Parking Space Occupancy Verification
Improving Robustness using a Convolutional Neural Network

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Abstract: With the number of privately owned cars increasing, the issue of locating an available parking space becomes apparant. This paper deals with the problem of verifying if a parking space is vacant, using a vision based system overlooking parking areas. In particular the paper proposes a binary classifier system, based on a Convolutional Neural Network, that is capable of determining if a parking space is occupied or not. A benchmark database consisting of images captured from different parking areas, under different weather and illumination conditions, has been used to train and test the system. The system shows promising performance on the database with an overall accuracy of 99.71 %.

1 INTRODUCTION

In recent years the amount of cars on the roads has increased, this development not only leads to a higher demand on the trafic network, but also in the number of parking spaces.

This is further evidenced by (Shoup, 2006), who in 2006 researched the cruising time and distance driven when searching for a curb parking in Los Angeles. They discovered that the average cruising time and distance covered was 3.3 minutes and about 0.8 km, respectively. (Shoup, 2006) also argue that the average search time is 8.1 min. and that the average share of cars in traffic, searching for a parking space, was 30 %, these assertions are based on previous research, conducted in business districts between 1927 and 2001.

(Zheng and Geroliminis, 2016) also investigates the issue of cruising-for-parking and creates corresponding models, while suggesting it can be reduced by varying the parking price.

The increased strain on the road network, the time wasted and extra fuel used, makes it interesting to consider, if all these aspects can be reduced by providing the drivers with information about the nearest vacant parking space.

One approach, that is currently being used, is placing signs at focal points, which indicates the number of available parking spaces at specific parking areas. The issue is that the driver is not informed about the exact location of a vacant parking space. At parking areas where such specific information is available, the solution normally is to locate sensors in each parking space, e.g. infrared sensors, which can be placed both above of or in the parking space, or magnetometers buried under the asphalt. Using several, possibly battery powered, sensors results in increasing price and maintainance as the size of the parking area increases.

One solution to this, could be to use a vision based system, where cameras are placed, such that they monitor a larger parking area, one sensor can thereby be used to deliver information about several parking spaces.

2 RELATED WORK

Previous effort have been put into developing vision based systems, with the intend of determining the vacancy of parking spaces.

In (Funck et al., 2004), they used several images of an empty parking area, under different illumination conditions, to create an average image. Principal Component Analysis (PCA) was used to create an eigenspace representation. Reconstructing any new image from the eigenspace representation, yields a reference image with the current illumination, any difference between the new image and its eigenspace reconstruction is then defined as an object. The system only estimates the occupancy of the whole park-

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ing area and tests showed an average error rate of 10 %.

(True, 2007) used manually labeled Regions-Of-Interest (ROI). The system is divided into two parts, in the first part a color histogram is created for each parking space and is then classified using either k-Nearest Neighbour (kNN) or a Support Vector Machine (SVM). In the second part Harris corner detection is used on each parking space. A feature vocabulary is then created and classification is done by comparing the feature vocabulary from the test image with the one from the training set. Using colour histogram and either kNN or SVM they achieved an error rate around 10 %, while the feature detection had an error rate of 51 %.

(Bhaskar et al., 2011) combined rectangle detection and Scale Invariant Feature Transform (SIFT). They worked with the assumption that a parking space is a rectangle of pixels in an image, the images used was captured from an aerial camera. Using a threshold based classifier, they achieved an accuracy of 96.9 %, since the system depends on the lines of the parking space to function, it is affected by partial or full occlusion of these, while also being dependent on the parking spaces being rectangular.

In (Masmoudi et al., 2014) a Homography transformation was used to change the point of view of the parking area, in order to reduce the effect of perspective distortion. The parking spaces are defined by using two corners of the first parking space and defining a width. A Gaussian Mixture Model (GMM) was used for background subtraction and they then only considered objects that overlap with the parking space model. For feature extraction they achieved the best results using Speeded Up Robust Features (SURFs), combined with SVM for classification. Their method achieved an accuracy above 92 % for all their tests, but was not robust against occlusion.

(Tschentscher et al., 2015) tested using both various colour histograms and Difference-of-Gaussian (DoG) for feature extraction, combining them with either kNN, Linear Discriminant Analysis (LDA) or a SVM for classification. They achieved the most robust results using DoG and SVM, with an average accuracy of 96.42 % on a never seen parking area.

(Huang and Vu, 2015) proposed using a cube model for each parking space, each of the six surfaces of the cube is then normalized and the classifier is trained on all of the patches separately. For feature extraction. They used Histogram of Oriented Gradients (HOG), LDA for feature reduction and Naive Bayes Classifier (NBC) for classification. The performance of the system was tested under various weather conditions and achieved more than 99 % accuracy in all of them.

(Klosowski et al., 2015) proposed using 2D separable Discrete Wavelet Transform (DWT) and then applying morphological operations. Since they don't manually mark the parking spaces or automatically detect them, they count the pixels and thereby calculate the occupied percentage.

In (De Almeida et al., 2015) a database consisting of 12,417 images was introduced, including images from two different parking areas, one of them from two different angles. Besides introducing the database, two systems were also proposed to solve the issue of vacancy verification. Both systems used textural descriptors, one Local Binary Pattern (LPB) and the other Local Phase Quantization (LPQ), both systems used SVM for classificaton. When training and testing the system on the same database, they achived an average error rate around 0.5 %. When testing on parking areas that was not used for training, the lowest achieved error rate was 11 %.

(Baroffio et al., 2015) proposed a system using wireless cameras, connected in a network. The system assumes that the region of the parking spaces is known, these regions are then extracted, converted to HSV colorspace and the hue is then used to create a histogram, which is used as local features. For classification they used a linear SVM, based on normalized histograms. To test the accuracy, the authors used the PKLot database presented in (De Almeida et al., 2015), they achieved respective accuracies of 96 % on UFPR04, 93 % on UFPR05 and 87 % on PUCPR.

In (Masmoudi et al., 2016) a modified 3D model of the parking spaces was used, where the part in focus is the surface tangent to the street, in order to solve the issue of occlusion. They then track the objects in the scene, using GMM for background subtraction and the cars are chosen based on their dimensions, tracking is performed using a Kalman filter. Using SURFs and SVM, they detect the current state of each parking space. Combining tracking of the cars and local features from SURFs, they use a decision tree to make the final decision of the occupancy of the parking spaces. They achieved an accuracy of 94.23 % in the used data.

(Sukhinskiy et al., 2016) applied perspective transformation on the images and manually marked the parking spaces. By continously getting a new frame of the parking area, they were able to compare the new frame to the old frame and thereby determine if the state of the parking space had changed. A pretrained neural network was used for the final classification.

When solving computer vision based problems, the traditional way is to use handcrafted features, extracted from e.g. SIFT, HOG etc., combined with a classifier, SVM for example. In later years Convolutional Neural Networks (CNN) have gotten more attention, as they have shown great potential in pattern recognition tasks. An example of the impact CNN's have had can be seen in the ImageNet contest, where the winning system in 2011 had an error rate around 25 % and the year after it was reduced to 16 %, when AlexNet won (Russakovsky et al., 2015) (Krizhevsky et al., 2012). Since then, CNNs have become an integral part in our every day life, used in our Digital Personal Assistants, auto tagging our photos and translating languages.

In (Valipour et al., 2016) a pre-trained CNN, VGGNet-f was used and fine-tuned to work on the PKLot database presented in (De Almeida et al., 2015). They used Stochastic Gradient Decent (SGD) with learning rate and weight decay and a mini-batch size of 128. They report their results using Area Under Curve (AUC), arguing that their method is 3 to 5 % better than the results in (De Almeida et al., 2015).

(Huang and You, 2016) propose using 3D point clouds, acquired by a Lidar, segment unwanted information, buildings, ground and curb and use three Orthogonal-Views as input to a CNN which test. Using the method they achieved an accuracy of 83.8 %.

(Ahrnbom et al., 2016) focuses on creating a fast classifier for detecting vacant parking spaces, which was tested on the PKLot database. For feature extraction they use 10 feature channels (LUV color space, gradient magnitude and quantized gradient channels), which are used with two classifiers, SVM and Logistic Regression with Elastic Net Regularization (LR). The best results were achieved using LR, the results are presented as AUC, with a mean value of 0.98, slightly better than the results from (De Almeida et al., 2015).

This paper will move away from the feature based approaches used in most of the work described above, instead a binary classification system using a CNN will be presented. Focus will also be put into designing a system, that is capable of delivering robust results, even on parking spaces that the system has not been trained to recognize.

In section 3 a short description of CNN will be presented and focus will then be on the proposed system and its elements. Section 4 will describe the resources, database, framework and hardware that was used to conduct the work presented. Section 5 will describe the tests conducted and reports corresponding results, while section 6 will discuss these and further work that could be looked into.

3 THE PROPOSED SYSTEM

CNNs are effective at processing data in the form of arrays, e.g. images, which makes it ideal for computer vision tasks (Lecun et al., 2015). CNNs are based on Multilayer Perceptrons (MLP), since these consist of fully connected layers, they do not scale well with image sizes. In contrast a CNN tries to take advantage of the spatially local correlation in images, by stacking the feature maps and only connecting each neuron to a small region of the input volume, this is also called the receptive field of the convolutional layer. For each feature map, the weight and bias will be shared, this is possible by assuming that a feature which is useful to compute at one position, is also useful to compute at another spatial position

In general it can be said that the convolutional part of the method, creates a feature map, based on a feature extractor and the Neural Network part is the classifier and is used for updating the systems internal parameters, based on past experience.

A CNN normally consist of several convolutional layers, an activation function, pooling layers and lastly the classification layer, which is normally a fully connected Neural Network.
Figure 1 shows an overview of the proposed CNN.

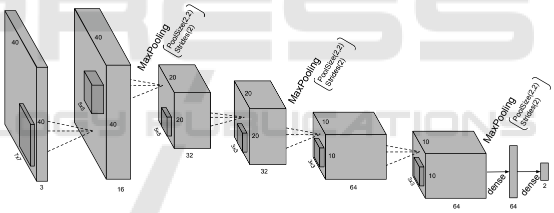


Figure 1: Illustration of the proposed CNN.

The proposed CNN follows a standard simple architecture, consisting of an initial convolutional layer, followed by a max pooling layer and then repeatedly two convolutional layers, followed by a max pooling layer. The depth of the feature maps increases after every max pooling layer, but reduces the spatial size. The network has a total 198,576 parameters and all the weights in the network are initialized randomly while Glorot Uniform is used as initialization in all layers.

Convolutional Layer. As explained above, each convolutional layer consist of stacked feature maps, these are created by convolving a kernel over the input, together with the neurons parameters (weight and bias). The depth of the convolutional layer is the amount of feature maps that are stacked. As can be seen on Fig. 1, the proposed system starts with a convolutional layer with an ouput having a depth of 16 and then increases the depth in the later layers, as the spatial size of the feature map decreases. As the size of the feature map decrease, the kernel size is also reduced, the stride for the kernels is always 1 and 1 pixel zero-padding is used at each convolutional layer.

In the early layers a CNN normally detects simple features, such as edges, then corners. In the later layers, the network starts to learn more complex features, which might seem abstract to the human eye.

Activation Function. CNNs are constructed of neurons, these have learnable weights and biases and can be expressed as the linear function:

$$y = w \cdot x + b \tag{1}$$

Where w is the weight, x the input and b is the bias. The activation function is an optional part of the nodes, it introduces a non-linearity to the output of the node, which is important in order to not create a linear decision boundary. The proposed system uses Recified Linear Unit (ReLU) as the activation function, which can be expressed as:

$$f(x) = max(0, x) \tag{2}$$

ReLU, is used since it is computational efficient, resulting in less training time. It doesn't have an issue with vanishing gradients and has shown to greatly accelerate convergence (Glorot et al., 2011).

Pooling Layer. A pooling layer is added between every second convolutional layer. The function of it, is to reduce the spatial size and thereby reduce the amount of parameters. This also helps to control overfitting. The reasoning behind it, is that the exact position of a found feature is not as important, as the its position relative to other features are. The proposed system uses max pooling with a 2x2 filter and a stride of 2.

Optimization. The last part of the network, is the fully connected Neural Network followed by a loss layer. The fully connected layer performs classification while the loss layer tries to find the error. The idea is that the network learns by its mistake and then updates the parameters, weights and bias, throughout the system.

The proposed system uses softmax at the output layer and Cross-entropy to calculate the error, which is then used by backpropagation in order to calculate the gradient for each weight. Lastly gradient descent is used to compute the changes that needs to be applied to the weights throughout the network, before starting over. Choosing a correct learning rate can be essential, a higher learning rate results in faster learning, but it might not end up at the ultimate minimal loss. choosing a too low value can result in very slow convergence, while a too high value can result in oscillation (Wilson and Martinez, 2003).

In this case AdaGrad is used to calculate the gradient descent. AdaGrad is a modified version of Stochastic Gradient Descent, which updates the parameters individually by using different learning rates for every parameter. When using Adagrad the learning rate needs to be initialized at start, for this system it has been set at a value of 0.0001. The learning rate is then updated throughout training at every time step t and based on the parameters past computed gradient.

The weakness with using AdaGrad is, that since it automatically updates the learning rate, it continously becomes smaller and the system might therefore learn slower or stop learning altogether. Compared to e.g. AdaDelta it is more robust to the initial learning rate, while converging is close to the same.

The minibatch size was set to 128, epoch size was set to all the sample in the training set. In order to validate the system while training, the training set was split, such that 1/6 of the images was used as a validation set, the system ran for a total of 500 epochs.

Data augmentation was also introduced, in order to get more value out of the data and at the same time introduce a bit of distortion into the data. Data augmentation can have positive effects on both accuracy and reduce overfitting, as explained in (Glorot et al., 2011) and (Simard et al., 2003). For this system both horizontal and vertical flipping was introduced, together with both vertical and horizontal shifting. Some slight rotation of the images was also used.

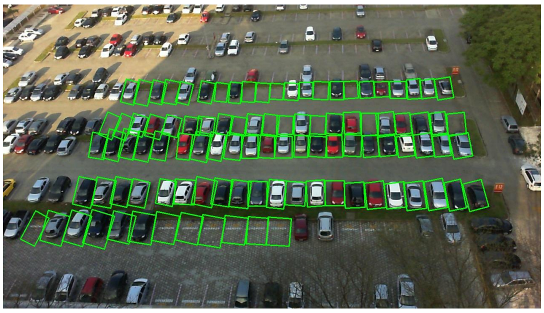
4 RESOURCES

This section will look at the resources used in the project, this involves the database and the framework used.

4.1 Database

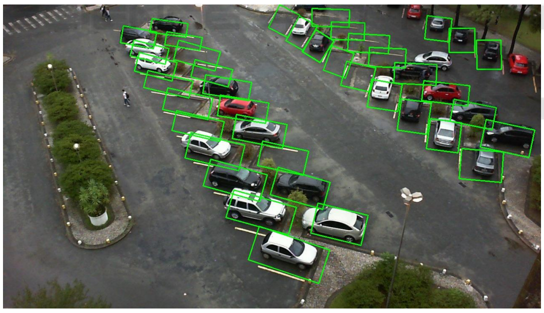
The PKLot database that was introduced by (De Almeida et al., 2015), will be used for this work. The reason for this, is that it provides a basis for comparison. This database consists of 12.417 images of the three parking areas, captured at a resolution of 1280x720 px. In total there is 695.900 images of parking spaces captured throughout the day and in three weather conditions, sunny, rainy and cloudy. The ground truth of the parking spaces is available as an XML file for each image of the parking area. It contains information about the state of the parking spaces and their pixel location and size.

An example of the three parking areas, together with bounding boxes for the segmented parking spaces, can be seen in Fig. 2.



(a)

(b)



(c)

Figure 2: The three parking areas and their segmented spaces shown: a) PUCPR b) UFPR04 and c) UFPR05.

For each image, chosen parking spaces have been labeled. Each parking space have also been segmented and rotated, such that they match each other. As the focus of the work presented here, is to verify the vacancy of the parking spaces and in order to be able to compare the systems, these images will be used.

Examples of segmented images, showing both occupied and vacant parking spaces can be seen in Fig. 3.

Figure 3: Example of the segmented images of both occupied and vacant parking spaces.

Before being fed to the CNN, the segmented images are all scaled to 40x40 px and normalized, this is done by simply dividing each RGB pixel value by 255.

Together with the PKLot database follows guidelines for how the training- and testing set should be created, they suggest dividing them 50/50 for each parking area. They also recommend having images captured the same day in the same set, such that the same car is not used for both training and testing. These guidelines have been followed, such that the achieved results are comparable to the ones presented in (De Almeida et al., 2015).

4.2 Framework

As explained earlier, CNNs have become an increasingly popular topic, which have resulted in a plethora of readily available frameworks. The most popular include TensorFlow by google, CNTK by Microsoft, Theano and Keras.

To realize the system described above, it has been chosen to use Theano combined with Keras as they support python bindings, allowing for rapid prototyping (Theano Development Team, 2016). Theano is a library made for numerical computations and seamlessly uses the GPU, while Keras is a Neural Network library, capable of running on top of both Theano and TensorFlow which adds support for CNNs in Theano.

4.3 Hardware

The computer used for the tests, described in the next section, had the following specifications:

- Ubuntu 16.04 LTS
- Intel Core i7-860 @ 3.2 GHz
- NVIDIA GTX 780
- 8 GB RAM

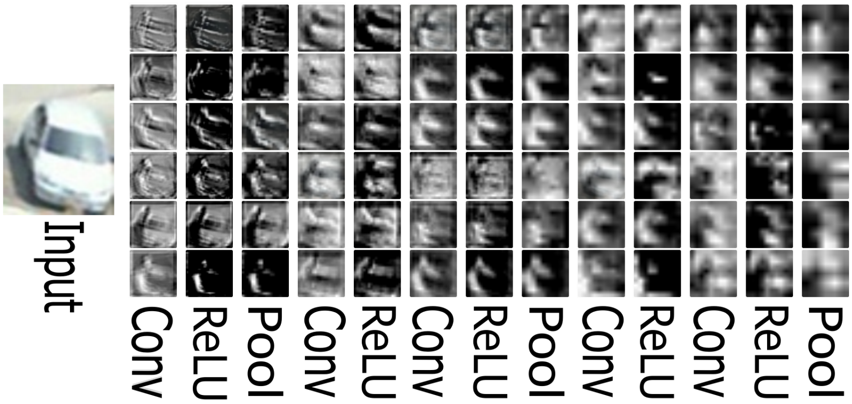


Figure 4: The CNNs activations throughout the system, when given the input image on the left. For each layer the first six feature maps are illustrated.

5 EXPERIMENTAL RESULTS

As described above the system have been trained and tested on the PKLot database. This section will look at the results achieved during testing.

Figure 4 illustrates the activations of the six first feature maps in each layer throughout the system, when given the segmented image of an occupied parking space shown on the left as input. As can be seen the first layers are still recognisable as they work as an edge detector, while the later layers are hard to interpret.

Robustness of the System. In order to test the robustness of the system, it have been trained on the individual parking areas and then tested on both the same parking area and on parking areas that have not been seen.

The accuracy can be seen in Fig. 5, as can be seen, when training and testing on the same parking area, the accuracy is in all cases above 99.70 %. The lowest accuracy achieved is 95.45 % when training on the UFPR05 and testing on UFPR04.

The robustness of the system will be seen as how well the system performs, when being tested on parking areas that it was not trained on. This means that a margin will be defined as being the difference between the accuracy achieved when testing on the same parking area, and the accuracy when tested on the two unseen parking areas. The robustness will then be the average of these margins, with lower percentages showing a more robust system.

Applying this to the results achieved in (De Almeida et al., 2015), using the highest accuracy regardless of the method used, reveals an average margin of 11.95 %.

Testing \ Training	UFPR04	UFPR05	PUCPR
UFPR04	99.70 %	95.46 %	96.91%
UFPR05	95.96 %	99.76 %	96.72 %
PUCPR	98.70 %	97.30 %	99.90 %

Figure 5: The results of the test, when training and testing on separate parking areas.

Table 1: Comparison of error rate, when training and testing on the same parking area.

	P. De Almeida	Proposed CNN
PUCPR	0.39 %	0.10 %
UFPR04	0.36 %	0.30 %
UFPR05	0.70 %	0.24 %

Applying the same method on the results achieved by the proposed system, reveals an average margin of 2.96 %. The system has therefore shown to be significantly more robust than the competing system.

Table 1 shows a comparison of the error rate between (De Almeida et al., 2015) and the proposed system, when they have been trained and tested on the same parking area. As can be seen, the proposed system greatly improves the results on especially the PUCPR and UFPR05 parking area.

Overall Accuracy of the System. In order to get a feeling of the systems overall performance, it has

Figure 6: An example of a classification performed on an image from the PUCPR parking area. Green being a occupied parking space and red a vacant parking space.

been trained on all the available training data. It has then been tested on the training data from the three parking areas individually and all the available testing data.

The result from this test can be seen in table 2.

Table 2: Training on all the parking areas and testing on the different parking spaces and all of them.

UFPR04	UFPR05	PUCPR	All
99.74 %	99.20 %	99.88 %	99.71 %

All of the conducted tests, have an accuracy above 99 %, with the overall accuracy being 99.71 %.

An example of the classification can be seen on Fig. 6, the example shows the PUCPR parking area with corresponding bounding boxes, red being vacant and green occupied parking spaces. Only parking spaces with metadata created by (De Almeida et al., 2015) have bounding boxes.

6 DISCUSSION

One of the more difficult tasks in parking space verification, is to design a system that is able to perform reliably, when shown new parking spaces compared to what it was trained on. The goal have been to design a system that is robust and can deliver good performance when being tested under there circumstances.

This paper has introduced a system, based on a Convolutional Neural Network, that is able to verify the vacancy of a parking space.

The proposed CNN has shown promising performance in these cases, with a robustness margin of 2.96 %, which is about 4 times better than previous efforts. It have other than that, shown high accuracy when introduced to new parking spaces, with the lowest accuracy achieved being 95.45 % and the highest 98.70 %.

Besides this the system have shown to perform well under different illuminations, as the results when training the system on all of the training data have shown. During these tests the accuracy was all above 99 %, with an overall accuracy at 99.71 %, when testing on all of the testing data.

6.1 Future Work

The PKLot database used in this work, does not provide images at dusk or night time and it could be interesting to see how the system would handle these more extreme situations. One prerequisite for this would be, that the parking area were lit by e.g. street light though.

Other improvements to the system could involve automatic detection of the parking spaces, as this would ease the process of installing the system at a new location. One method to do this, could be by assuming all parking spaces are bound by two easy identifiable lines and are parallel to each other.

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