

Pro-Natalism Policy, Demography and Democracy - Nonlinear Dynamics Analysis

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Abstract: Pro-natal policies, namely promoting human reproduction, are common in many societies, often due to religious, nationalistic, or ethnic reasons. If successful, such policies can have detrimental environmental impact, increase demand for natural and economical resources, and may lead to demographic changes with political consequences. Hence, high fertility rates deserve careful analysis. Here we study, using linear dynamics and non-linear dynamics tools, how high fertility rates may impact demography in Israel. We further comment on potential implications of such scenarios for democracy in Israel.

1 INTRODUCTION

There have been many cases in recent history where social and ethnic groups encouraged their members to increase their fertility (pro-natality) for religious, nationalistic, or political reasons.

In Quebec, Canada, for example, the “revenge of the cradle” (“*La Revanche du berceau*” in French) policy was encouraged by Quebec patriots in order to increase their influence and power and attain their nationalistic and political aims (Morland, 2016); we note that after these aims were largely achieved, in the 1960s, this policy subsided. Similarly, in Iran following the Islamic revolution and especially in the wake of the Iran-Iraq war, pro-natal policies were enacted explicitly to grow the nation and foster its strength (Baktiari, 1995); we note that after realizing their economic cost, Iran switched to promoting family planning (Abrahamian, 2008), until recently rescinding support for such policies for cultural as well as practical reasons.

“Demographic warfare” of this sort is also prevalent in Israel, in various ways. In the context of the Israeli-Palestinian conflict, both sides have explicitly and often officially encouraged natal policies to increase their relative population size (Morland, 2016). The Palestinian leader Yasser Arafat famously said that “the womb of the Arab woman is my strongest weapon”, for example, while Israel’s founder Ben Gurion explicitly set out a natality committee to bolster Jewish reproduction (Schiff, 1981). Such de-

mographic warfare may be especially effective in explaining the high Palestinian fertility rates (Fargues, 2000).

However, not all demographic shifts of this sort are due to an officially-declared or intentional demographic “warfare”. In Lebanon, population sizes change rapidly, with a high Shiite fertility rate being a key factor, but there is no indication that this is by design (Faour, 2007). Similarly, it has been argued that in the USA the high fertility of the religious-right compared to the liberal left will in the future shift the politics of the nation to the right (Kaufmann, 2010; Morland, 2016).

In this paper, we would like to focus on another pro-natal case in Israel. The fertility rates of the Jewish population in Israel are sharply divided along religious lines (Levi, 2016). Among the ultra-Orthodox Jews, in particular, religious, social, and political factors combine to form a strong pro-natal effect, producing a high fertility rate of approximately 6.9 (Friedlander, 2009). The rest of Israeli Jewish society has a much lower fertility rate (2.84 on average). This disparity could potentially lead to a major demographic change. In the past, this did not occur as the large difference in fertility rates was balanced (over the last half century) by immigration, and in particular by a massive post-Soviet immigration (consisting mainly of secular Jews and non-Jews with Jewish roots) in the decade around 1990-1995. Given the current worldwide Jewish demographics, such a massive immigration is unlikely to repeat itself (DellaPergola,

2016).

Our goal in this paper is to explore the expected future demographic changes implied by these high fertility rate differences using the tools of dynamical analysis.

2 DEMOGRAPHICS OF ISRAELI ULTRA-ORTHODOX (HAREDIM)

In Israeli society, fertility rates vary greatly between religious subgroups. The largest population group (secular Jews) has a total fertility rate of 2.1 (as of 2014; see (Levi, 2016)), while the ultra-Orthodox Jews, or "Haredi", population has a fertility rate of 6.9. Notwithstanding causal models of fertility rate dynamics (Berman, 2000; Berman et al., 2012; Manski and Mayshar, 2003), all Jewish religiosity subgroups had a fluctuating but fairly stable fertility rate for the past four decades or so (Levi, 2016), so it is not likely that these fertility rates will vary greatly in the near future. It is this unique stability and contrast of the fertility rates that motivates our study. The tools of complex-systems and nonlinear-dynamics can be used to provide a more careful analysis of the dynamics than a simple projection based on the fertility rates.

We note that a fuller analysis of Israel's future demographics will need to consider also the sizable Arab minority within Israel (comprising about 20% of the population). As their fertility rates have not been stable, however, such an analysis is beyond the scope of this position paper, and we focus only on the more stable Jewish populations.

Further information on the Israeli population can be obtained from the Israeli Central Statistics Bureau. It divides the Jewish populace to five levels of religiosity: Nonreligious or secular ("Hiloni"), traditional but nonreligious (Masorti Lo-Dati), traditional religious (Masorti Dati), religious (Dati), and ultra-orthodox or Haredi. As can be seen in Table 1, most Israeli Jews consider themselves to be Secular or Traditional-nonreligious.

These statistics relate to the respondent's current level of belief. However, a person's current level of religious observance may not be the same as the one he was raised in. The Israeli Central Bureau of Statistics has also provided, for limited years (2002-2012), information about the level of religiosity of the household the person was raised in at age 15. The data for the 2012 census is shown in Table 2. Note that the CBS data does not include the last two entries in the

Table 1: Religiosity levels of Israeli Jews.

Current Religiosity	Tot Pop
Total	3896707
Haredi	366851
Religious	389707
Trad. Rel.	533096
Trad. Nonrel.	885071
Secular	1721982

Haredi Home column, but does imply 5707 people in these categories (without noting how they are distributed between the two, since the numbers are very small and hence the relative uncertainty in them is too high). Our analysis below is unaffected by the specific distribution.

The CBS survey indicates a high retention rate for the extreme levels of religiosity (secular and ultra-orthodox), and a marked tendency for the society as a whole to secularize. If all religiosity groups grew at similar rates, then, Israeli society would be projected to become more secular but more polarized in time, with a large secular and Haredi population and few in between.

As already discussed in the introduction, the fertility rates of the religiosity subgroups vary significantly. According to (Levi, 2016), the latest (2014) total fertility rates of the different groups are 6.9 Haredi, 4.2 Religious, 4 Traditional Religious, 3 Traditional Nonreligious, and 2.1 Secular. While the fertility rates fluctuate in time, they remained fairly stable for the past decades, and we shall assume them to be constant for our initial (linear) analysis. If we assume the total fertility rate to be (twice the) growth rate per single generation, this implies that on average two secular people will become 2.1 people in the next generation, while two Haredi people will become 6.9 people. The Haredi relative population size can hence increase very substantially even within a single generation.

For simplicity, we shall divide the population into two groups: the Haredi, and everyone else (denoted as populations H and E below). (We note other divisions are possible, e.g. (Pew, 2016) used a division of Haredi, Religious, Traditional, and Secular; we leave such analysis to the journal version of this paper). Normalizing for group size, according to the above table, gives a total fertility rate of 6.9 for Haredi and 2.84 for everyone else. These rates correspond to a change per generation, or growth rate, of $r_H = 6.9/2 - 1 = 2.450$ for Haredi, and $r_E = 0.422$ for everyone else.

The above table can also be used to derive the retention and transition rates between the religious

Table 2: Religiosity of Israeli Jews, by the religiosity of the household they were raised in at age 15. The first column denotes the current level of religiosity, the next the total population at the level of religiosity, then the number of people raised at a Haredi household (at age 15), the ones raised in a Religious household, those raised in a Traditional-Religious household, a Traditional Nonreligious household, and finally a secular or non-religious household. Data is from the 2012 Social Survey by the Central Bureau of Statistics of Israel; note that two entries are missing from the table, see discussion in the main text.

Current Religiosity	Tot Pop	Haredi Home	Rel. H.	Trad.-Rel. H.	Trad. Nonrel. H.	Secular H.
Total	3896707	287057	590818	713571	725369	1576014
Haredi	366851	257989	32554	31375	14927	30006
Rel.	389707	12485	274574	57449	20080	24517
Trad. Rel.	533096	10876	106243	320440	55549	39330
Trad. Nonrel.	885071	-	124182	206257	391211	161028
Secular	1721982	-	53264	98049	243602	1321133

groups. The Haredi retention rate per generation is $c_{H \rightarrow H} = 0.899$, while their transition rate is $c_{H \rightarrow E} = 0.101$. For everyone else, the retention rate is $c_{E \rightarrow E} = 0.970$ and the transition rate is $c_{E \rightarrow H} = 0.030$.

3 OUR BASIC EQUATIONS

We consider populations changing in time according to dynamical (ordinary differential) equations, such as (for the case of linear equations)

$$\dot{H} = (r_H - c_{H \rightarrow E})H + c_{E \rightarrow H}E \quad (1)$$

$$\dot{E} = c_{H \rightarrow E}H + (r_E - c_{E \rightarrow H})E \quad (2)$$

Here the dot indicates the derivative with respect to time. These equations describe the dynamics of the populations of the Haredi (H) and everyone else (E) in units of generation time. With the above choice of parameters, our basic equations are:

$$\dot{H} = 2.349H + 0.030E \quad (3)$$

$$\dot{E} = 0.101H + 0.392E \quad (4)$$

These linear equations can be solved exactly analytically. Starting from the initial populations $H = 366,851$ and $E = 3,529,856$ in 2012 (according to Table 2), the projected populations fit very well to the limited historical record (stretching from 2002 to 2015).

Under this model, the Haredi population quickly explodes so that it surpasses everyone else within about 1.1 generation (see Fig. 1; for the mathematical derivation leading up to it, see Sections 4 and 5). This is due to its high fertility rate. As can be seen from the figure, conversion between the religiosity currents has little effect on this growth. The difference in fertility rates is thus expected to completely overwhelm the secularization process in Israel.

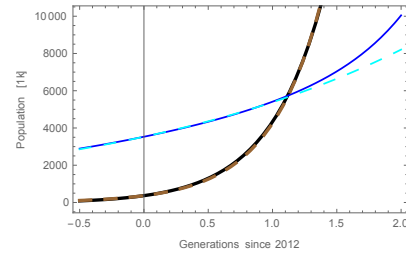


Figure 1: The Jewish religious populations of Israel from -0.5 to +2.0 generations from 2012. The solid thick (black online) line marks the Haredi population, while a solid thin (blue online) line marks the E population. The H population becomes dominant approximately 1.1 generations after 2012. Dashed (brown and cyan online) lines mark the population dynamics in a model without conversion between the two populations. For both populations, the effect of religious conversion is initially so small as to be unnoticeable, so that the dashed lines lie on top of the solid lines.

4 DYNAMIC ANALYSIS IN GENERAL

For any dynamic system, one generally wants to obtain a global understanding of its dynamics. This can be done by considering the system's fixed or stationary points (the points at which the system stays constant, unchanging), and seeing how the system behaves near them so as to obtain a global view of how it flows between fixed points. One therefore always wishes to explore the dynamics near the stationary points ($\dot{H} = 0, \dot{E} = 0, \dots$), and from that deduce the overall picture of the dynamics (see, e.g. (Glendinning, 1994)). For a one-dimensional system $\dot{x} = f(x)$, the system will have a *stable* stationary point if the derivative df/dx is negative at the stationary point, i.e. when $df/dx|_{x^*} < 0$ with x^* being a stationary point so that $\dot{x}|_{x^*} = f(x^*) = 0$.

Similarly, for a multi-dimensional problem we now have multiple f functions, $\dot{x} = f_1, \dot{y} = f_2, \dots$. We need to look at the Jacobian of the problem, namely

the matrix $J_{ik} = \partial f_i / \partial x_k$. Intuitively, directions where these partial derivatives are less than zero at the stationary points will be the stable directions; but we need not be limited to just these axes, so it is better to consider the problem more abstractly and look for the eigenvalues of the Jacobian matrix. If all are negative at the stationary point, then it is stable.

5 LINEAR HOMOGENEOUS CASE WITH TWO POPULATIONS

For our basic, linear, equations, we have only two dimensions, and the equations are linear so each is a line. The Jacobian is simply the matrix of coefficients

$$J = \begin{pmatrix} 2.349 & 0.030 \\ 0.101 & 0.392 \end{pmatrix} \quad (5)$$

Analysis of dynamical equations can be done by considering the "null-clines", i.e. the lines where the derivatives are zero. Where these intersect, all the derivatives are zero so the point is stationary. In our case, the nullclines are not within the physical range, which is that of positive populations. Nevertheless, considering them will allow us to better understand the dynamics as a whole. Nullclines will also be more useful in the following, non-linear, analysis.

To understand the dynamics, we can consider the vector field of the population change; we have drawn it explicitly for this case in Fig. 2. The line of $\dot{H} = 0$ (the H "nullcline") intersects the line of $\dot{E} = 0$ (the E-nullcline) at a single point, which is the origin (0,0). This is the sole stationary point. At this point both f_i increase with E and with H , having positive derivatives, so clearly the point is unstable.

The realistic case, with positive populations, is above both nullclines, so that both E and H are increasing indefinitely. If we include negative populations in our analysis, however, we can understand the dynamics better. In the area below the H-nullcline, H is decreasing, and in the area below the E-nullcline E is decreasing. Overall, the dynamics is a "running away" from the origin (zero populations)

How can immigration be incorporated into such a dynamic analysis? In the 90's, Israel saw a large influx of primarily-secular-Jews from the former USSR. Such a scenario can be simplified to a sudden jump of approximately 1 million in E (everybody who is not Haredi), or by explicit time-dependent models. Analysis of the impact of the such immigration will be provided in the full (journal) paper.

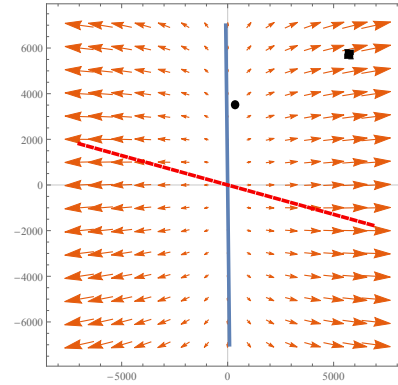


Figure 2: The population change field (\dot{H}, \dot{E}) as a function of H and E (H on the x-axis, E on the y-axis), for the simple (linear) model. The H-nullcline is the (blue) line near the horizontal axis, and divides the area where H increases from where it decreases; the E-nullcline is the dashed (red) line, and delimits where E decreases or increases. The nullclines meet at a single point (at the origin), which is the only stationary point. It is unstable, with populations "running away" from it. The circle indicates the current populations (as of 2012), and the square indicates the point where the Haredi population will start to surpass everyone else according to the linear model.

6 NON-LINEAR DYNAMICS

Constant high birth rates are ultimately not sustainable. There is good reason to think they are unsustainable even in the relatively near future (Biello, 2009). But even if they will decline, how will they do so? We speculate that the relative fraction of the population may be a major factor, and introduce a simple change to the basic dynamic equations which reflects this. According to this (non-linear) model, the Haredi birth rate will drop as their fraction in the population grows,

$$\dot{H} = (r'_H - c_{H \rightarrow E} - \delta \frac{H}{H+E})H + c_{E \rightarrow H}E \quad (6)$$

$$\dot{E} = c_{H \rightarrow E}H + (r_E - c_{E \rightarrow H})E \quad (7)$$

Our speculated (nonlinear) model assumes, for simplicity, that the drop in fertility is linear with $H/(H+E)$. For this model, we need to choose the parameters r'_H and δ so as to maintain two constraints - to preserve an effective Haredi growth rate (including the δ term) of 2.45 at 2012, and to receive some desired effective Haredi growth rate at the large H limit ($H \gg E$). We will choose the latter, large H limit, growth rate to be equal to the non-Ultra-orthodox growth rate r_E . With this choice, the dynamical equations become:

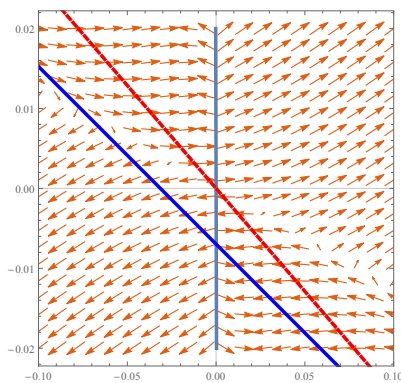


Figure 3: A distorted view of the populations change field (\dot{H}, \dot{E}) as a function of H and E , for the non-linear model, zooming in on the origin. The size differences of the arrows has been reduced to allow one to see arrows of vastly different sizes. Note that the finite size of the arrows makes the dynamics near the singularities (which are close to the horizontal axis) unclear; separate plots for each component will be provided in the journal version to clarify this region.

$$\dot{H} = 2.560H + 0.030E - 2.239 \frac{H^2}{H + E} \quad (8)$$

$$\dot{E} = 0.101H + 0.392E \quad (9)$$

In these equations, we have set $r'_H - \delta(H/(H + E)) = r_H$ for the initial populations, and $r'_H - \delta = r_E$.

The H-nullcline ($\dot{H} = 0$) in this case is no longer a line; as can be seen in Fig. 3, it consists of two lines. The equations are not defined at the origin, but the origin effectively serves as an unstable saddle point. The full paper will provide a more detailed analysis of the behavior of this system in the negative-populations domain (such an analysis which is vital to ensure it doesn't behave unexpectedly in the positive-populations domain, and potentially interesting on its own from a complex-systems point of view). Focusing on the real (positive populations) part (Fig. 4), one can see that the dynamics is still quite similar to the linear case.

The main point of difference is that the growth of the Haredi population is slower at long times, so that the point at which the Haredi overpass everyone else is delayed to approximately 1.4 generations (instead of 1.1 generation in the linear model), as can be seen in Fig. 5.

7 DISCUSSION

Due to their past consistency (Levi, 2016) and the stability of the political factors (Friedlander, 2009), we have suggested the Haredi population will maintain their high fertility rates in the foreseeable future. Our analysis indicates that this implies that under a linear

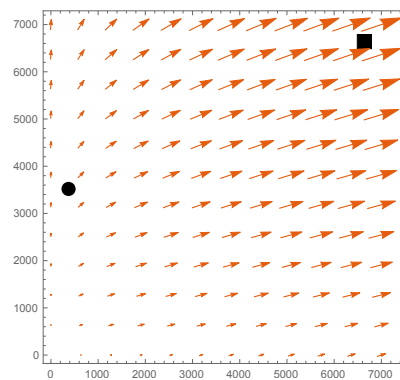


Figure 4: The populations change field (\dot{H}, \dot{E}) as a function of H and E , for the non-linear model, for positive populations. The dynamics are overall similar to the linear model, with the same initial position in 2012 (circle). The point where the two populations meet (square) is more distant in comparison to the linear model.

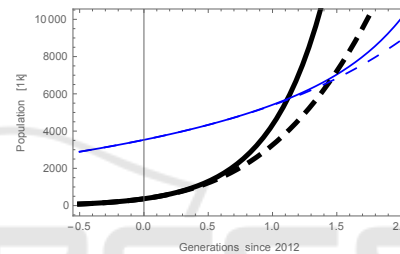


Figure 5: The populations of H and E in time, in units of generations from 2012. The nonlinear model is the dashed line, the linear model the solid line; H population is the thick (black) line, E is the narrower (blue) line. The H population overtakes the E population later in the non-linear model.

model the ultra-Orthodox will become the majority group within 1.1 generations from 2012; assuming a generation time is about 30 years, this is circa 2040.

We have shown that the dynamics are not significantly affected by whether or not religious conversion is included in the model, as religious conversion is too slow to affect them appreciably. The high religious fertility completely overwhelms the secularization process in Israel.

Due to the past stability of the fertility rates it is likely that any model will not drastically change the projection for the near (0.5-1 generations) future. It is difficult to make predictions beyond this point, especially as the examples of Iran or Quebec show that after the goal of a pro-natalism policy is reached, fertility rates may drop and change drastically. Therefore, due to the examples of Iran and Quebec, we have chosen a speculative and simple non-linear model with a fertility rate that drops with time. This model pushed the point of transition into an ultra-Orthodox dominance from 1.1 to 1.4 generations (i.e. from 2040 to 2055 or so).

We note that in all the above calculations, error bars and error analysis were excluded due to uncontrolled uncertainty in the model (the assumption of constant fertility rate, for example, would certainly be strictly false). In particular, we choose a non-linear model which is linear in $H/(H+E)$ as a demonstrative example, but there is no guarantee that this choice would be a good parametrization. We believe data from other places (such as Iran and Quebec) is insufficient in order to justify a specific model. The estimated numbers in Table 2 are also highly uncertain, with some uncertainties in the range of 30% or more. Despite all this, we expect our results to be relatively stable to actual plausible errors (past history suggests, for example, that the fertility rates will only fluctuate slightly, and a different non-linear model will behave similarly in the near future).

The extension of the dynamics into the domain of negative populations allows us to utilize the tools of dynamical analysis to understand the system. We have seen that in the simple, linear, model zero populations is the sole unstable stationary point, which the populations "run away" from. The addition of nonlinearity considerably complicates the dynamics in the non-positive domain, and may complicate the nullclines and dynamics there. For our non-linear model the origin effectively serves as an unstable saddle stationary point, attracting the populations in part of the negative-populations domain. The origin remains unstable in the positive-populations domain, for both the linear and the non-linear domain.

The political views of the ultra-Orthodox are strikingly different from those of the currently-dominant secular Jewish group in many respects, especially those related to democratic and religious values. For example, 69% of the ultra-Orthodox believe Israel is too democratic, whereas 59% of the secular Jews believe it is too religious (Hermann et al., 2016). As the Haredi population will become dominant, more and more of its values may be made into laws and policy. These include favoring religious law over secular law, favoring government actions to promote (orthodox) Jewish religion, and imposing religious law (halacha) as the state law (Pew, 2016), as well as excluding women from public life (never has a woman been nominated to parliament by an ultra-Orthodox party, for example), privileging Jewish citizens (Hermann et al., 2016), and more. According to our demographic model, then, Israel is set to undergo substantial legal and political change in the coming decades, and will become a less democratic and more religious state.

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