FzMEBN: Toward a General Formalism of Fuzzy Multi-Entity Bayesian Networks for Representing and Reasoning with Uncertain Knowledge

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Abstract: Good representing and reasoning with uncertainty is a topic of growing interest within the community of artificial intelligence (AI). In this context, the Multi-Entity Bayesian Networks (MEBNs) are proposed as a candidate solution. It's a powerful tool based on the first order logic expressiveness. Furthermore, in the last decade they have shown its effectiveness in various complex and uncertainty-rich domains. However, in most cases the random variables are vague or imprecise by nature, to deal with this problem; we have to extend the standard Multi-Entity Bayesian Networks to improve their capabilities for good representing and reasoning with uncertainty. This paper details a promising solution based on fuzzy logic; it permits to overcome the weaknesses of classical Multi-Entity Bayesian networks. In addition, we have proposed a general process for the inference task. This process contains four steps, (1) Generating a Fuzzy Situation Specific Bayesian Networks, (2) Computing fuzzy evidence, (3) Adding virtual nodes, and (4) finally, the fuzzy probabilistic inference step. Our process is based on the virtual evidence method in order to incorporate the fuzzy evidence in probabilistic inference, moreover, approximate or exact algorithms can be used, and this choice of inference type depends to the contribution of the domain expert and the complexity of the problem. Illustrative examples taken from the literatures are considered to show potential applicability of our extended MEBN.

1 INTRODUCTION

Bayesian networks (BNs) (Pearl, 1988; Delcroix *et al.,* 2013) have been applied successfully to model and resonate with the problems where uncertainty is prevalent. it's a hybrid model in which it is a combination between the graph theory and the probability theory, they can represent a qualitative knowledge such as (dependencies between random variable) and a quantitative knowledge (probabilities), currently they have been widely used in lots of fields such as medical diagnosis, risk analytic…etc. Furthermore, in the last few years various researchers tried to improve the classical BNs by proposing new extensions such as the Multi-Entity Bayesian Networks (MEBNs) (Laskey, 2008), Object oriented Bayesian networks (Koller, 1997; Liu *et al.,*2016)…etc., these extensions have been proposed to enhance and enrich the classical BNs to be expressive enough in order to better represent the real world's problems and complex systems.

On one hand, Multi-Entity Bayesian Networks are proposed as an extension of the classical Bayesian networks by integrating the first order logic (FOL) in this later, in order to face the randomness. But they are unable to represent the vague and imprecise knowledge.

On the other hand, fuzzy logic and fuzzy sets theory (Zadeh, 1975) were introduced to deal with vague and imprecise knowledge. But they are unable to represent and deal with the randomness.

Nowadays, the real world problems are not only complex in its large structure but also, in the knowledge's nature which involved within, where the uncertainty is indispensible in many cases. Furthermore, the most of real world problems involve several kinds of the uncertainty at the same time such as randomness, vagueness and imprecise knowledge. It seems very important to develop a hybrid models for good representing and reasoning with such complex systems and real world's problems, where lot of kinds of uncertainty appear simultaneously, for this reason, we propose a new extension of the MEBNs we named FuZzy MEBN

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(FzMEBN) using fuzzy logic and fuzzy sets theory to benefit of the advantages of two models. Moreover, as key feature of our FuZzy MEBN, is very powerful model due to its capability to express and reasoning over several kinds of uncertainty at the same time, which is inherently present in lots of real world problems.

The rest of this paper is organized as follow: Section 2 explores the theoretical background. Section 3 presents the related work. Then section 4 introduces a detailed presentation of the proposed FuZzy-MEBN, including its detailed structure, and the inference mechanism in this later. Finally, we aim to conclude this paper and present some perspectives and future works.

2 BACKGROUND

In this section we are going to give an outlook on some background knowledge, we start by Multi-Entity Bayesian Networks (Section 2.1), here after we present the fuzzy logic (Section 2.2)

2.1 Multi-Entity Bayesian Networks

Multi-Entity Bayesian Networks extends Bayesian networks to achieve the high level of expressivity of First Order Logic (FOL). Where the capability of BN to model uncertainty is combined with the expressivity of FOL, in MEBN knowledge's are represented as a collection of MEBN Fragments (MFrags) and a set of MFrags are organized into MEBN Theories (MTheories).

An MFrag contains a collection of random variables (RVs) and the dependencies among these RVs are represented into fragment graph. In addition, an MFrag can be considered as a template or pattern to represents repeatable piece of knowledge fragments of a Bayesian network; an MFrag is defined as **F = (C; I; R; G; D)** (Laskey, 2008). It includes three types of nodes (RVs): resident, input and context nodes. The local conditional distribution also called local probability distribution (LPD) for Resident nodes is defined in the home MFrag, while an input node has its LPD defined in another MFrag (it is a resident node defined in another MFrag). The context nodes defined in order to represent a set of logical constraints that must be satisfied for the distributions represented in the MFrag be valid. Furthermore, G represents an MFrag graph, and D is a set of local distributions.

MEBN theory represents a coherent probability distribution; while Bayes theorem provides a mathematical foundation for learning and inference, the inference in MEBN consists to instantiated it, i.e. generate a Situation Specific Bayesian Network (SSBN) in order to models the situation that has been observed as evidence. Hence, this instantiation overcomes the non-flexibility nature of Bayesian networks where the structure kept fixed in the classical Bayesian networks. Thus, the generated SSBNs can use regular BN inference engines to answer the query.

In (Laskey, 2008) the author presents a Bottom-Up algorithm to generate SSBNs. More recent work presented in (Santos *et al*., 2016), a new algorithm to generate SSBNs based on the Bayes-Ball method, this solution overcome the limitation presented in the Bottom-Up algorithm, by focusing on the scalability problem.

2.2 Fuzzy Logic

In the classical logic the variables are binary where each variable can belong or not to a set, however, in the fuzzy logic and fuzzy set theory an element can belong in more than one set at the same time with some degrees. This property allows to an element to more or less strongly belong to a set, fuzzy logic and fuzzy set theory were proposed by Zadeh (Zadeh, 1975) to manage imprecise and vague knowledge. It is proposed as an extension of the binary logic, this logic does not consist to be precise in the affirmations, for example let ask this question" is the speed of the car fast? "in the classical logic to answer to this question we have to say "yes" if the speed of the car is fast or "no" if the speed of the car slow, however, in the fuzzy logic we can represent the cases when the speed of the car is **too slow, slow, medium, fast, and too fast.**

In fuzzy logic the **variable** speed can take many values, and if we interpreted this variable as "linguistic variable" the corresponding values "*linguistic values* "are {**too slow, slow, medium, fast, and too fast**}.

Each linguistic variable called fuzzy variable and the linguistic value can be seen as a label to a fuzzy sets.

The fuzzy sets can be represented with a membership function μA.

$$\mu A(x) \colon x \rightarrow [0,1].$$

Where μA(x) = 1 if x is belongs totally in A, μA(x) = 0 if x does not belongs to A, and 0< μA(x) < 1 to represent the partial belonging of x in the fuzzy set A.

Examples of membership functions presented in Figure 1.

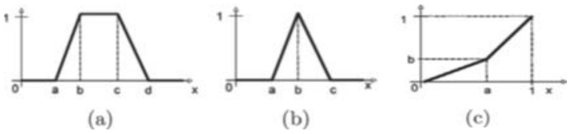

Figure 1: membership functions (a) trapezoidal function (b) triangular function (c) linear function.

3 RELATED WORK

The fuzzy Bayesian networks have been applied successfully in many field such as fuzzy Bayesian classification (Moura *et al.,* 2015) and the Risk analysis (Zhang *et al.,* 2016)… etc. However, combining of fuzzy logic with Bayesian networks is a very difficult task due to the difference between the two formalisms. In addition, the proposed approaches are completely different because each author used different notations from the others thus there is no unified model to define the fuzzy Bayesian networks as the classical Bayesian networks, which makes this model very difficult to understand and to work with.

To incorporate the fuzzy logic in Bayesian networks several approaches have been proposed, an approach based on weighted method, another approach based on Fuzzy Probability Distribution, and finally the virtual evidence method.

In the Weighted method (Tang *et al.,* 2007; Mrad *et al.,* 2012), the main idea is to extend the different rules used in the Bayesian networks by associating a membership degree value to each rule as weight; then the fuzzy Bayesian rules can be defined to support the fuzzy Bayesian inference in FBN model. As a limitation of this approach, the algorithms of inference in Bayesian networks must be also changed and updated. Because these algorithms based on the standard Bayesian Equation.

In Fuzzy Probability Distribution method (Fogelberg *et al.,* 2008; Ryhajlo *et al.,* 2013) the fuzzy membership integrated directly in the probability distribution, where in the first step the fuzzy membership degree must be represented like a probability distribution, then this later will be integrated in the probability distribution in order to generate the Fuzzy Probability Distribution, where the Fuzzy Probability Distribution is a hybrid representation of the fuzzy membership degree and the probability distribution.

The virtual evidence it's a method proposed in (Pearl, 1988), in order to incorporate external knowledge such as the uncertainty of evidence into Bayesian networks as it mentioned in (Li, 2009). Hence, this technique is used in (Pan *et al.,* 1999) in order to incorporate fuzzy membership values into a Bayesian network. It consists to add a new node in the DAG called virtual evidence node. And then we can incorporate the fuzzy evidence in this later, the fuzzy evidence will be represented as a probability distribution in the conditional probability table (CPT) of the virtual node. After adding the virtual nodes and constructing the CPT of the virtual nodes a standard Bayesian inference can be applied in order to calculate the fuzzy inference, we can apply a classical inference. We use this method in the step of inference in the proposed FzMEBN due to its effectiveness and its simplicity. An illustrative example is considered in (section 3.2).

Furthermore, the matter of how to extend the Multi-Entity Bayesian Networks is already devoted in (Golestan *et al.,* 2013; Golestan *et al.,* 2015), where the authors tried to enhance the classical MEBN to support fuzzy logic and apply this later in the context of the data-fusion, in their extension they replaced the First Order Logic by the First Order Fuzzy Logic (FOFL) when defining the contextual and semantic constraints. Moreover, they extend the definition of MFrag by adding fuzzy rules "if-then rules" in this extension where the crisp MFrags were slightly modified by annotating each MFrag **F** by a set of fuzzy if-then rules that are used by a Fuzzy Inference System (FIS).

The inference process in the Fuzzy MEBN is also been discussed, after the generation of the SSFBN (Situation Specific Fuzzy Bayesian networks), we can apply the new version of the modified Clique Tree (CT) algorithm to tackle inference in FBNs. In addition the modified algorithm based on the belief propagation where the authors treated three cases and all these three cases based on weighted formulas using both the membership degrees and the probability distributions. As drawback of this modification, the inference must be done only by the modified Clique Tree. However, in our extension we use the virtual evidence method in order to allow the possibility to the use of both an exact and approximate inference, without any changes in these algorithms. And as we know that the inference is an NP-complete problem (Cooper, 1999). So, if we are limited in one exact algorithm as their extension we may risk of a computational explosion when the complexity of the problem arise. Moreover, in the contrast to FMEBNs cited above, in our extension we focus on extending the components of the classical MEBN, especially the most important one the MEBN fragments (MFrags). Where in our extension a Fuzzy-Mfrag can be seen as template for repeatable Fuzzy small knowledge, Based on hybrid Mfrags (a combination of fuzzy and crisp nodes) and in the step of the inference and after the instantiating we will have a hybrid directed acyclic graph (DAG) and this later contains two type of node (fuzzy, and crisp). The next section will be devoted to the presentation of our solution.

4 THE PROPOSED FUZZY MULTI-ENTITY BAYESIAN NETWORKS (FzMEBN)

The contribution in this paper is to propose a new extension of the classical Multi-Entity Bayesian networks (MEBN) as an answer to the need of representing randomness, vague and imprecise knowledge at the same time, we have chosen to extend this model due to their expressiveness and power of reasoning. The idea behind our extension is to add new types of nodes in Multi-Entity Bayesian Networks to represent the vague and imprecise knowledge's as it illustrated in the figure 2.

4.1 Modeling with FzMEBN

FuZzy Multi-Entity Bayesian Network is enhancement of the classical Multi-Entity Bayesian Network to benefit the power of the fuzzy logic, thus improve the expressiveness of classical MEBNs in order to well represent knowledge's of the real world's problems and situations. As the classical MEBN the FuZzy Mutli-Entity Bayesian Network contains a set of FuZzy MFrags (FzMfrags) organized into MTheories.

4.1.1 FuZzy MFrags

Fuzzy MFrag (FzMFrag) is an extension which enables the crisp MFrags to deal with vague and imprecise knowledge's, a Fuzzy MFrags F it's a hybrid template in which contains both the vague and crisp nodes.

FzMFrag is a 7-tuple**<C, R, I, F_R, F_I, G, D>**:

- **C** is a set of crisp context nodes,
- **R** is a set of crisp resident nodes,
- **I**is a set of crisp Input nodes.
- **F_R** is a set of fuzzy resident nodes.
- **F_I** is a set of fuzzy Input nodes.
- **G** represents a hybrid MFrag Graph (Fuzzy MFrag Graph), and
- **D** represents the local distributions.

Context Nodes: these types of nodes are Boolean random variables representing conditions and constraints that must be satisfied to make a distribution in an FzMFrag valid.

Input Nodes: these nodes can be seen as «foreign nodes" or "pointers" referring to a resident node defined in another FzMFrag. Its own distributions defined in its home FzMFrag.

Fuzzy input Nodes: fuzzy input nodes are fuzzy resident nodes defined in another FzMFrag, the fuzzy input nodes can also influence the probability distribution of the resident nodes, but its probability Distribution and own membership functions are defined in its own home FzMFrag.

Resident Nodes: Resident Node can be defined as Function, Predicate, or Formula of First Order Logic (FOL), and this node is attached by a probability distribution.

Fuzzy resident Nodes: are extensions of the classical resident nodes enabling the FzMFrag to cope the vagueness and imprecise knowledge. As the classical resident nodes the fuzzy resident nodes are attached with a probability distribution. In addition, they can represent the vagueness by using the membership functions.

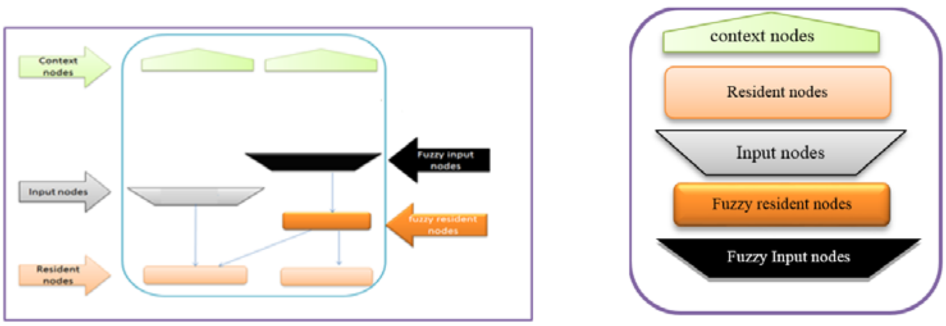

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Figure 2: structure of the proposed Fuzzy MFrag.

Formally a fuzzy resident node is 4-tuple **<T, P, Sfs, M>,** where:

- **T** can be predicate or function or a first order logic expression,
- **P** represents the probability distribution of the fuzzy resident node,
- **Sfs** represent a set of fuzzy states of the resident node, and
- **M** is a mapping rule which map every fuzzy state of each fuzzy resident node to a fuzzy set. Per each **state ∈Sfs** is attached with its own membership function.

An example of Danger MFrag belongs to ***Vehicle Identification MTheory*** (Park *et al.*, 2013) is presented in Figure 03.The Danger MFrags contains:

a) Context nodes, where **isA(obj, vehicle)** and **isA(rgn, Region)** are used in order to represent the types of the ordinary variables (**obj and rgn**), **rgn=location (obj)** represent a condition about the variable **rgn** must be satisfied, and

b) An Input node called VehiculeType(obj) its defined in another Mfrag, and

c) A resident node named Danger-level (rgn) to represent the danger level of a region and this later depends on the type of the vehicle located in this region

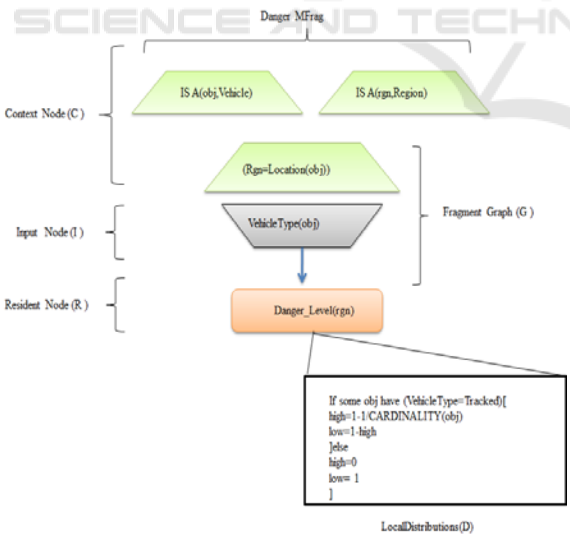

Figure 3: Danger MFrag.

The Danger-level (rgn) node is fuzzy by nature and it can take this set of Fuzzy states {high, low}.

The member ship functions to represent the fuzzy states {**low, high**} presented in Figures 4, and 5.

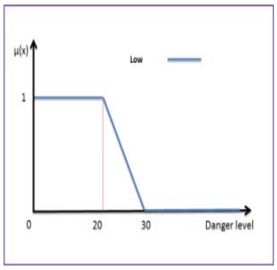

Figure 4: Low danger member ship function.

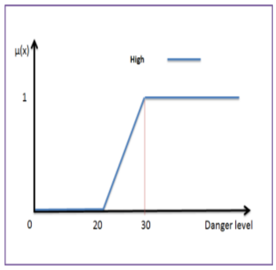

Figure 5: High danger member ship function.

4.2 Fuzzy Probabilistic Inference in FzMEBN

We believe strongly that the success of our extension depends on it capability to provide a strong mechanism for Fuzzy probabilistic reasoning. Thus in this section we will explain how the fuzzy probabilistic inference can be done in the FuZzy Multi-Entity Bayesian networks (FzMEBN).

The inference in our extension consists to generate a Fuzzy Situation Specific Bayesian Networks (FSSBN) where the FSSBN is a fuzzy Bayesian network contains the crisp and fuzzy nodes, and then a fuzzy probabilistic inference based on the virtual evidence method can applied in this later in order to answer the queries. The process of the inference in the FuZzy Multi-Entity Bayesian Networks illustrated in the Figure 6.

Step 1 - Generating Fuzzy SSBN

The purpose of this step is to generate a Fuzzy SSBN by executing a query, thus the generation of the FSSBN achieved with the same manner as the classical SSBN using Laskey algorithm (Laskey, 2008). Furthermore, each instance of a fuzzy Resident node must be attached with a set of membership functions and these last are similar to the membership functions attached to the resident node in which it belong.

The generation of the Fuzzy SSBN is held according to algorithm 1.

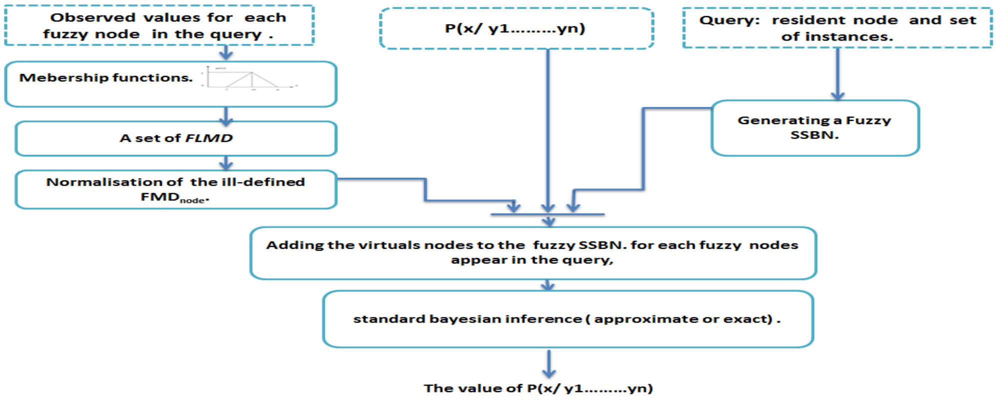

Figure 6: The inference process in the FuZzy MEBN.

Algorithm 1: Generating Fuzzy SSBN.

Inputs : <query List: L, knowledge base: k, FuZzyMEBN: m, Fuzzy resident nodes list: FL >

Output: Probabilistic Network **Net;**
Begin:
01: Net=**LaskeySSBNGenerator**.**generateSSBN** (L, k, m);
02: **For all** N ∈ Net **do**
03: R←**getResidentNodeName (N)**
//Get the resident name in which this node was instantiated.
04: **If** (R ∈ FL) **Then**
//If the resident node is fuzzy.
05: Attach N with the same membership functions than R.
06: **Endif**
07: **Done**
End

Example of a Fuzzy Situation Specific Bayesian Network FSSBN is presented in figures 7.

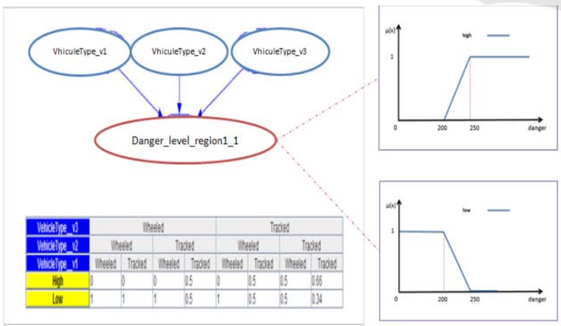

Figure 7: Fuzzy Situation Specific Bayesian Network (FSSBN) (given v1, v2, and v3 as vehicle, and region1_1 as region).

Step 2- Computing Fuzzy Evidence

For each fuzzy node appear in the query the agent must give the observed value, hereafter, the degrees of membership of every state of every fuzzy node can be calculated using the membership functions attached to each fuzzy node.

Let **LFMD** be a list of fuzzy membership degrees for all fuzzy node in the query i.e. The fuzzy evidence, so **FLMD** can be defined as:

LFMD={ $\mathbf{FMD_{node1}}$, $\mathbf{FMD_{node2}}$............$\mathbf{FMD_{noden}}$ } where **$\mathbf{node_i}$** 1< i<n are a fuzzy nodes and

$\mathbf{FMD_{nodei}}$=<μ_{state1}, μ_{state2},............ μ_{statem}> represent the membership degrees for a fuzzy *node* $_i$.

Occasionally, the sums of the membership degrees of a linguistic variable are not well-defined as it's discussed in (Waltman *et al.*, 2005). To deal with this problem we normalize each membership degree **μstatej(x)** as flows:

$$\bar{\mu}\ \mathrm{satatej}(x) = \frac{\mu\mathrm{statej}(x)}{\sum_{i=0}^{|n|} \mu\mathrm{statei}(x)} \quad (1)$$

For example the agent executes this query on the generated Fuzzy SSBN:

P (VhiculeType_v2=tracked| Danger_level_region1_1=high).

In this case the danger level node is fuzzy and it appear in the query, so let assume that the observed level of danger is 27 %, so $\mu_{high}(27)=0.7$and $\mu_{low}(27) = 0.3$ calculated using "Low", and "High" membership functions attached to the Danger_leve_region1_1 node.

LFMD= {**FMD** $_{\mathbf{Danger\_leve\_region1\_1}}$ }={ <μ_{low} , μ $_{high}$ >}={<0.3,0.7>}.

The computation of the fuzzy evidence is held according to the algorithm 2.

Algorithm 2: Computing the Fuzzy evidence

Input : < list of observed values: L, Fuzzy resident nodes list: FL >
Output: **LFMD** Lmd.
Begin:
01**:** Lmd= Φ.
02: i ← 1.
03: **While** i < = |FL| **do** //Get the observed value for the node i 04: ObservedVi ←getVlaueL(i);
//Calculate the membership degrees for each state i ∈ node i
05: FMD_{nodei}=<μ_{state1}(ObservedVi)…… μ_{statem}(ObservedVi)>;
06: **If** (FMD_{nodei} is not normalized) **Then**
07: Normalize each state using equation **(1);**
08: Update FMDnodei;
09: **Endif**
10: ADD ($FMD_{nodei,}$ Lmd) ;
11**: Done**
End

Step 3- Adding the virtual nodes

For each fuzzy node appear in the query of the agent, a child node will be added automatically, then the normalized membership degrees calculated using **step 2** will be incorporated in the CPT of the virtual node as probability distributions.

The "Danger_level_region1_1" node appears in the query of the agent and it is fuzzy. So a virtual node will be added as it illustrated in the figure 8. The step of adding the virtual nodes is held according to the algorithm 3.

Algorithm 3: Adding the virtual nodes.

Input :< **FLMD**: Flm,Fuzzy resident nodes list: FL , FSSBN : BN>
Begin:
01**: For all** nodei ∈ **Fl do**
//Get the membership degrees for the node i
02: FMD_{nodei}=getFMD(Flm,i);
//Create a virtual node of the node i as a child
03: child ←create_Child (nodei, BN);
04: Incorporate FMD_{nodei} in the CPT of child node;
05: ADD (child, FMD_{nodei});
06: **Done**
End

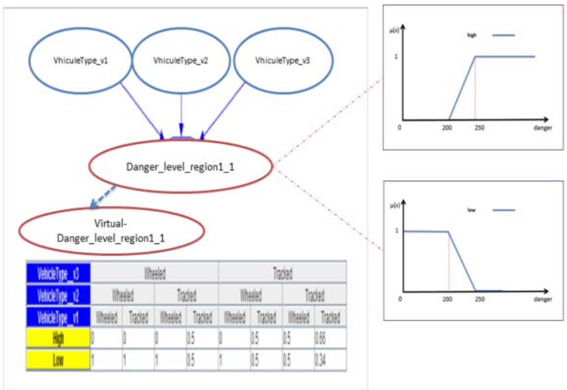

Figure 8: Fuzzy Situation Specific Bayesian Network (FSSBN) with a virtual node.

Step 4- Fuzzy Probabilistic Inference

The fuzzy probabilistic inference can be done by substituting each fuzzy node appears in the query by its virtual node. Then a classical probabilistic inference approximate or exact will be tackled.

The fuzzy probabilitic inference can be done using the algorithm 4.

Algorithm 4: Fuzzy probabilistic inference.

Input :< query nodes X, Fuzzy Evidence Y, Fuzzy resident nodes list: FL, FSSBN BN >
Output: Probability value;
Begin:
01: **If(** X ∈ FL) **Then** // if the query node is fuzzy
02: V←getVirtualNodeName(X, BN);
03: Substitute X by V;
04 **Endif**
05: **For all** N ∈ Y **do**
06: **If** (n ∈ FL) **Then** {if the node is fuzzy}
//Get the virtual node name of the node N from FSSBN
07: V←getVirtualNodeName(N, BN);
08: Substitute N by V in Y;
09**: Endif**
10: **Done**
11: Run a classical Bayesian inference using X and the new evidence Y;
End.

In our example the Danger_level_region1_1 is fuzzy so it will be substituted in the evidence by its chilled virtual node "VirtualDanger_level_region1_1".To make the notation easy we note VhiculeType_v2 as V2, Danger_level_region1_1 as D1, Virtual_Danger_level_region1_1 as V_D1, High as H and Low as L.

Then the new query taken V_D1 as evidence:

P(V2 = Tracked| D1 = H) = P (VT2 = Tracked| V_D1 = H).

$$= \frac{P(V\_D1=H|V2=Tracked)*p(V2=Tracked)}{P(V\_D1=H)}.$$

$$= \frac{P(V\_D1=H|V2=Tracked)*p(V2=Tracked)}{P(V\_D1=H|D1=H)*P(D1=H)+p(V\_D1=H|D1=L)*p(D1=L)}.$$

Noting that $P(V\_D1 = H|D1 = H)$ and $P(V_{D1} = H|D1 = L)$ represents the membership degrees incorporated in **step 3** in the CPT of the virtual node respectively μ_{high} and μ_{low}.

5 CONCLUSIONS

The overall goal of this paper is to develop a solution to deal with vague and imprecise knowledge in MEBNs, thus dealing with tow kind of uncertainty at the same time, for this, we have introduced a new extension of the Multi-Entity Bayesian Networks based on fuzzy logic in order to improve the classical MEBN by extending the classical MFrags to a FuZzy MFrags, our approach based on a strong probabilistic graphical model enabling the reasoning with uncertainty under a complex problems. Moreover, we have proposed a complete process to do the fuzzy inference in the extended MEBNs, where the inference task in FzMEBN is divided in four steps, the first one consist to generate a minimal fuzzy Bayesian networks (Fuzzy SSBN) capable to answer the query as the classical MEBN using Laskey algorithm . The second one consists to computing the fuzzy evidence, the third consist to incorporate the fuzzy evidence in the Fuzzy SSBN and finally, fuzzy Bayesian inference can be done using classical Bayesian inference on the generated Fuzzy SSBN.

Currently, we are focusing on evaluating the ability of the proposed FuZzy Multi-Entity Bayesian Networks by apply it on a complex real world problems, thus in our next work we are interesting to evaluate the performance of our solution taking the diabetes disease as a case of study.

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