# A Lower Bound on the Number of Nodes with Multiple Slots in Wireless Sensor Networks with Multiple Sinks

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Abstract: Wireless Sensor Networks (WSNs) once deployed, are left unattended for extended periods of time. During this time, the network can experience a range of faulty scenarios. If a sink node fails, data gathered by sensor nodes may not be delivered to a destination. One way to increase the reliability of such WSNs is to deploy with more than one sink. In this paper, we formalise the problem of many-to-many data aggregation scheduling in WSNs with multiple sinks and establish a lower bound on the number of nodes with multiple slots.

## 1 INTRODUCTION

Typically, WSNs, once deployed, are left unattended for extended periods of time. During this time, the network can experience a range of faulty scenarios: (i) any sensor node may fail due to energy exhaustion, (ii) links may fail due to interference or (iii) the sink may fail to communicate due to some reasons such as link failures, node and sink failures. For example, in (Paritosh et al., ), the authors observe that 4 of the 7 correctly working nodes had communication failure with the sink over time, furthermore, they observe sink outage due to power failure. In a deployment (Polastre et al., 2004), a crash of the database running on the sink node resulted in the complete loss of data for two weeks. Likewise, in a deployment (Tolle et al., 2005), two weeks of data were lost due to a sink outage. In (Szewczyk et al., 2004), authors observed a sink outage due to harsh weather. In such situations, as mentioned, the loss of the single sink results in the loss of the network.

One way to increase the reliability of such WSNs is to deploy with more than one sink. Moreover, deploying more than one sink may improve network throughput and prolong network lifetime by balancing energy consumption, and may address fault tolerance issues (Lee et al., 2005; Valero et al., 2012; Sitanayah et al., 2012).

A number data aggregation scheduling (DAS) algorithms have been proposed for WSNs with a single sink. There also exist DAS algorithms for WSNs with multiple sinks. For example, DAS algorithms for a WSN with multiple sinks have been presented in (Kawano and Miyazaki, 2008; Bo and Li, 2011). However, those works present DAS algorithms where many nodes send data to only one sink, whereas this paper considers the problem of data aggregation scheduling where many nodes send to many sinks.

In (Saginbekov et al., 2016), the authors focused on the same problem. However, their proposed algorithm performs data aggregation from many nodes to two sinks. Their algorithm also works for more than two sinks, but in sub-optimal ways. The main idea of that algorithm was to develop a backbone that connects two sinks and then schedule nodes transmission.

The same idea, i.e., building a backbone can be used in WSNs with multiple sinks, which is directly related to the problem of developing a Steiner tree (Gilbert and Pollak, 1968). We are interested in developing a minimum Steiner tree, which is known to be NP-complete (Karp, 1972), as it reduces the number of nodes with multiple slots.

In this context, we make the following novel contributions:

- We formalise the problem of DAS scheduling in a WSN with multiple sink.
- We prove a lower bound for solving a variant of DAS called weak DAS.

The rest of the paper is organized as follows. In Section 2, we present an overview of related work.



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In Section 3, we formalise the problem of Data Aggregation Scheduling (DAS) in a WSN with multiple sinks. Then, in Section 4, we show a lower bound for solving a variant of DAS called weak DAS. Finally, in Section 5, we discuss future work and conclude the paper.

# 2 RELATED WORK

Communication protocols that have been developed for WSNs with multiple sinks can be found in (Mottola and Picco, 2011; Kawano and Miyazaki, 2008; Bo and Li, 2011; Thulasiraman et al., 2007; Tuysuz Erman and Havinga, 2010; Hui Zhou and Xu, 2012). A data collection protocol that tries to decrease the number of redundant transmissions has been proposed in (Mottola and Picco, 2011). This protocol uses information about the neighbourhood nodes to reduce transmissions while collecting data from many nodes to many sinks.

In (Thulasiraman et al., 2007), the authors propose an algorithm that builds two node-disjoint paths from every node to two different sinks. If one path fails, the other is used to route the data. In (Tuysuz Erman and Havinga, 2010), the authors propose a routing protocol with hexagon-based architecture. Nodes in the network are grouped into hexagons according to their locations. A routing protocol proposed in (Hui Zhou and Xu, 2012), is based on trees. In this protocol, different trees rooted at different sinks are used to forward data.

The schemes that have been proposed in (Kawano and Miyazaki, 2008; Bo and Li, 2011) have more relevance to our work. In (Kawano and Miyazaki, 2008), the authors propose two algorithms: an algorithm that builds shortest path trees rooted at each root and a scheduling algorithm that exploits a graph colouring algorithm to allow nodes to forward their messages to their closest sink without message collisions. The authors of (Bo and Li, 2011), propose two data aggregation scheduling algorithms for multiple-sink sensor networks. The first algorithm is Voronoi-based scheduling where the sensing area is divided into regions forming $k$ forests, one forest for each sink. Then the algorithm assigns slots to nodes. The second algorithm is Independent scheduling which differs from the first one in forest construction. However, in both of these algorithms different portions of sensor nodes send their data to a single different sink, i.e., many-to-one communication, whereas we consider the case where many nodes send their data to many sinks.

# 3 PROBLEM FORMULATION

We present the following definitions that we will use in this paper.

**Definition 1 (Schedule)** *A schedule* $\mathbb{S} : V \to 2^{\mathbb{N}}$ *is a function that maps a node to a set of time slots.*

**Definition 2 (DAS-label)** *Given a network* $G = (V, E)$*, a sink* $\Delta$*, a schedule* $\mathbb{S}$ *and a path* $\gamma = n \cdot m \dots \Delta$*, we say that* n is DAS-labeled under $\mathbb{S}$ on $\gamma$ for $\Delta$ *if* $\exists t \in \mathbb{S}(n) \cdot \exists t' \in \mathbb{S}(m) : t' > t$.

We call the node $m$ on $\gamma$ the $\Delta$-parent of $n$ and $\gamma$ the DAS-path for $n$.

**Definition 3 (Strong and Weak schedule)** *Given a network* $G = (V, E)$*, a sink* $\Delta \in V$ *and a schedule* $\mathbb{S}$*,* $\mathbb{S}$ *is said to be a* strong DAS schedule for $\Delta$ *for a node* $n \in V$ *iff* $\forall$ *path* $\gamma_i = n \cdot m_i \dots \Delta$*, n is DAS-labeled under* $\mathbb{S}$ *on* $\gamma_i$ *for* $\Delta$*.* $\mathbb{S}$ *is a* weak DAS schedule for $\Delta$ *for n if* $\exists$ *path* $\gamma = n \cdot m_i \dots \Delta$ *such that n is DAS-labeled under* $\mathbb{S}$ *on* $\gamma$ *for* $\Delta$*.*

*A schedule* $\mathbb{S}$ *is strong DAS (resp. weak DAS) for G iff* $\forall n \in V$*,* $\mathbb{S}$ *is strong DAS schedule (resp. weak DAS schedule) for* $\Delta$ *for n.*

A strong schedule, which is impossible to develop (Jhumka, 2010), in essence, is resilient to problems that occur in the network such as radio links not working or node crashes during deployment. On the other hand, a weak schedule is not resilient and, any problem happening, will entail that a message from node $n$ to $m$ will be lost.

Given a network with $n$ sinks $\Delta_1, \Delta_2, \cdots, \Delta_n$ we wish to develop a weak schedule for all sinks. There are different ways to achieve this. In general, to develop a weak schedule, several works have adopted the approach whereby a tree is first constructed, rooted at the sink, and then slots assigned along the branches to satisfy the data aggregation constraints. A trivial solution is to construct $n$ trees, each rooted at a sink, and then to assign slots to nodes along the trees. This means that nodes can have $n$ slots, i.e., meaning that nodes may have to do $n$ transmissions for the same message. Thus, to reduce the number of transmissions, we want to reduce the number of slots for nodes to transmit in.

## 3.1 DAS Scheduling

We model our problem as follows:

We capture slots assignment with a set of decision variables.

$$t_n^{\mathbb{S}} = \begin{cases} 1 & t \in \mathbb{S}(n) \\ 0 & \text{otherwise} \end{cases}$$

A set value assignment to these variables represent a possible schedule. The number of slots used, which equates to the number of transmission by nodes, has to be reduced for extending the lifetime of the network.

We capture the number of nodes with multiple slots as follows:

$$f_n^{\mathbb{S}} = \begin{cases} 1 & |\mathbb{S}(n)| > 1 \\ 0 & \text{otherwise} \end{cases}$$

However, such a schedule may not assign a slot to a given node, so we need to rule out some schedules with a constraint:

$$\forall n \in V \cdot \exists t : t_n^{\mathbb{S}} = 1$$

The above constraint means that all nodes in the network will be assigned at least one slot. We also rule out schedules $\mathbb{S}$ that assign the same slot to two nodes that are in the two-hop neighbourhood, i.e,

$$\forall m, n \in V : t_m^{\mathbb{S}} = 1 \wedge t_n^{\mathbb{S}} = 1 \Rightarrow \neg 2HopN(m,n)$$

Nodes can get information about slots assigned to nodes of two-hop neighbourhod by exchanging messages. Finally, we require to generate weak DAS schedules $\mathbb{S}$, i.e.,

$$\forall i \in N, \forall m \in V \cdot \exists n \in V, (m \cdot n \ldots \Delta_i) : t_m^{\mathbb{S}} = 1 \Rightarrow \exists \tau > t : \tau_n^{\mathbb{S}} = 1$$

Thus, there are different ways to generate a collision-free weak DAS schedule for all sinks. For instance, one may want to minimise *numSlots* to reduce the number of slots during which nodes transmit. Another way is to reduce the number of times any node can transmit, in some sort of load balancing. Thus, our goal is to solve the following problem, which we call the *EECF-N-DAS* problem (for energy-efficient collision-free N-sinks DAS):

EECF-N-DAS problem: Obtain an $\mathbb{S}$ such that

minimise $\sum_{\forall t} \sum_{\forall n \in V} f_n^{\mathbb{S}}$ subject to

1. $\forall n \in V \cdot \exists t : t_n^{\mathbb{S}} \neq 0$
2. $\forall i \in N, \forall m \in V \cdot \exists n \in V, (m \cdot n \ldots \Delta_i) : t_m^{\mathbb{S}} = 1 \Rightarrow \exists \tau > t : \tau_n^{\mathbb{S}} = 1$
3. $\forall m, n \in V : t_m^{\mathbb{S}} = 1 \wedge t_n^{\mathbb{S}} = 1 \Rightarrow \neg 2HopN(m,n)$

The EECF-N-DAS problem consists of two subproblems: (i) The first two conditions amount to what we call the *weak DAS problem* and (ii) the fourth condition ensures that any weak DAS schedule is collision-free. Collision freedom is guaranteed by ensuring that no two nodes in a 2-hop neighbourhood share the same slot.

# 4 THEORETICAL CONTRIBUTIONS

In (Saginbekov et al., 2016), authors proved that it is impossible to have a weak schedule in a network with more than one sink where all nodes have only one slot. Therefore, there should be a certain number of nodes that require at least two slots.

In this section, we investigate how small can the number of nodes with multiple slots be to generate an energy efficient collision-free weak schedule in a network with multiple sinks.

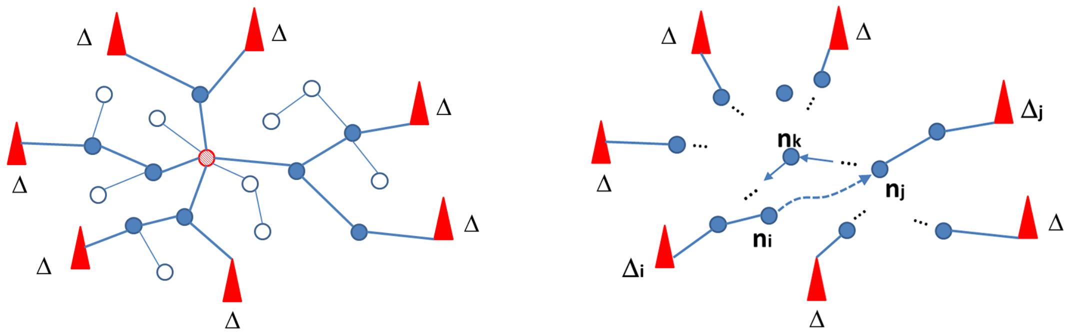
## 4.1 All Nodes Have Multiple Slots ($\sum_{\forall n \in V} f_n^{\mathbb{S}} = |V|$)

A trivial solution to this is as follows: generate $k$ trees, each rooted at a different sink, where $k$ is the number of sinks. For sink $\Delta_1$, starting with slot $|V|$, assign, in decreasing order, slots to nodes using Breadth-first search algorithm (BFS) (Cormen et al., 2001). The same process is repeated with other trees rooted at $\Delta_{i \leq k}$, except they start with slot $i * |V|$. This sets an upper bound for collision-free weak schedules for WSNs.

## 4.2 Towards Minimizing the Number of Nodes with Multiple Slots ($\sum_{\forall n \in V} f_n^{\mathbb{S}}$)

One way of building a network that solves the weak DAS problem is to assign two slots to the nodes on a tree that connects all $\Delta_{1 \leq i \leq k}$, and assign one slot to all other nodes like shown in Figure 1(a).

The scheme works as follows: Initially, the scheme builds a backbone tree that connects all sinks. Then one node on the backbone tree is selected as a *super virtual root* (shaded node in Figure 1(a)). Then, the scheme starts to build trees rooted at each node, called a *virtual root*, on the backbone tree (filled circles in Figure 1(a)). There exist many tree-building algorithms. For example, BFS algorithm can be used to build such trees. After building trees rooted at the virtual roots, nodes that are not on the backbone tree (circles in Figure 1(a)) forward their data to their virtual root. On the way towards a virtual root, each node aggregates received data from its children. After receiving aggregated data, each virtual root forwards data to the super virtual root. The super virtual root, in turn, aggregates received data and sends the final aggregated data to the sinks using the backbone tree. In other words, only virtual roots forward the data towards the sinks. Thus, in this scheme, only the nodes that are on the backbone tree send twice, except the super node. The super node sends only once.




(a) (b)

Figure 1: (a) An example of network that solves weak DAS. Tree (bold lines) shows the backbone in which the nodes (virtual roots) have at least two slots. (b) An illustration of the proof of Theorem 1.

Now, the question is does the scheme above minimize the number of nodes that send the same message more than once? In other words, does the scheme reduce the number of nodes with multiple slots? In the scheme above, the number of nodes with multiple sinks is equal to the number of nodes on the backbone tree minus one (the number of virtual roots except the super root ). Can we reduce it further? The answer is no, if the backbone tree is Steiner minimal tree.

**Corollary 1** *Given a network $G = (V,A)$ with k sinks $\Delta_1$, $\Delta_2$, $\ldots$, $\Delta_k$, where $k \geq 2$, and a Steiner minimal tree, with m nodes, that connects all k sinks, then there exists a weak DAS $\mathbb{S}$ for G, $\sum_{\forall n \in V} f_n^{\mathbb{S}} = m-1$.*

Since we know that it is possible to obtain a weak DAS schedule $\mathbb{S}$ that assigns two or more slots to at most $m-1$ nodes, the objective is to determine the minimum number of such nodes with at least two slots. This is captured in the following result (Theorem 1):

**Theorem 1** *Given a finite network $G = (V,A)$ with k sinks $\Delta_1$, $\Delta_2$, $\ldots$, $\Delta_k$, where $k \geq 2$, and a Steiner minimal tree T, with m nodes, that connects all k sinks. Then, there exists no weak DAS schedule $\mathbb{S}$ for G such that $\sum_{\forall n \in V} f_n^{\mathbb{S}} \leq m-2$.*

From Corollary 1, we know that it is possible to have a weak DAS schedule with at most $m-1$ nodes that assigned more than two slots. Now, we prove that it is impossible to have such a schedule with less than $m-1$ nodes.

**Proof.**

We assume that there exists a weak DAS $\mathbb{S}$ for finite network $G$ such that $\sum_{\forall n \in V} f_n^{\mathbb{S}} \leq m-2$ under $\mathbb{S}$, and show a contradiction.

Let $T \subset G$ be a Steiner minimal tree which spans all sinks in $G$ $\Delta_1, \Delta_2, \ldots, \Delta_k$. Since there are only $m-2$ nodes in $G$ that have more than one slot, there exist at least two nodes in $T$ that have only one slot, say $n_i$ and $n_j$ with, without loss of generality, $\mathbb{S}(n_i) < \mathbb{S}(n_j)$. Note that, as $T$ is a tree, a path $P_{ij}$ between $n_i$ and $n_j$ connects at least two sinks, say $\Delta_i$ and $\Delta_j$. Assume that $n_i$ is closer to $\Delta_i$ than to $\Delta_j$ (See Figure 1(b) for illustration).

Now, as $n_j$ should send its packet to $\Delta_i$ and cannot use $P_{ij}$ to deliver its packet to $\Delta_i$ (as $\mathbb{S}(n_i) < \mathbb{S}(n_j)$), there should exist a node $n_k$ (See Figure 1(b)), other than $n_i$, such that $\mathbb{S}(n_j) < \mathbb{S}(n_k)$. If $n_k$ is assigned more than one time slot, then we are done, as the number of nodes with more than one slot becomes $m-1$. Otherwise, as $n_j$ has only one slot and $\mathbb{S}(n_j) < \mathbb{S}(n_k)$, there should exist another node $n_r$ such that $\mathbb{S}(n_k) < \mathbb{S}(n_r)$ to deliver the packet of $n_k$ to $\Delta_j$. If $n_r$ is assigned more than one time slot, then we are done. It continues like this until there exists a node with at least two time slots. However, if there is no such a node, then as $G$ is finite, it is impossible to have a weak DAS schedule for $G$ that has $m-2$ nodes with at least two time slots.

□

## 5 CONCLUSIONS AND FUTURE WORK

In this paper, we formalised the problem of data aggregation scheduling in WSNs with multiple sinks. We then established a lower bound on the number of nodes with multiple slots. We proved that the number of nodes that send one message more than once cannot be less than the number of nodes in minimal Steiner tree that connects all sinks. As a future work we plan to develop an approximation algorithm that solves the DAS problem in WSNs with multiple sinks, and conduct simulation and testbed experiments to evaluate its performance.

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