

# MP-ABT: A Minimal Perturbation Approach for Complex Local Problems

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**Abstract:** The ability of Distributed Constraints Reasoning (DCR) to solve distributed combinatorial problems brings the DCR to have a considerable interest in multi-agent community. Hence, many DisCSP algorithms have been proposed in order to solve such distributed problems. The major limit of these algorithms is the simplification assumptions. The scientists assume that each agent is a simple one; it handles just one variable. But in the complex local problem case; where each agent has more than one variable; two methods are used: The compilation and the decomposition. These methods transform the original problem so as to make it as a simple one. In this paper, we propose a new protocol: MP-ABT (Minimal Perturbation complex local problems in the Asynchronous Backtracking). It is a resolution algorithm of DisCSPs with complex local problems. It is based on the ABT algorithm and the Dynamic CSP. Each complex agent is seen as a Minimal Perturbation Problem (MPP) and any received message is considered as a new intra-constraint perturbation event. The complex local problem is updated and a new MPP local solution is reported. The MP-ABT is presented and compared to three ABT families. Our experimental results show the MP-ABT effectiveness.

## 1 INTRODUCTION

The growing interest in Distributed Constraint Reasoning (DCR) (Yokoo et al., 1992) has prompted researchers to propose several distributed approaches, whether the global problem is naturally distributed among agents. These agents have to communicate in order to revise their local solutions. The distributed Constraint Satisfaction Problem formalism (DisCSP) has been, widely, used in this context.

The DisCSP can be formalized as: a set of agents, a set of variables, a set of intra-agent constraint (constraints that link two variables of the same agent) and inter-agent constraints (constraints that link two variables of two different agents) and a function that associates each variable an agent. Several resolution algorithms have been developed. The pioneer approaches are: Asynchronous Backtracking (ABT) (Yokoo et al., 1992), Asynchronous Forward Checking (AFC) (Meisels and Zivan, 2007) and Nogood-Based Asynchronous Forward Checking (AFC-ng) (Wahbi et al., 2013). Authors often concentrate on simplification assumptions; where each agent owns only one variable. They assume that the existing algorithms are

easily extended to the most general case. Although, this is not completely true, because ignoring local resolution strategy can lead to a global costly resolution.

However, managing the trade off between complex local problems and distributed search effort can give a way to great improvements. There are several real complex local problems that can benefit from this trade off strategy, as the meeting scheduling problem, the road traffic and the multi-robot exploration.

In the literature, several techniques have been proposed. They reformulate the complex local CSP problems, so that there is exactly one variable per agent. The two most known approaches are: (i) Compilation that defines, for each complex agent, a single new abstract variable whose domain is the set of solutions to local CSP problem. Hence, to find the whole solutions set will be impossible, once granularity of local problems becomes larger. However, this method is used by a few number of researchers as the ABT-cf (Ezzahir et al., 2007; Ezzahir et al., 2008). (ii) Decomposition that creates a virtual agent, for each variable, in order to manage its domain. Several methods have used this method, as the multi-AWC (Yokoo, 1995), the Multiple local variables Distributed Bre-

about (Multi-DB (Hirayama and Yokoo, 2002)) and the Multi Dynamic Priorisation with Penalties algorithm (Multi-DynApp) (Magaji et al., 2014). The big issue that arises with this method is the loss of the centralization nature of the local problems, and several additional messages, that have not to be exchanged locally.

In (Burke and Brown, 2006), the authors proposed some improvements to the compilation method of Yokoo (compilation). It is based on the interchangeability principle; the external variables (i.e., variables that are constrained with other agents) in a local problem are the only variables that have a direct effect on the resolution process; by: (i) removing interchangeable solutions from the new constructed domain and (ii) identifying interchangeable solutions by considering the inter-agent constraints, in order to speed up the search. In (Ezzahir et al., 2007; Ezzahir et al., 2008), the authors present the ABT-cf algorithm (ABT with compilation formulation), that consists of integrating the compilation formulation and the interchangeability in the ABT algorithm. These methods (i.e., compilation, decomposition and compilation with interchangeability) are not suitable in the context of a realistic use. This is due to the load of extra messages when using the decomposition formulation and to the computational run time in the context of the compilation.

Our contribution is focused on how to choose the local solutions, after receiving a new message. However, a carefully chosen solution may allow neighbor agents to avoid unnecessary messages. Whereas, if each new solution is totally different from the previous one, then an avalanche batch of messages may be generated. So, in this paper, we propose a new algorithm, named MP-ABT, that improves the original ABT in the context of complex local problems. MP-ABT keeps the centralized nature of the local problems, without having neither to compile solutions, nor to decompose variables. It is based on Minimal Perturbation Problem formalism (MPP). When a new solution is needed, it looks for an optimal solution to the newly generated CSP, that it is as close as possible to the former complex local problem solution.

Inspiring from reality, each agent is seen as an MPP problem. It considers each received message as a new constraint perturbation of its complex local problem. Afterward, the agent has not to prepare any solution beforehand. It reacts in real time. It looks for the nearest solution to its former assignment (the solution that satisfies all new constraints and minimizes the number of changed values). In the MP-ABT algorithm, we use the HS-MPP approach locally (as an MPP algorithm), in order to reduce the number of

changed local variables, hence the number of disturbed neighbor agents.

In the following, we present the Asynchronous Backtracking ABT (Section 2.1), The Hybrid Search for Minimal Perturbation Problems HS-MPP (Section 2.2), then we describe the MP-ABT algorithm and show some properties of this new algorithm (Section 3). And then, in order to better understand the MP-ABT algorithm, we apply it on a simple example. Finally, we show experiment results that illustrate the efficiency of our newly developed algorithm (Section 5).

## 2 BASIC ALGORITHMS

### 2.1 Asynchronous Backtracking

The Asynchronous Backtracking algorithm (ABT) was developed by Yokoo et al. in (Bessière et al., 2005). Yet, in this communication protocol, the problem is solved asynchronously, by each agent. The priority order of the agents is set alphabetically. In general it is put alphabetically (i.e.,  $A_i$  is higher priority than  $A_j$  if  $i < j$ ). The inter-agent constraints are directed from higher priority agents to the lower priority agents. Each agent stores the variable assignments of other agents (higher priority agent assignments) in the AgentView structure, and the conflicts (nogoods) of low priority agents in the NogoodStore structure.

When the protocol starts, each agent attributes a value to its variable and sends it to the lower priority agents  $\Gamma^+(self)$  via OK? messages. After receiving a new OK? message, the receiver has to update its AgentView and NogoodStore (by storing the new assignments and removing the incoherent nogoods), then it has to check the consistency of its value with the AgentView. If it is inconsistent, self should look for a new consistent value. But if self can not find a consistent value, it generates a new nogood (that contains the assignments of higher priority agents  $\Gamma^-(self)$ , saying that this assignments subset can not be a part of a global solution) and sends a nogood message to the low priority agent (backtrack). A nogood  $ngd$  contains two sides. The left hand side  $lhs(ngd)$  is a conjunction of values chosen by the highest priority agents. The right hand side  $rhs(ngd)$  contains the assignment of the lowest priority agent. The set of values that form the nogood constitute the partial assignment that can not be part of the global affection (i.e., the problem solution). When a nogood is sent to the lowest agent (i.e., the owner of the assignment that exists in the  $rhs$  of the nogood), that

means that as long as the higher priority agents do not change the values which exist in the *lhs*, the receiver can not choose the value that exists in the *rhs* of the nogood.

When an agent receives a nogood message, it has to store it in its NogoodStore, and return a new value that is unremoved by the NogoodStore and is consistent with the AgentView. If the nogood contains values of agents that are not constrained with the nogood message receiver (i.e., they do not exist in  $\Gamma^-(self)$ ), it requests them via AddLink messages. The latter, should add a new link with self (*adl*), aiming to inform self by their assignments, whenever they change their values. After receipt of either an OK? message or a nogood message. If this message requires a change of its value, the ABT agent chooses its value by browsing its domain sequentially. If the reviewed value is inconsistent with its AgentView, it creates a nogood to be stored in its NogoodStore, and spends to the next value. A value is selected if it is consistent with the AgentView and not removed by its NogoodStore. If it does not find a consistent value, it creates a new nogood, by solving the nogoods which exist in its NogoodStore. The algorithm is made up to solve simple problems, where each agent possesses just one variable. But in the complex case, where agents handle multiple variables, it assumes to do a simple transformation either with the compilation or the decomposition. For each complex local CSP, the compilation creates a new abstract variable whose domain is the set of all the local solutions. The decomposition creates, for each variable, a virtual agent.

## 2.2 Hybrid Search for Minimal Perturbation Problems

The Hybrid Search for Minimal Perturbation Problems (HS-MPP) (Zivan et al., 2011; EL Graoui et al., 2016) is an approach that solves minimal perturbation problems (MPPs).

**MPP.** *A Minimal Perturbation Problem (MPP) is a solved CSP problem that is altered. Where the main task is to find a new solution in such a way that this latter does not differ much from the solution of the original CSP. The MPP can be formulated as a CSP  $P$ , an initial solution  $S_1$  of  $P$  and a distance function  $f$  defining the distance between any two assignments. A solution of an MPP problem is a solution  $S_2$ , such that the distance  $f(S_1, S_2)$  between  $S_1$  and  $S_2$  is minimal.*

The HS-MPP method aims to minimize the Hamming distance as a distance function.

**Hamming distance.** *The hamming distance is a mathematical concept that computes the number of positions where two entities, with the same length, differ. In our case, on two different assignments of the same local problem (i.e., the same agent), this measure computes the number of positions where the variable values differ. For example, a local problem of an agent  $A_i$ , with three local variables  $X_{i,1}$ ,  $X_{i,2}$ , and  $X_{i,3}$ , had an initial solution  $S_1 = \{X_{i,1} = 1, X_{i,2} = 1, X_{i,3} = 1\}$ . After a constraint modification, the solution becomes  $S_2 = \{X_{i,1} = 1, X_{i,2} = 2, X_{i,3} = 2\}$ . So, the Hamming distance of  $S_1$  and  $S_2$  is equal to 2.*

The HS-MPP algorithm takes variables, domains, constraints and the previous solution as parameters, and returns a closest solution or an empty one, saying that there is no solution to this MPP problem.

## 3 MP-ABT

### 3.1 Description of the Algorithm

The main contribution of MP-ABT is highlighted when an agent receives a new Ok? or Nogood message. In the ABT algorithm, if such message requires a new assignment, the receiver chooses a consistent local solution randomly. This assignment is chosen without considering the former local solution. In this approach, the MPP formalism is used in order to benefit from the former solution, to minimize the local perturbations and to reduce the number of disturbed neighbor agents.

The MP-ABT merges the Asynchronous Backtracking algorithm ABT, and the Hybrid Search for Minimal Perturbation Problems algorithm HS-MPP. This algorithm extends the ABT algorithm, in order to tackle DisCSPs with complex local problems, with less perturbations. The idea is to consider each local complex problem as an MPP problem and each new received message as a new constraint perturbation. The aim is to find a new solution that is as close as possible to the former solution.

The local search of ABT can not be directly replaced by the HS-MPP approach. Because in the local search of the ABT protocol, the ABT agent stores nogoods during the search of a local consistent solution. So, if it can not find a consistent local solution, it has justifications to create a new nogood. But in the case of HS-MPP, the local consistent value is searched in a single execution. Therefore, if the returned value is empty (i.e., there is no consistent local solution), the agent has no justifications to construct the nogood. That is why, we have to make several modifications to the original pseudo code of ABT. In the following,

Algorithm 1: MP-ABT algorithm.

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1: procedure MP-ABT myvalue  $\leftarrow$  empty;
2: end  $\leftarrow$  false;
3: CheckAgentView();
4:   while  $\neg$ end do
5:     msg  $\leftarrow$  getMsg();
6:     Switch (msg.type)
7:     Ok? : ProcessInfo(msg);
8:     ngd : ResolveConflict(msg);
9:     stp : end  $\leftarrow$  true;
10:    Adl : SetLink(msg);
11:  end while
12: end procedure

13: function CHOOSEVALUE(VariableDomains, NO-
GOODSTORE)
14:   if (one of VariableDomains is empty) then
15:     return empty
16:   end if
17:   Create a MPP problem with the same variables of
the initial local problem;
18:   Attribute VariableDomains to the variables;
19:   Add the non redundant rhs(nogood) of the Nogood-
Store as a not equal constraint of the MPP problem;
20:   return MPP.getSolution
21: end function

22: procedure UPDATE(myAgentView, newAssig)
23:   add(newAssig, myAgentView);
24:   for { each ng  $\in$  myNogoodStore } do
25:     if ( $\neg$ Coherent(lhs(ng), myAgentView)) then
26:       remove(ng, myNogoodStore);
27:     end if
28:   end for
29:   for { each ng  $\in$  Justifications } do
30:     if ( $\neg$ Coherent(lhs(ng), myAgentView)) then
31:       remove(ng, Justifications);
32:     if (rhs(ng) is not removed by a nogood in
Justifications) then
33:       restore rhs(ng).value to the
currentDomains;
34:     end if
35:   end if
36: end for
37:   for each local variable do
38:     Remove inconsistent values from
currentDomain;
39:     Add a nogood in justifications for each remo-
ved value;
40:   end for
41: end procedure

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we are going to describe how the MP-ABT algorithm is running. The Algorithm 1 provides the procedures and functions executed by each MP-ABT agent, that do not exist in the ABT pseudo code, or they are changed.

As the ABT agent, each MP-ABT agent assigns values to its variables, sends them to the correspon-

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40: procedure BACKTRACK
41:   newNogood  $\leftarrow$  solve(myNogoodStore  $\cup$ 
Justifications);
42:   if newNogood = empty then
43:     end  $\leftarrow$  true;
44:     sendMsg: stp (system);
45:   else
46:     sendMsg:ngd(newNogood);
47:     Update (myAgentView, rhs(newNogood)  $\leftarrow$ 
unknown);
48:     CheckAgentView();
49:   end if
50: end procedure

```

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ding agents, and then switches to the listening position, in order to respond to incoming messages (ABT pseudo-code (Bessière et al., 2005)).

After the reception of an Ok? message, for each local variable of its local problem, the receiver filters the variables domains. it removes all values that are inconsistent with sender values from the variable domain. For each filtered value, it adds a nogood to its 'Justifications' structure (Update procedure, line 89). This structure contains the deleted value and the variable that cause this value removal. The resulting nogood contains a shorter partial assignment that causes the inconsistency, and not the whole assignment of the local problem. Hence it enjoys the benefits of the interchangeability to speed up the resolution process. During the filtering process, the agent tests the whole domain even if, it may contain values that are already deleted. These redundant justifications aim to save all suppression causes of each value. Foremost, self updates the 'Justifications' and NogoodStore structures, by removing the nogoods that become obsolete (Update procedure, lines 76 and 81). Then, it restores values to the variables domains (Update procedure, line 83). A value is restored to its domain if and only if all the corresponding justifications are removed (Update procedure, line 82). Finally, it chooses a new local solution (ChooseValue procedure).

For this, it follows the succeeding steps:

- It checks if there is an empty filtered domain (line 66).
- If so, it returns an **empty** value, in order to send a nogood message, without looking for a new local solution.
- Otherwise, it declares a new MPP problem. The latter contains its local problem variables which are defined on their new corresponding filtered domains, its local constraints (intra-agent constraints), and then it adds the valid nogoods, that are compatible with the other agent value, which



exist in its NogoodStore as new constraints (lines 68, 69 and 70).

- Finally, it looks for a new closest solution to its current solution, using the HS-MPP algorithm.
- If it finds a new solution, it will send this solution to the corresponding agents (CheckAgentView procedure).
- Otherwise, it generates a new nogood, using the stored justifications and the received nogoods (Backtrack procedure, line 55).

After the reception of a nogood message, the agent stores it in its NogoodStore, and chooses a new closest solution, using the same manner described previously.

### 3.2 MP-ABT Properties

The MP-ABT has the same properties as the ABT: soundness, completeness and termination.

*MP-ABT is sound.*

**Proof:** Since the local search of ABT is replaced by the MPP approach in MP-ABT. The only risk that may cause the unsoundness of MP-ABT is that it will have no way to record the deletion reasons of each local solution. But as it records justifications during the filtering operation, therefore it remains also sound, as the ABT algorithm.

*MP-ABT is complete.*

**Proof:** The MP-ABT uses the filtered domain, the existing intra-agent constraints and the valid received nogoods as inputs to the MPP algorithm. Since the HS-MPP algorithm is sound, the returned solution will satisfy all constraints (the original constraints, and the received nogoods). So the local solution of each agent, satisfy its inter-agent and intra-agent constraints. Then the algorithm is complete.

*MP-ABT can always terminate.*

**Proof:** The filtering process speeds up the search, and can never be trapped in an infinite loop, since the domains are finite. In addition, since the HS-MPP algorithm finds the solution in a limited time, so the whole problem can be solved also in a finite time. On the contrary, it speeds up the search, because it gives the local decision in just one loop, without doing the compilation nor the decomposition process.

## 4 EXAMPLE

The figure 1 illustrates a complex DisCSP example containing three agents  $A_1$ ,  $A_2$  and  $A_3$ , and two va-

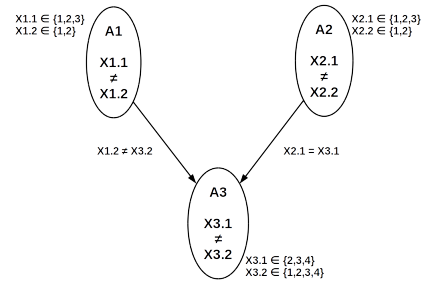


Figure 1: Example of DisCSP with complex local problems.

riable per agent:  $X_{1.1}$ ,  $X_{1.2}$ ,  $X_{2.1}$ ,  $X_{2.2}$ ,  $X_{3.1}$  and  $X_{3.2}$ . The domains of variables are  $D(X_{1.1}) = D(X_{2.1}) = \{1, 2, 3\}$ ,  $D(X_{1.2}) = D(X_{2.2}) = \{1, 2\}$ ,  $D(X_{3.1}) = \{2, 3, 4\}$ , and  $D(X_{3.2}) = \{1, 2, 3, 4\}$ . The intra-agent constraint is the not equal constraint (all local variables should be not equal) and the inter-agent constraints are  $X_{1.2} \neq X_{3.2}$  and  $X_{2.1} = X_{3.1}$ . We assume that the priority order of agents is lexicographic ( $A_1 > A_2 > A_3$ ).

When the MP-ABT protocol starts, each agent chooses its first assignment and communicates it to the lower priority agents. Agents  $A_1$  and  $A_2$  choose the values (1,2) (i.e., the first variable is equal to 1 and the second is equal to 2), and send OK? messages, carrying its affectations, to the Agent  $A_3$ . After receiving the OK? message from  $A_1$ , the agent  $A_3$  stores the received values in its AgentView, removes inconsistent values from its domains and stores a justification for each removed value. It deletes the value 2 from the  $X_{3.2}$  domain (because  $X_{1.2} = 2$  and  $X_{1.2} \neq X_{3.2}$ ), adds the nogood  $X_{1.2} = 2 \rightarrow X_{3.2} \neq 2$  as a deletion justification to its 'Justifications' structure, and checks if its values remain consistent. the assignment (1,2) still consistent, so  $A_3$  switches to the listening state. When  $A_3$  receives the second OK? message from  $A_2$ , it does the same treatment. It updates its AgentView with the received values, deletes the values 2, 3, and 4 from the  $X_{3.1}$  domain (because  $X_{2.1} = 1$  and  $X_{2.1} = X_{3.1}$ ). So, the domains become:  $D(X_{3.1}) = \{\emptyset\}$ , and  $D(X_{3.2}) = \{1, 3, 4\}$ . In the other hand, it adds these nogoods to its 'Justifications' structure  $X_{2.1} = 1 \rightarrow X_{3.1} \neq 2$ ,  $X_{2.1} = 1 \rightarrow X_{3.1} \neq 3$ , and  $X_{2.1} = 1 \rightarrow X_{3.1} \neq 4$ .  $A_3$  finds that the  $X_{3.1}$  domain is empty, so it sends a nogood  $\emptyset \rightarrow X_{2.1} \neq 1$  to  $A_2$ .

After receiving the nogood message,  $A_2$  stores it in its NogoodStore. It creates a new MPP problem whose variables are its local variables ( $X_{2.1} \in \{1, 2, 3\}$  and  $X_{2.2} \in \{1, 2\}$ ) and constraints are  $X_{2.1} \neq X_{2.2}$  and  $X_{2.1} \neq 1$ ; the second constraint is taken from the nogood that exists in the  $A_2$  NogoodStore; retrieves the HS-MPP local solution (3,2), which is clo-

sest to its current solution (1, 2), and sends it to the agent  $A_3$  via an OK? message.  $A_3$  removes the no-goods from its 'Justifications' structure, because they become obsolete, and restores the values 2, 3, and 4 to the  $D(X_{3,1})$ . Then it filters out the values 2 and 4 from  $D(X_{3,1})$ , and stores the nogoods  $X_{2,1} = 3 \rightarrow X_{3,1} \neq 2$  and  $X_{2,1} = 3 \rightarrow X_{3,1} \neq 4$  in the 'Justifications' structure. The current values (2, 1) of  $A_3$  does not remain valid, so  $A_3$  generates a new MPP problem with  $X_{3,1} \in \{3\}$ ,  $X_{3,2} \in \{1, 3, 4\}$ , and  $X_{3,1} \neq X_{3,2}$ . The returned local solution by the HS-MPP approach is (3, 1).

We saw, by this example, one of the advantages of the MP-ABT, that is the nogood content, which is very pointed. It allows to know the real cause of a conflict and to implicate the responsible variable. Not just the responsible agent, as is done in the ABT algorithm.

Note that, even the filtering process, helps to detect the conflict without the need of looking for a new solution.

Finally, if the variable  $X_{2,2}$  of the agent  $A_2$  was constrained with other external variables,  $A_2$  will not disrupt the other agents. The same thing with the variable  $X_{3,2}$  of the agent  $A_3$ . So we minimize the number of the global perturbation, by minimizing it locally.

## 5 EXPERIMENTAL RESULTS

In this section, we evaluate the performance of our new algorithm MP-ABT, by comparing it with the algorithms: ABT-comp, the ABT-decomp, and the ABT-cf. The checked constraints during the compilation and the decomposition are also computed.

In the ABT-comp algorithm, each agent searches all its local problem solutions, prepares its new domain, that contains the found solutions in the compilation process and starts the ABT protocol, considering each agent as a simple one (i.e., that has just one local variable).

The ABT-decomp agent creates a virtual agent, for each local variable. The original problem will be distributed between the created agents. So the problem becomes simple, and ABT algorithm is applied.

The ABT-cf is an extension of ABT-comp. After preparing the new domain (using the compilation), The ABT-cf agent applies the interchangeability principle in order to remove interchangeable solutions, and speed up the search, by considering the inter-agent constraints.

The algorithms are evaluated on Random Complex DisCSPs. The assessment is made against the number of exchanged messages (# MSGs) that measures the number of the perturbations and the

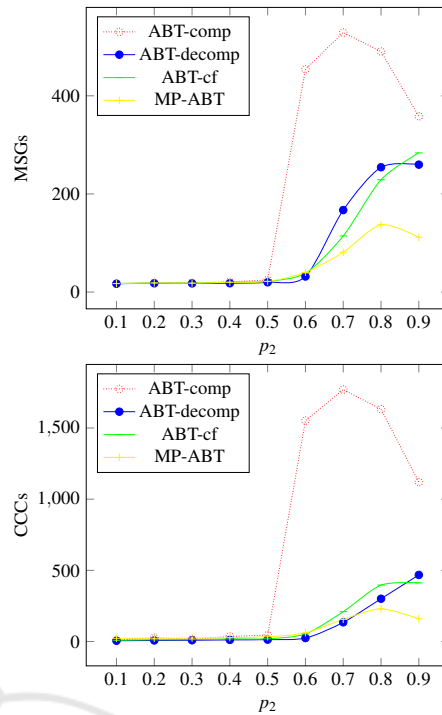


Figure 2: Benchmarking with  $\langle 0.3, 0.7, 0.3, 5, 5, 2, p_2 \rangle$ .

communication load, and the Concurrent Constraint Checks (# CCCs), it is a metric used in distributed constraint solving, which simulates the computation time and computes the computation effort. All random Complex DisCSP problems are characterized by  $(ia, ib, c, n, d, v, p_2)$  where,  $ia$  is the intra-agent density,  $ib$  the inter-agent density,  $c$  the connection density,  $n$  the number of agents,  $d$  the domain size,  $v$  the number of variables per agent and  $p_2$  is the tightness of constraints. We select problems in four most representative areas classes of the constraints:  $\langle 0.3, 0.7, 0.3, 5, 5, 2, p_2 \rangle$ ,  $\langle 0.7, 0.3, 0.7, 5, 5, 2, p_2 \rangle$ ,  $\langle 0.3, 0.7, 0.3, 5, 5, 4, p_2 \rangle$ , and  $\langle 0.7, 0.3, 0.7, 5, 5, 4, p_2 \rangle$ . The tightness  $p_2$  is varied from 0.1 to 0.9 by the step 0.1. For each fixed set  $(ia, ib, c, n, d, v, p_2)$ , we generated 10 instance, and we took their average.

The figure 2 shows the performances of ABT-comp, ABT-decomp, ABT-cf, and MP-ABT, functioning on the first class of constraints  $\langle 0.3, 0.7, 0.3, 5, 5, 2, p_2 \rangle$ . The figure 3 exhibits the behaviors of the four algorithms, against the second class of constraints  $\langle 0.7, 0.3, 0.7, 5, 5, 2, p_2 \rangle$ .

In the two classes where each agent handles two variables, we observe that, in general, the ABT-comp is the less-performance algorithm. It exchanges more messages (more disturbance) and checks more constraints concurrently. It is due to the size of the new constructed domain by the compilation. The ABT-cf

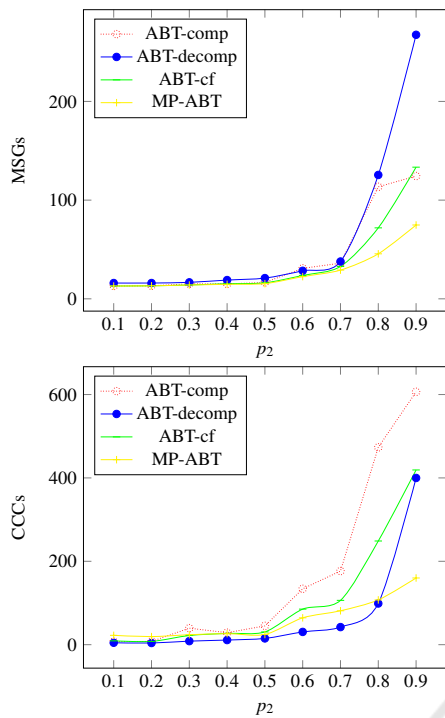


Figure 3: Benchmarking with  $\langle 0.7, 0.3, 0.7, 5, 5, 2, p_2 \rangle$ .

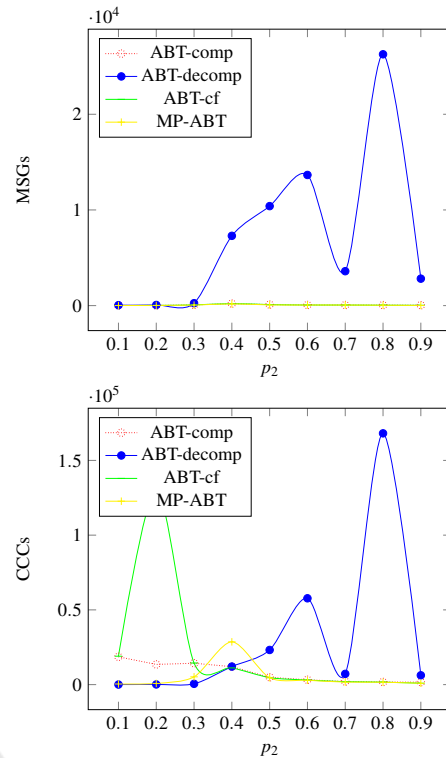


Figure 5: Benchmarking with  $\langle 0.7, 0.3, 0.7, 5, 5, 4, p_2 \rangle$ .

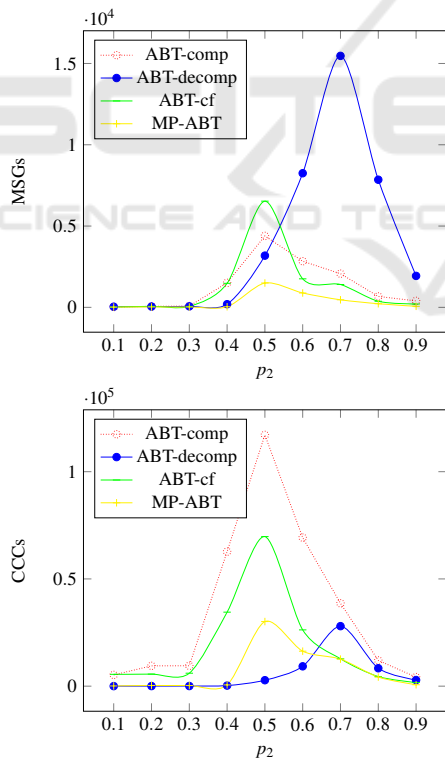


Figure 4: Benchmarking with  $\langle 0.3, 0.7, 0.3, 5, 5, 4, p_2 \rangle$ .

outperforms the ABT-comp by the deletion of the interchangeable solutions, but still not exceed the MP-ABT, because the content of nogoods, in the ABT-cf, does not speed up the search. It involves a solution

of the local problem of the agent, not just the variables that cause the conflict. So, instead of deleting the responsible variable values, the algorithm deletes just a solution, and the next sent value of a variable may be the same that already caused the problem. In the ABT-decomp algorithm, even if the domain size is still the same, it is also less efficient than MP-ABT, because of the creation of the virtual agents, that increases the size of the global problem. And so, there is more exchanged messages without any importance, and more checked concurrent constraints.

For 4 variables per agent, the evaluation results are shown in figures 4 and 5. In this level, the performance of the MP-ABT algorithm becomes more important. In contrary of the first results (i.e., 2 variables per agent), the out-performance began to be important in the problems with low tightness too. In this case, the local problems become more constrained. So, it is too hard and complex to find the whole compiled domain, in ABT-comp and ABT-cf. So, the domain contains several values. Therefore more constraints are tested. For the ABT-decomp which is based on the decomposition, it makes a big effort. It can loop several times in order to find one local consistent solution.

Our aim is to minimize the number of perturbations (i.e., the number of exchanged messages). while, such results exhibit that our principal aim is achieved, in the case of two variables per agent as well as in the case of four variables per agent. In addition

of our main goal, we have also reduced the number of concurrent constraint checks, in most tightness values, especially when the constraint network becomes dense.

The different evaluation results demonstrate that the MP-ABT is a very effective way to decrease the number of messages and therefore the number of perturbations. We observe that, the more variables we have, the less disturbance MP-ABT does. And even if problems become insolvable. The MP-ABT algorithm remains better, due to the nogood content. The nogood used by the MP-ABT contains just the solution parts that cause the failure (contrary to ABT-comp and the ABT-cf that reports all the solution with the different variables). So, instead of deleting just one solution after the reception of a nogood message, we can delete a group of solutions. In addition, the filtering process used in the MP-ABT, minimizes the number of checked constraints.

## 6 CONCLUSION

In this paper, we have proposed a new complete algorithm: Minimal Perturbation complex local problems in the Asynchronous Backtracking **MP-ABT**. It is an upgrade of the ABT algorithm. It is able to solve DisCSP problems with complex local problems, while minimizing the perturbations, without any transformation of the original problem, as it is done by the compilation and decomposition methods.

The MP-ABT algorithm considers each complex local problem as an MPP problem, and each received message as a perturbation of the intra-agent constraints. After a message reception, the MP-ABT agent tries to find a closest local solution to its current values, using the HS-MPP algorithm.

The experimentations show that the MP-ABT algorithm outperforms the different ABT versions, in terms of the number of exchanged messages and therefore the number of perturbations, while minimizing the computational effort, especially when problems are dense and contain more variables per agent.

We perceive to generalize the method of the minimal perturbation in the complex local problems, in order to be integrated in the existing DisCSP algorithms, as the AFC and AFC-ng algorithms.

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