is perfect if its ETX value is 1. Moreover, the route ETX is the sum of the ETX of every link in the route. Hence, a two-hop route of perfect links has an ETX of 2. As we can see, the larger the ETX value the less reliable the links. ETX has several significant features, such as that it affects throughput, since it it depends on delivery ratios. Also, it detects link asymmetry by employing bidirectional ratios, uses precise link loss ratio measurements, and penalizes routes with more hops, which have lower throughput due to interference between different hops of the same path (Li et al., 2001). In addition, ETX may implicitly lower the energy consumption per packet, as each transmission or retransmission may increase a node's energy consumption.

At this point we will provide the relationship of ETX with transmission power of each node. Note that we consider a Rayleigh channel (Rappaport et al., 1996). For a wireless link (i, j), the Packet Reception Ratio $PRR_{i,j}$ is defined as the ratio of the number of packets received by node j over the number of packets sent by node i. This is the PRR_{frwd} and similarly, $PRR_{j,i}$ is the PRR_{bkwd}. It can be expressed by approximation as

$$PRR_{i,j} = (1 - \xi)^l \tag{5}$$

where l is the packet length in bits. The Bit Error Rate (BER), which we denote as $\xi_{i,j}$, is given by the following formula (Fu et al., 2012)

$$\xi_{i,j} = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_{i,j}}{1+\gamma_{i,j}}}\right) \tag{6}$$

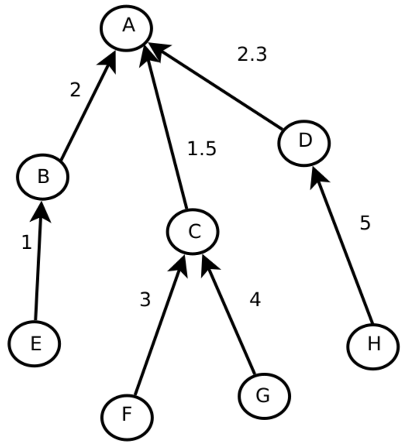


Figure 1: ETX valued network.

where $\gamma_{i,j}$ is the Signal-to-Interference-plus-Noise Ratio (SINR) of the transmission from node i to node j. $\gamma_{i,j}$ is given by

$$\gamma_{i,j} = \frac{H_{i,j} p_i}{\sum_{t \neq i, t \neq j} p_t H_{t,j} + N_0} \tag{7}$$

We model the wireless channel as a log-distance shadowing path loss channel, defined as

$$PL(d) = PL(d_0) + 10\eta log(\frac{d}{d_0}) + X_\sigma \tag{8}$$

where $PL(d)$ is the path loss at distance d, $PL(d_0)$ is the path loss, which is known, at reference distance d_0, η is the path loss exponent and X_σ is the zero-mean Gaussian random variable with standard deviation σ. Our setting is based on a dynamic environment, possibly indoor, where the nodes transmit or receive in a multipath manner. This setting is equivalent to node mobility. Shadow fading and distance based attenuation vary; hence, the instant value of the magnitude of the received signal is a random variable with Rayleigh probability density function. Here, we assume that the channel coefficients are constant for a codeword, but there are independent and identically distributed (i.i.d) for different blocks. For a set of different links, the channel fading coefficients are statistically i.i.d., which is a reasonable assumption, since nodes are spatially deployed (Sadek et al., 2006).

State-of-the-art wireless routing protocols, such as CTP (Gnawali et al., 2009), encapsulate ETX to encapsulate the routes packet reception quality by making it additive. An example can be seen in figure 1. As we can see, the ETX values of the links originating at the basestation are being added as the network goes downwards away from the basestation. Each node selects its parent node that it transmits to depending on the link quality of the available parents. For instance, node G has an ETX value of 5.5 to reach node

A, which is the basestation. This implicitly provides us substantial information regarding the hop-count of the nodes in the network. Throughput of the network may be optimised, since we can attempt to optimise the link quality in entire routes.

Hence, we consider the additive route ETX that we call AETX, which is given in equation (9). We denote R is the set of nodes that create the links of a route to the basestation for every node $i \in R$.

$$AETX = \sum_{i=1}^{R} ETX_i \tag{9}$$

4.1 Relationship between AETX and Capacity

We know that the capacity of the link is given by the Shannon's formula, which is a function of the SINR between nodes i, j in equation (7) as

$$f(\gamma_{i,j}) = W \log_2(1 + \gamma_{i,j}) \tag{10}$$

We can derive from equation (10) that the capacity of the link is maximised when the SINR of the link is maximum. From equation (5), it is straightforward to see that when SINR increases PRR increases as well, except when interference is too large, whereby the node has its maximum PRR by using a transmission power level lower than the maximum value (Hackmann et al., 2008). From equation (4) we can derive that the smaller the value of AETX, the larger the capacity of the wireless link. Furthermore, it is intuitive that we have a sweet spot of ETX that maximises the capacity between the two nodes by using one of the finite set of available transmission power levels. Furthermore, we see that in a status of competition for the wireless medium there is a value of the transmission power that is able to maximise the end-to-end throughput of the nodes route to the basestation given in equation (9) and maximise capacity at the same time.

Additionally, we are required to show the relationship between the AETX with the delay of the packet transmission due to interference, which will present in the following part of this paper. Our aim is to show that the selection of the most appropriate transmission power level will increase throughput and minimise the delay of packet transmission as well.

4.2 Relationship between AETX and Delay

In our network scenario we wish to satisfy certain QoS requirements, such as keeping the transmission and queuing delay smaller than an upper bound. We denote the upper bound of the delay as d. We assume that the incoming traffic in our network is Poisson distributed with average packet arrival rate λ_i and packet length of N bits. Thus, we have the rate of the source, which is given by $r_i = M\lambda_i$. We assume that the packets are queued in a FIFO queue. The time required for each packet transmission is given by

$$t_i = \frac{M}{R_i} \tag{11}$$

where R_i is the transmission rate of the queued packets.

We encapsulate a similar approach with Meshkati et al. (Meshkati et al., 2006) in our model, to indicate the relationship of the transmission delay with AETX and provide a QoS constraint. We consider an M/G/1 queue with Poisson traffic with the aforementioned parameter λ_i and service time s_i. We transform AETX to be normalised to $[0, 1]$, in order to make it resemble with AETX probability. To this end, the formula of AETX, which we denote as $AETX_i^{norm}$ is given by

$$AETX_i^{norm} = \frac{1}{1 - AETX_{max}} * AETX - \frac{AETX_{max}}{1 - AETX_{max}} \tag{12}$$

where $AETX_{max}$ is the maximum value that $AETX$ can take give the hops that node i exists away from the basestation. Hence, we have the Probability Mass Function (PMF)

$$Pr\{s_i = mt_i\} = AETX_i^{norm}(1 - AETX_i^{norm})^{m-1} \quad \text{for } m = 1, 2...$$

Thereafter, the service rate μ_i can be given by

$$\mu_i = \frac{AETX_i^{norm}}{t_i} \tag{13}$$

and the load is given by $\rho_i = \frac{\lambda_i t_i}{AETX_i^{norm}}$

The average packet delay as given by Meshkati is given by

$$\bar{d} = t_i \left(\frac{1 - \frac{\lambda_i t_i}{2}}{AETX_i^{norm} - \lambda_i t_i} \right) AETX_i^{norm} > \lambda_i t_i \tag{14}$$

From equation (14), we can see that a larger $AETX_i^{norm}$ leads to smaller delay. Hence, a smaller $AETX$ offers as quicker transmission of our packets. Thus, we find that we can define a utility function to create a game-theoretic algorithm that will aim to optimise the end-to-end throughput in a hierarchically routed network.

5 GETOA

We consider a game-theoretic formulation of the network $\Gamma = (N, A, u)$, where N is the number of players/nodes, A is the set of available strategies to a player and u represent the utility functions of the players.

We define the strategies of the players $A = p_1, ...p_A$ as a set of finite values that correspond to the transmission power settings of a wireless module. Furthermore, we define the utility function of a player i as

$$u_i = -AETX + c_i p_i \quad (15)$$

This a common definition of a utility function that utilises pricing, in order to make the game more efficient (Spyrou and Mitrakos, 2015a; Tsiropoulou et al., 2012). We transform the utility function of (15) in the following manner. We use the negative AETX to the power of two, in order to make the first term concave. The second term includes the variable $c_i > 0$, which is the cost of using the transmission power p_i and is assumed to be set to 1.

$$u_i = -(AETX)^2 + c_i p_i \quad (16)$$

We formulate our game as an exact potential game as shown below

Lemma 1. *The game Γ described above with utility functions as in (16) is an exact potential game and its potential function is given by $V = -\sum_i^N (AETX)^2 + \sum_i^N c_i p_i$.*

Proof.

$$V(p_i, p_{-i}) - V(p_i^{'}, p_{-i}) = u_i(p_i, p_{-i}) - u_i(p_i^{'}, p_{-i}) + \sum_{m \in N, m \neq i}^{N} (u_m(p_m, p_{-m}) - u_m(p_m^{'}, p_{-m}))$$

Since only one node can deviate $\sum_{m \in N, m \neq i}^{N} (u_m(p_m, p_{-m}) - u_m(p_m^{'}, p_{-m})) = 0$. Hence, we conclude that Γ is an exact potential game. We can derive the above from the proof of Monderer and Shapley where $\frac{\partial V(\mathbf{p})}{\partial p_m} = \frac{\partial u_m(\mathbf{p})}{\partial p_m}, m \in N$. □

5.1 Equilibrium Analysis and Best Response Dynamics

Remark 3.1: The potential function is significant since its maximisation, when a specific policy is played, results in this policy being an equilibrium of the designed game.

Since we have a finite strategy set, the potential function needs to satisfy the Larger Midpoint Property (LMP) (Ui, 2008). The converse is true as well; As such, if a policy is an equilibrium, it maximises the potential function. Our function in equation (16) is concave. Thus, Schur concavity is ensured and specific majorisation properties are satisfied (Olkin and Marshall, 2016). We will show that in the next part of this section.

We consider two n-dimensional vectors $\delta(1), \delta(2)$.

Definition 1: (Marshall et al., 2010) A vector $\delta(2)$ majorises $\delta(1)$, which we denote as $\delta(1) \prec \delta(2)$, if $\delta(2)$ is more "unregular" in the following fashion:

$$\begin{cases} \sum_{i=1}^{k} \delta_{[i]}(1) \leq \sum_{i=1}^{k} \delta_{[i]}(2), k = 1, 2, ..., n-1 \\ \sum_{i=1}^{k} \delta_{[i]}(1) = \sum_{i=1}^{k} \delta_{[i]}(2) \end{cases} \quad (17)$$

where $\delta_{[i]}(m)$ is a permutation of $\delta_i(m)$ satisfying the condition $\delta_{[1]}(m) \geq \delta_{[2]}(m) \geq ... \geq \delta_{[n]}(m), m = 1, 2$.

From equation 16 we can derive that the largest element of $\delta(2)$ is larger than the largest element of $\delta(1)$. Similarly, the smallest element of $\delta(2)$ is smaller than the smallest element of $\delta(1)$. Thereafter, we proceed in showing that we can accomplish the global optimum by investigating the Schur convexity properties of majorisation.

Definition 2:: A function $f : \mathbb{R}^n \to \mathbb{R}$ is Schur concave if $\delta(1) \prec \delta(2)$ suggests $f(\delta(1)) \geq f(\delta(2))$.

Definition 1 dictates that there is strong majorisation; Proposition $C.2$ of (Marshall et al., 2010) suggests that a function $f : \mathbb{R}^n \to \mathbb{R}$ is Schur-concave if it is symmetric and concave, . In the next part we will show that the potential function V is Schur-concave.

Lemma 2. *Function V is concave in N*

Proof. It is obvious that the function is concave. This can be concluded since if the second derivative of the potential function includes two terms, of which the first a concave term and the second will be set to 0. □

Proposition 1. *If the function $u(p)$ is concave then the function $V(p)$ is Schur concave.*

Proof. The proof is given by using the following corollary from (Marshall et al., 2010). □

Corollary 1. *Let* $\phi(x) = \sum_{i=1}^{n} g(x)$ *where* g *is concave. Then* ϕ *is Schur-concave*

Theorem 1. *The GETOA algorithm reaches the glocal optimum*

Proof. These majorisation properties can be utilised to show that our function follows LMP. Since our function is Schur concave the global optimum p^* majorises another potential p'^*. Since $V(p)$ is Schur concave it follows by definition that $V(p'^*) \geq V(p^*)$. Since, $p*$ maximises the potential, this is only possible when $V(p'^*) = V(p^*)$. Hence, p^* is the global optimum. Moreover, when the maximiser is reached we set the variable c_i of the potential function (16) equal to the derivative of its first term $-\sum_i^N (AETX)^2$ as in (Candogan et al., 2010). Thus, we have $\frac{\partial u_i}{\partial p_i} = 0$. Using this result and the majorisation of the function, which is concave, we conclude that our approach reaches the global maximum. □

In our non-cooperative game formulation we introduce the class of best-response dynamics, in which every node updates its strategy, in order to maximise its utility, given the strategies of the other nodes. We denote the best-response dynamics with β_i for the i^{th} node, which satisfies

$$\beta_i(\mathbf{p}_i) = \arg\max_{p_i \in P_i} u_i(p_i, \mathbf{p}_{-i}) \tag{18}$$

We are investigating finite actions for the nodes' strategies; hence, the best response dynamics may be addressed using the discrete-time fictitious play (DTFP) (Brown, 1951; Robinson, 1951). Fictitious play has been proven to converge in finite potential games (Monderer and Shapley, 1996).

In this paper, we are concentrating in the study of the dynamical properties of our potential game. To this end, we wish to show that the Nash Equilibrium that our approach is converging to is Lyapunov stable (Khalil and Grizzle, 1996).

Theorem 2. *Let* $\Phi = V_{max} - V_{min} > 0$ *a Lyapunov function. The Nash Equilibrium that our algorithm converges to is Lyapunov stable.*

Proof. We know that the maximiser of our potential function is the globally optimal Nash equilibrium $V^* = V_{max} = 0$. We have that $\Phi(0) = 0$. Hence, we only need to show that $-V'_{min} \leq 0$. If we take the derivative of $-V_{min}$ we have

$$\begin{aligned} -V'_{min} = -\sum_i^N (-(p_i^{1/2} l(H_{i,j}/ \\ (\sum_{i\in N,k\neq i} H_{k,j} p_k^2 (p_i H_{i,j}/(\sum_{i\in N,k\neq i} H_{k,j} p_k) + 1)) - H_{i,j}^2 p_i/ \\ (\sum_{i\in N,k\neq i} H_{k,j}^2 p_k^3 (H_{i,j} p_i/ \\ (\sum_{i\in N,k\neq i} H_{k,j} p_k) + 1)^2)))/(2((p_i^{1/2}(H_{i,j}/ \\ (\sum_{i\in N,k\neq i} H_{k,j} p_k (H_{i,j} p_i/ \\ \sum_{i\in N,k\neq i} (H_{k,j} p_k) + 1)))^{1/2})/2 + 1/2)^{2l+1} ((H_{m,i} p_m/ \\ (\sum_{s\in N,s\neq i\neq m} H_{s,i} p_s (H_{m,i} p_m/ \\ (\sum_{s\in N,s\neq i\neq m} H_{s,i} p_s) + 1)))^{1/2}/2 + 1/2)^{2l} (H_{i,j}/ \\ (\sum_{i\in N,k\neq i} H_{k,j} p_k H_{i,j} p_i/(\sum_{i\in N,k\neq i} H_{k,j} p_k) + 1)))^{1/2}) - c_i) \end{aligned} \tag{19}$$

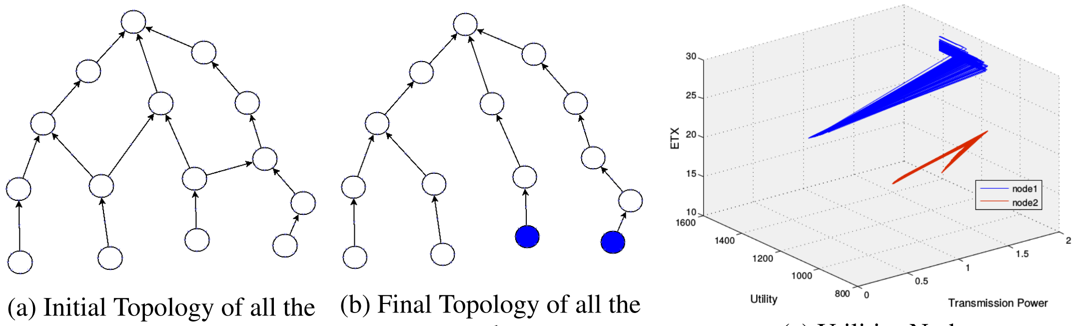
Our transmission power strategies are $p_i > 0$, the channel gain is negative $H_{i,j} < 0$ and l is an even number. In order to simplify equation (20) we denote

- $f_1 = p_i H_{i,j} < 0$
- $f_2 = \sum_{i\in N,k\neq i} H_{k,j} p_k < 0$
- $f_3 = \sum_{i\in N,k\neq i} H_{k,j}^2 p_k^3 > 0$
- $f_4 = \sum_{i\in N,k\neq i} H_{k,j} p_k^2 < 0$
- $f_5 = H_{i,j}^2 p_i > 0$
- $f_6 = \sum_{s\in N,s\neq i\neq m} H_{s,i} p_s < 0$
- $f_7 = H_{m,i} p_m < 0$
- $f_8 = p_i^{1/2} > 0$

Thus, we have

$$\begin{aligned} -V'_{min} = -\sum_i^N (-(l f_8 (H_{i,j}/ \\ (f_4(f_1/(f_2)+1)) - f_5/(f_3(f_1/(f_2)+1)^2)))/ \\ (2((f_8(H_{i,j}/(f_2(f_1/f_2+1)))^{1/2})/2+1/2)^{2l+1} \\ ((f_7/(f_6(f_7/(f_6)+1)))^{1/2}/2+1/2)^{2l} \\ (H_{i,j}/(f_2 f_1/(f_2)+1)))^{1/2}) - c_i) \end{aligned} \tag{20}$$

Even though the derivative is complex, we can see that $-V'_{min} \leq 0$ in our strategy set. We also verified the sign of the derivative through extensive simulations



(a) Initial Topology of all the nodes (b) Final Topology of all the nodes (c) Utilities Nodes

Figure 2: Initial, Final Topologies and Utilities, Transmission Powers, ETX values of the Coloured Nodes.

on MATLAB. This means that the Nash Equilibrium that our algorithm is reaching is stable. □

6 RESULTS

We consider the network of 15 nodes, which is shown in figure 2 (a). The arrows represent that the wireless nodes operate in a hierarchical routing mode, where each node unicasts its data to its selected parent. The parent selection process is undertaken by evaluation the ETX values of each node's neighbours and choosing the one with the least ETX value.

After the formation of the network and the operation of GETOA, the final topology is shown in figure 2 (b). We can see that a route towards the basestation has been formed for every mode based on the ETX values of the upstream nodes.

The transmission power values that are selected as strategies of the nodes come from the CC2020 datasheet (Datasheet, 2006) and are given in table 1. The reason behind the selection is the fact that want to see the operation of our algorithm in the gray areas as identified in (Son et al., 2004). The operation of GETOA initiates by selecting a random transmission power level and then starts to perform the fictitious play part. We can see the utility function over time of the blue nodes in figure 2 (c). Specifically, we can see the convergence of the blue nodes to the transmission power 15.2 after they fluctuate to transmission power 13.9. Node 2 accomplishes a utility of approximately 1180, an ETX value of 18.43. On the other hand, node 1 achieves a utility of 1419 approximately and its ETX value is approximately 26.

The utility of node 1 is decreasing from the initial one, since node 1 switched to transmission power 2; thus, increasing interference on node 1. Hence,we can see in figure 3 that the ETX of node 1 is fluctuating, since its utility is decreasing for the aforementioned reason. On the other hand, the ETX value of node 2 is slightly increasing by taking advantage of the medium transmission power of node 1. Note that both the nodes do not utilise transmission power 17.4, since it creates more interference to other nodes; thus making their utilities smaller, which can be verified in (Hackmann et al., 2008).

Table 1: Tx power strategies.

Tx Power Levels	p_1	p_2	p_3
Values	16.5	15.2	13.9

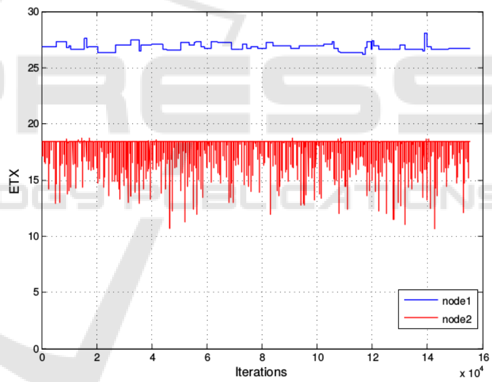


Figure 3: ETX Values of Blue Nodes.

7 CONCLUSIONS

In this paper we addressed the end-to-end throughput optimisation using game theory. We established a utility function based on the additive ETX from the sink to each node and we priced it with a concave price utilising the transmission power used for data transmission. We showed that there is a relationship between ETX with capacity and packet transmission delay. Furthermore, we formulated our game theoretic model with finite strategies and showed that it is a potential game. Moreover, we proved that the Nash equilibrium we located is the global optimum as well, using Schur concavity. Finally, we proved that the

Nash equilibrium our GETOA algorithm approached to is Lyapunov stable.

The demonstration of our approach consisted of a network of 15 nodes. We analysed the two blue nodes that exist further down the network to check their behavior under our GETOA algorithm. We found that one of the nodes increases its utility, while slightly decreases its ETX value; thus exhibiting better end-to-end reception. The other node, has to decrease its utility function, since it was suffering from interference. Both nodes converged to transmission power $15.2mA$, out of the three available transmission power levels.

Our future work includes the packet transmission delay as a constraint to our utility function. This will give us interesting results on the utility function. Furthermore, we aim to compare our approach with a log-linear learning algorithm (Monderer and Shapley, 1996) to investigate the differences in performance and convergence.

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