Sampling Density Criterion for Circular Structured Light 3D Imaging

Deokwoo Lee[1] and Hamid Krim[2]

[1] *Youngsan University, Yangsan, Gyeongnam, South Korea*

[2] *Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, U.S.A.*

dwoolee@ysu.ac.kr, ahk@ncsu.edu

Keywords: 3D Reconstruction, Structured Light Pattern, Sampling Theorem, Geometry.

Abstract: 3D reconstruction work has chiefly focused on the accuracy of reconstruction results in computer vision, and efficient 3D functional camera system has been of interest in the field of mobile camera as well. The optimal sampling density, referred to as *the minimum sampling rate* for 3D or high-dimensional signal reconstruction, is proposed in this paper. There have been many research activities to develop an adaptive sampling theorem beyond the *Shannon-Nyquist Sampling Theorem* in the areas of signal processing, but sampling theorem for 3D imaging or reconstruction is an open challenging topic and crucial part of our contribution in this paper. We hence propose an approach to sampling rate (lower / upper bound) determination to recover 3D objects (surfaces) represented by a set of circular light patterns, and the criterion for a sampling rate is formulated using geometric characteristics of the light patterns overlaid on the surface. The proposed method is in a sense a foundation for a sampling theorem applied to 3D image processing, by establishing a relationship between frequency components and geometric information of a surface.

1 INTRODUCTION

3D Imaging research has been extensively performed since a few decades in the areas of computer vision, image processing and pattern recognition for diverse applications. In 3D reconstruction, a number of methods have been proposed for the problem and the passive (Kolev et al., 2014) and the active ((Geng, 2011), (Lei et al., 2013)) methods have been widely applied in practice. Both methods are based on establishing a geometrical relationship of high dimensional signal (or 3D object points) and a low dimensional one (or 2D image projected on the image plane). Readers also can refer to about recent research of the passive and the active methods and a hybrid method that combines the passive and the active method for the further improvement on the quality of the 3D reconstruction results ((Chan et al., 2008), (Song et al., 2014)). Consider $f \in \mathbb{R}^3$ and $f' \in \mathbb{R}^2$, the problem is to then recover f from given information of f', where f' is determined by a transformation $\mathcal{P}$, such that

$$f' = \mathcal{P} f, \tag{1}$$

where $\mathcal{P}$ is usually a projection operator (Faugeras et al., 2001), and the operator yields an approximately perfect reconstruction of f. In the field of image processing and computer vision, the general principle space in passive methods is triangulation using two or more 2D image planes generated from a number of cameras. The relative geometric relationship between image planes located at different positions and a target object in 3D domain, and a successful correspondence matching results provide sufficient information required to recover 3D coordinates of an object surface. The passive methods, however, sometimes fails high quality reconstruction results because there exists a inherent limitation and disadvantages due to the occlusion problem, low textured surface, high computational complexity of the correspondence matching, etc. To alleviate the limitations of the passive method, active method using structured light patterns has been employed for 3D reconstruction . The active method, an alternative to the passive methods, also solves the reconstruction problems based on the geometrical relationship between the components (optical center, 3D points on the surface, location of a light source) by replacing one camera with a light source generating structured light patterns. It is obvious that data acquisition procedure and efficient use of data is crucial in reconstruction work in both passive and active methods. To achieve this efficiency, reconstruction is closely related to sampling rate represented as the optimal number of light patterns. This paper proposes an approach to a sampling theorem for high dimensional signals and a sampling rate de-

Lee D. and Krim H.
Sampling Density Criterion for Circular Structured Light 3D Imaging.
DOI: 10.5220/0006147504780483
In *Proceedings of the 12th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications (VISIGRAPP 2017)*, pages 478-483
ISBN: 978-989-758-227-1

termination for structured light 3D imaging applications. In particular, we determine the lower and the upper bound criterion of a sampling rate for the reconstruction using structured circular light patterns. The active method on the basis of circular structured light patterns provides simple, efficient reconstruction algorithm, and the active method (Geng, 2011), in general, achieve accurate reconstruction result in contrast to the passive method. Once we have one dimensional continuous signal, we are interested in the minimum sampling rate (f_s) which is represented as $f_s = 2f_{max}$ or the maximum sampling interval (T_{max}). Akin to 1D signal, once fully reconstructed, the surface which is a (image) signal of interest, represented as 3D Euclidean coordinates (triangle meshes composed of vertices and faces), is considered a continuous signal, which yields a natural question about an optimal number of circular light patterns necessary for reconstruction of surfaces. The spatial sampling density (or sampling rate), is defined as *the minimum number of circular patterns* to be projected on a surface. The minimum number of circular patterns leads to the maximum sampling interval between neighboring circular patterns. Determination of the minimum number of the patterns is proposed in this paper. The sampling rate determination in the field of 3D reconstruction using the structured light pattern has not gained much attention in contrast to the research for improving accuracy of reconstruction itself. In this paper, the sampling rate is ultimately obtained from the spatial maximum frequency component which is closely related to geometric characteristics of an object surface. This paper proposes the approach to derivation of the upper and the lower bound of a sampling rate for the surface reconstruction and the simulations results are also shown to verify the proposed approaches.

2 SAMPLING RATE

This section details the derivation of the minimum sampling rate which leads to sufficient representation of 3D characteristics of a target object surface. This paper uses curvature information to determine the minimum sampling rate, and frequency components of the surface is derived by establishing a relationship between curvature and frequency. Intuitively, required number of circular patterns is determined by geometric surface shape, for instance, larger number of the patterns is required for the surface which has frequent variation of a surface shape. As previously stated, surface shape deforms a set of concentric circular patterns. In other words, deformation can be

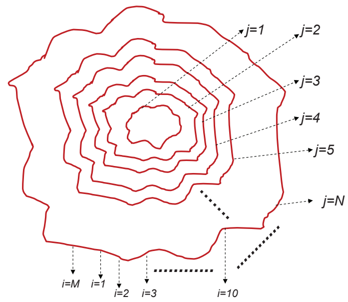

Figure 1: i and j respectively index the number of points(M) in each patterns, and the pattern's position.

represented quantitatively by measuring curvatures of the patterns projected onto the surface (i.e., original circle has a constant curvature, but the curvatures are not constant in deformed circles). These curvatures can lead to developing a sampling criterion for a surface reconstruction if the relationship between spatial frequency components and curvature information is established. This section establishes the relationship to achieve sampling rate determination. In practice, given small amount geometric information (i.e., highest curvature that is corresponding to a point of highest variation in the surface), sampling rate is directly determined and leads to efficient reconstruction (approximate geometric reconstruction) system. In this paper, we assume that we are given the initial number of circular patterns that are sufficient to represent 3D object (i.e., oversampled) to theoretically derive a sampling criterion for a surface reconstruction. Once 3D coordinates of a surface are provided, we denote the surface and the discretely sampled surface (sampled by N concentric circular patterns) by S_w and S_3, represented as

$$S_w = \{x_w, y_w, z_w\}, \quad (2)$$
$$S_3 = \{x_{ij}, y_{ij}, z_{ij}\}, \quad (3)$$
$$i = 1,2,\ldots,M, \; j = 1,2,\ldots,N,$$

where where i and j respectively index the number of points in each patterns, and the pattern's position (Fig. 1). Since the present work aims at deriving the minimum number of the patterns, this paper chiefly deals with pattern-wise sampling rather than point-wise sampling under the assumption that M is sufficiently large. Fig. 2, we view $j = 1,2,\ldots,N$ as a time index, and the signal of interest is given by

$$S_3(t) = \{x_i(t), y_i(t), z_i(t)\}, \quad (4)$$
$$i = 1,2,\ldots,M, \; j = 1,2,\ldots,N.$$

A sufficiently large N makes $S_3(t)$ dense and close to a continuous signal, which we in turn use to introduce sampling technique using a 1D signal. $S_1(t)$ is defined

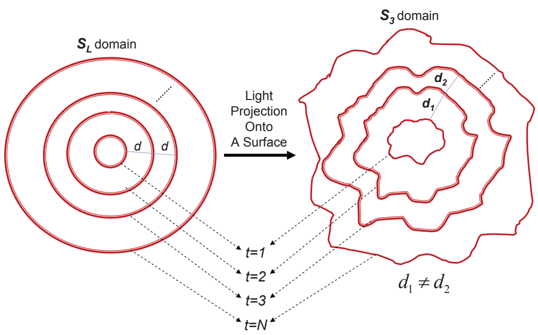

Figure 2: Indices of the location of each curve alternatively represented as time t to introduce a concept of a frequency component of a surface.

in the continuous time and $S_1[n] = S_1(nT)$ is defined is the discrete time domain. The *Fourier transform* of $S_1(t)$, $S_1 : \mathbb{R} \rightarrow \mathbb{R}$, is denoted by

$$S_1(\omega) \quad = \quad \int_{-\infty}^{\infty} S_1(t)e^{-j\omega t}dt, \tag{5}$$

where $\omega = 2\pi f$, is a radian frequency. If we assume that $S_1(t)$ is bandlimited, has a finite energy (square-summable or square-intergrable in Lebesgue's sense) (Unser, 2000) and is sampled at the points $t_n = nT$, then $S_1(t)$ can be reconstructed with an appropriate T as follows :

$$S_1(nT) \quad = \quad \sum_{n=-\infty}^{\infty} S_1(t)\varphi_2(t-nT), \tag{6}$$

$$\begin{aligned} S_1(t) \quad &= \quad S_1(nT)\star\varphi_1(nT) \\ &= \quad \sum_{n=-\infty}^{\infty} S_1(nT)\varphi_1(t-nT), \end{aligned} \tag{7}$$

(8)

where '$\star$' denotes the convolution operator, t can be a *time* or a *index* of a sample (e.g., sampled pattern in Fig. 1, and $\varphi_1(t)$ and $\varphi_2(t)$ are orthogonal basis functions,

$$< \varphi_1(k), \varphi_2(k) >= \delta(k-l), \tag{9}$$

where '$<,>$' is an L^2 inner product operation and $\delta(t)$ is the *Dirac delta function*. $\varphi_1(t)$ and $\varphi_2(t)$ are selected appropriately to solve the reconstruction problem and are ideal lowpass filters in this paper (i.e. *sinc* function) ((Higgins, 2003), (Jerri, 1977), (Mallat, 1989), (Papoulis, 1977), (Unser, 2000)). The reconstruction process then can be written as

$$S_1(t) = \sum_{n=-\infty}^{\infty} S_1(nT)sinc(\omega_0(t-nT)), \tag{10}$$

where $sinc(\omega_0(t-nT)) = \sin(\omega_0(t-nT))/\omega_0(t-nT)$ is a *sinc function*. Recovery of $S_1(t)$ from $S_1(nT)$ is perfectly conducted by sampling the points from $S_1(t)$, or by selecting an appropriate sampling frequency f_s, where $f_s = 1/T$. Akin to determining the sampling frequency for a 1D signal, our reconstruction of a surface, $S_3(t)$ will seek the maximal frequency component f_{max}, and this approach can be applied to any type of signals. We assume that the signal of interest $S_3(t)$ is bandlimited or pre-processed by appropriate filters (Eldar and Pohl, 2009). The sampling rate f_s provides a sufficient criterion for a geometric signal recovery, and it is defined in the unit arc length or pixel. The minimum number of circular patterns N_s for the reconstruction is simply obtained as follows :

$$N_s = f_s \times N = 2f_{max} \times N, \tag{11}$$

where N is the initial number of circular patterns projected onto the surface to extract 3D coordinates, and N_s is the minimum number of patterns to reconstruct the surface. In the experiments with synthetic data, we initially determine N which can fully reconstruct (or represent) 3D geometric information of a target object surface.

2.1 Sampling Rate using the Two-Third Power Law

Quantifying a surface shape is tantamount to extracting sufficient geometric information (e.g., tangent vectors, curvatures, etc.) from a surface. To that end, curvature, the first derivative of a tangent vector, is geometric information for estimating spatial frequency components, and the curvature is better to describe geometric properties of 3D object because it is viewpoint invariant. The Two-Third Power Law (Lee and Krim, 2011), proposed by Paolo Viviani (Lacquaniti et al., 1983) explains a relationship between an absolute angular velocity and curvature for constrained movements (Fig. 3). In this method, the constrained movement of points comprises a curve, from which we can estimate an angular velocity and curvature of a curve composed of points. In Fig. 3, r is an Euclidean distance from the reference point to any point P, $\overrightarrow{T}$ is a tangent vector, and κ is a curvature based on an osculating circle where radius is R, and V is an angular velocity which satisfies the following equations (de'Sperati C and P, 1997):

$$\begin{aligned} V(t) &= K \cdot \left(\frac{R(t)}{1+\alpha R(t)}\right)^{1-\beta}, \\ V(t) &= r(t)\omega = 2\pi r(t)f, \end{aligned} \tag{12}$$

where K depends on the movement duration, called a velocity gain, α and β are parameters (α, $\beta \in \mathbb{R}$). Given $R(t)$, and $r(t)$, recall that our purpose here is

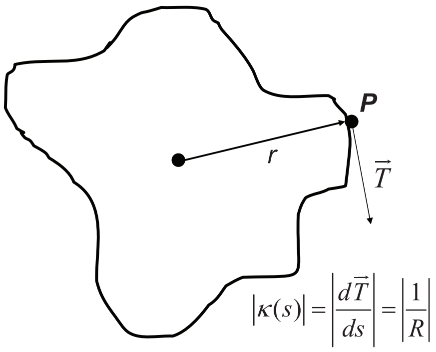

Figure 3: *The Two-Thirds Power Law* explains a relationship between a geometrical information (i.e. curvature) and an angular velocity of a curve. Angular velocity is composed of a radius r and an angular frequency ω.

to estimate the maximal frequency component, f_{max}, Eq. (12) is rewritten as follows :

$$f = \frac{1}{2\pi r(t)} K \cdot \left(\frac{R(t)}{1+\alpha R(t)}\right)^{1-\beta}. \quad (13)$$

Eq. (13) hence yields the maximal frequency component and a sampling frequency estimation based on a *Nyquist rate* :

$$f_{max} = \max\left[\frac{1}{2\pi r(t)} K \cdot \left(\frac{R(t)}{1+\alpha R(t)}\right)^{1-\beta}\right] \quad (14)$$

$$f_s \geq 2 f_{max}. \quad (15)$$

Having established the algorithm for a spatial sampling frequency estimation, sampling rate for 3D object surface is considered. Extended to a 3-dimensional signal, projected circular patterns on a surface have two tangential vectors (Fig. 5). Hence, two curvature components (κ_{1ij} and κ_{2ij}), the first derivative of tangential vectors, T_{1ij} and T_{2ij} are defined at P_{ij} with respect to the arc length. For a 3D signal (i.e., surface), at any point P_{ij}, if two orthogonal tangent vectors are selected, then two corresponding curvatures are defined as well.

We can hence acquire the frequency components corresponding to κ_{1ij} and κ_{2ij}, respectively, and *The Two-Thirds Power Law* for a 3D surface may also be written as :

$$\omega_{1ij} = 2\pi f_{1ij} = \frac{1}{r_{1ij}} \cdot \left(\frac{R_{1ij}}{1+\alpha R_{1ij}}\right)^{1-\beta}, \quad (16)$$

$$\omega_{2ij} = 2\pi f_{2ij} = \frac{1}{r_{2ij}} \cdot \left(\frac{R_{2ij}}{1+\alpha R_{2ij}}\right)^{1-\beta}, \quad (17)$$

$$r_{1ij} = r_{2ij} = r_{ij},$$

$$i = 1,2,\ldots,M, \ j = 1,2,\ldots,N,$$

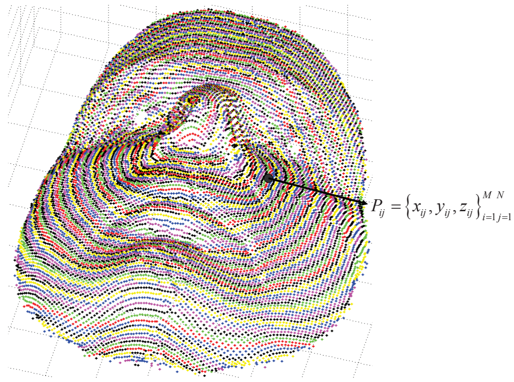

Figure 4: Example of 3D face model(fvinput data).

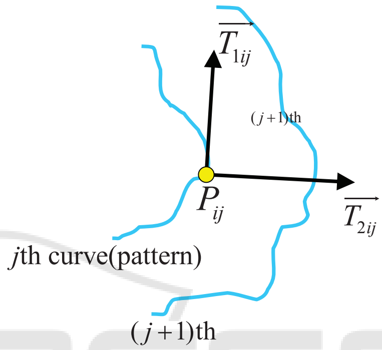

Figure 5: Vectors defined on a point in a facial curve.

where R_{1ij} and R_{2ij} are $1/\kappa_{1ij}$ and $1/\kappa_{2ij}$, respectively, and M is the number of points on each curve and N is the number of curves (patterns) on the surface. To determine the minimum sampling rate ($2 \times \max(f_{ij})$) consistent with the *Nyquist Rate*, the maximum frequency component, $\max(f_{ij})$ is required, and we define $\max(f_{ij})$ as

$$\max(f_{ij}) = \max[sup(f_{1ij}), sup(f_{2ij})]. \quad (18)$$

Using the relationship between the frequency component, f_{ij} and the corresponding r_{ij}, the maximum frequency is calculated. Prior to measuring the global maximum f_{ij}, local maximum $sup(f_{1ij})$ and $sup(f_{2ij})$ should be acquired, and each of which satisfies the following,

$$sup(f_{kij}) \leq \frac{1}{2\pi} sup\left(\frac{1}{r_{kij}} \cdot \left(\frac{R_{kij}}{1+\alpha R_{kij}}\right)^{1-\beta}\right), \ k = 1,2. \quad (19)$$

Prior to the measurement of curvatures and r_{ij}'s of all the points of the deformed circular patterns, a normalization of data points is carried out. The normalization yields the determination of the intrinsic characteristics of each curve projection on the surface.

2.2 Proposed Sampling Rate Determination

In case of any dimensional signal, sampling rate can be determined using curvature information. Using a *Fourier Series*, any signal can be represented as a combination of sinusoidal signal. Based on the principle of *Fourier Series*, a signal (or a curve) can ba represented as $\alpha(t) = [t, \cos\omega t]$,where ω is a frequency component of a signal. Once $\alpha(t)$ is determined, we use the following information to estimate a curvature:

$$\alpha'(t) = \frac{d\alpha(t)}{dt} = [1, -\omega \sin \omega t], \quad (20)$$

$$\alpha''(t) = \frac{d\alpha(t)^2}{d^2 t} = [0, -\omega^2 \cos \omega t]. \quad (21)$$

For any regular curves, curvature is defined as (Oprea, 2007)

$$\kappa = \frac{|\alpha' \times \alpha''|}{|\alpha'|^3}, \quad (22)$$

where $\alpha = \alpha(t)$, $\times$ represents the outer product between two vectors. To achieve the minimum sampling rate, we do not have to measure all curvature or frequency information. Because a maximal frequency component contributes to the minimum sampling rate. Thus, we represent the target signal as just $\alpha(t) = [t, \cos\omega t]$ or $\alpha(t) = [R(t), z(t)]$. If we use circular patterns each of which has a radius $R(t) = t$, $z(t)$ can be represented as a combination of sinusoidal signals, and ω can be a dominant frequency component, $\alpha(t) = [t, \cos\omega t]$ is sufficient representation. Curvature can be written as

$$\begin{aligned} \kappa &= \frac{|-\omega \cos \omega t|}{\left(1 + \omega^2 \sin^2 \omega t\right)^{3/2}} \\ &\leq \frac{\omega^2}{\left(1 + \omega^2 \sin^2 \omega t\right)^{3/2}}. \end{aligned} \quad (23)$$

Then, κ is bounded by $0 \leq \kappa \leq \omega^2$ and the frequency component can be written as

$$\begin{aligned} \kappa &\leq 4\pi f^2, \\ f &\geq \frac{1}{2\pi}\sqrt{\kappa}. \end{aligned} \quad (24)$$

Since κ is any curvature value,

$$f \geq \frac{1}{2\pi}\sqrt{\kappa_{max}}. \quad \blacksquare \quad (25)$$

Although this approach is very simple and straightforward, we only have minimum bound of a frequency component, which is a drawback of the direct method.

3 SIMULATION RESULTS

Having established representations for colored and geometric faces, a sampling criterion determines the minimum number of curves (patterns) to reconstruct surfaces. Using the algorithms proposed above, the sampling rate of the surface is shown in Table. I. 7 3D face models are used and they are composed of *vertices* and *faces*. The estimated minimum sampling rate, N_s, derived from algorithms in the previous sections, for 5 real face models and 2 synthetic ones, are shown in Table. I. As shown in Table. I, for instance, 147 circular patterns are projected onto the target 3D face and the reconstruction is carried out. Once reconstructed with 147 patterns, the minimum number of the patterns is derived based on Eq.s 19 and 25, and try to achieve reasonable reconstruction result that does not have significant loss of information

4 SUMMARY AND CONCLUSION

In this paper, sampling rate determination for 3D reconstruction using structured light patterns has been discussed. To the best of our knowledge, sampling rate determination has not been of much interest in the areas of computer vision and 3D imaging. Although there have been extensive research about accurate 3D reconstruction, the efficient reconstruction employing sampling theorem in 1D signal has been little investigated. The proposed approach is very efficient and simple from practical perspectives only by using curvature information defined in facial curves that are projected light patterns. In practice, there also have been extensive research about face recognition using facial curves and our approach also can be used for efficient 3D face recognition work. Deformation of a projected light pattern is important key to a reconstruction and a *sampling rate determination* in this paper. Although many works about reconstruction have been done, this is very challenging work to use a circular pattern and to analyze a relationship of deformation of a 2D and a 3D image. In aspect of efficiency, the optimal sampling rate was induced from a curvature estimation. Before determining sampling rate, we estimated curvatures of set of curves on the 3D object. 3D curves are exactly mapped to 2D curves and these are topologically equivalent to each other. The highest curvature or the biggest distorted light pattern on 3D surface is shown as the biggest distorted curve on 2D image. This implies that values of curvature on 3D are also mapped to the values of curvature on 2D image. Since a curvature estimation is a method of es-

Table 1: Estimated minimum number of patterns.

	N (Initial number of patterns)	$f_s[1/arc\ length]$ (Sampling rate)	N_s (*Minimum number of patterns*)	Size(*vertex*)	Size(*face*)
Brian	147	0.38	56	15926×3	23526×3
Eric	153	0.40	62	17266×3	23567×3
Greg	142	0.33	47	16257×3	28023×3
Jeff	221	0.49	109	16080×3	18522×3
Weihong	271	0.33	90	15769×3	26959×3
fvgallary	89	0.30	27	5031×3	9999×3
fvinput	93	0.44	41	16092×3	32116×3

timation of curves' shape, we used average curvature of each curve in each unit space. So, curvature information can tell us the characteristic of curves which is matched to signal function. Given a maximal curvature variation, we could find the sampling rate and determined the number of light patterns to be projected for reliable 3D surface reconstruction. In addition, we have presented an alternative algorithm to determine the sampling rate of a surface (or defining the minimum number of light patterns to be projected on a surface whose maximal curvatures may be known) subjected to an active light source probing. Such a rate, in turn plays a key role in the efficient representation of a surface and its subsequent reconstruction from these patterns. While our primary application of interest lies in the area of biometrics and face modeling, the two-thirds-based sampling criterion may be exploited in many different settings where surface representation and sampling are of interest (e.g. surface archiving). Although our sampling rate does not recover the surface perfectly as the *Shannon-Nyquist Sampling Rate* does for 1D signals, the sampling criterion we proposed does not show a considerable information loss to be recognized. In the future, there are some technical issues to be considered - quantifying the algorithm efficiency (i.e. computational complexity) and the reconstruction accuracy compared to the previous methods is needed.

REFERENCES

Chan, Y., Delmas, P., Gimel'farb, G., and Valkenburg, R. (2008). On fusion of active range data and passive stereo data for 3d scene modelling. In *Image and Vision Computing New Zealand, 23rd International Conference*. IEEE.

de'Sperati C and P, V. (1997). The relationship between curvature and velocity in two-dimensional smooth pursuit eye movements. *The Journal of Neuroscience*, 17(10):3932 – 3945.

Eldar, Y. C. and Pohl, V. (2009). Recovering signals from lowpass data. *CoRR*, abs/0907.3576.

Faugeras, O., Luong, Q.-T., and Papadopoulou, T. (2001). *The Geometry of Multiple Images: The Laws That Govern The Formation of Images of A Scene and Some of Their Applications*. MIT Press, Cambridge, MA, USA.

Geng, J. (2011). Structured-light 3d surface imaging: atutorial. *Adv. Opt. Photon.*, 3(2):128–160.

Higgins, J. (2003). A sampling theorem for irregularly spaced sample points (corresp.). *IEEE Transactions on Information Theory*, 22(5):621 – 622.

Jerri, A. J. (1977). The shannon sampling theorem - its various extensions and applications: A tutorial review. *Proceedings of the IEEE*, 65(11):1565 – 1596.

Kolev, K., Tanskanen, P., Speciale, P., and Pollefeys, M. (2014). Turning mobile phones into 3d scanners. *2014 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 00(undefined):3946–3953.

Lacquaniti, F., Terzuolo, C., and Viviani, P. (1983). The law relating the kinematic and figural aspects of drawing movements. *Acta Psychologica*, 54(13):115 – 130.

Lee, D. and Krim, H. (2011). A sampling theorem for a 2d surface. In *Scale Space and Variational Methods in Computer Vision - Third International Conference, SSVM 2011, Ein-Gedi, Israel, May 29 - June 2, 2011, Revised Selected Papers*, pages 556–567.

Lei, Y., Bengtson, K., Li, L., and Allebach, J. (2013). Design and decoding of an m-array pattern for low-cost structured light 3d reconstruction systems. In *Image Processing (ICIP), 2013 20th IEEE International Conference on*.

Mallat, S. G. (1989). A theory for multiresolution signal decomposition: The wavelet representation. *IEEE Trans. Pattern Anal. Mach. Intell.*, 11(7):674–693.

Oprea, J. (2007). *Differential Geometry and Its Applications*. Mathematical Association of America, 2nd edition.

Papoulis, A. (1977). *Signal analysis*. Electrical and electronic engineering series. McGraw-Hill, New York, San Francisco, Paris.

Song, Y., Glasbey, C. A., Polder, G., and van Heijden, G. W. (2014). Non-destructive automatic leaf area measurements by combining stereo and time-of-flight images. *IET Computer Vision*, 8(5):391–403.

Unser, M. (2000). Sampling—50 Years after Shannon. *Proceedings of the IEEE*, 88(4):569–587.