Modeling $\dot{V}O_2$ and $\dot{V}CO_2$ with Hammerstein-Wiener Models

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Abstract: $\dot{V}O_2$ and $\dot{V}CO_2$ measurements are central to methods for assessment of physical fitness and endurance capabilities in athletes. As measuring $\dot{V}O_2$ and $\dot{V}CO_2$ is difficult outside a lab, models with good prediction properties are necessary for online analysis and modeling in the field. Easier to measure are heart rate and during cycling also power. Thus, the here described models are based on either one of them or both. It is commonly accepted that the relationship between power and $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate can be described by a linear and a nonlinear component. The latter describes a drift over time without increase in workload. Thus, block-structured systems such as Hammerstein-Wiener models with linear and nonlinear elements can be employed for modeling and prediction. Modeling and prediction power of these models is compared with a dynamic model based on physiological evidence. Our findings show that the simpler Hammerstein-Wiener model performs slightly better for both modeling and prediction with the advantage of being easier to estimate and evaluate. Overall, both models performed with errors smaller than the range of the natural variability of the modeled quantities. Thus, such models allow for applications in the field where $\dot{V}O_2$ and $\dot{V}CO_2$ cannot be measured.

1 INTRODUCTION

Physiological quantities such as heart rate or respiratory gas exchange are important parameters to assess the performance capabilities of athletes in competitive sports. In particular the respiratory gas exchange is a valuable source of information since it allows for a non-invasive, continuous, and precise measurement of the gross oxygen uptake and carbon dioxide output of the whole body. Particularly in endurance sports, the metabolic rates of this substantial fuel and the degradation product of the exercising muscles are reflected in that rate.

For endurance sports like cycling the models for power demand due to mechanical resistance are well understood by Martin et al. (1998). However, the individual power supply model of an athlete is the bottleneck that has hindered the design of an individual adequate feedback control system that guides him/her to perform a specific task such as to find the minimum-time pacing in a race on a hilly track (Dahmen, 2012). For such purposes, a model for the prediction of gas exchange rates in response to load profiles given by a particular race course would be beneficial.

There are two kinds of approaches to get such a model. The first model type is directly based on concepts of physiology, such as exponential saturation functions with appropriate time-constants. Another approach are black box models without relation to physiology such as Hammerstein-Wiener models.

2 PREVIOUS WORK

A detailed review and historical account of the mathematical modeling of the $\dot{V}O_2$ kinetics for constant work rate has recently been given by Poole and Jones (2012), containing over 800 references. See also Jones and Poole (2005) and, for a clarification, Ma et al. (2010).

Artiga Gonzalez et al. (2015) generalized successfully the established constant work rate models towards a dynamic model for variable work rate. The result is a model that consists of two differential equations based on a steady-state function for oxygen demand. For completeness, we briefly review this dynamic model in the following and later compare the performance of Hammerstein-Wiener models with that of the dynamic model.

The steady-state oxygen demand is given by a

constant baseline component, the first, fast component, and the second, slow component with amplitudes $\dot{V}O_{2\text{base}}$, $A_1(P)$, and $A_2(P)$, respectively. In terms of formulas, the amplitudes are

$$A_1(P) = \min(s \cdot P, \dot{V}O_{2\text{max}} - \dot{V}O_{2\text{base}})$$

$$A_2(P) = \begin{cases} V_\Delta \cdot \exp(-(P_c - P)/\Delta) & P \leq P_c \\ \dot{V}O_{2\text{max}} - \dot{V}O_{2\text{base}} - A_1(P) & P > P_c \end{cases}$$

where s is the slope (or gain) for the fast component, P_c denotes the critical power, V_Δ is the maximal amplitude of the slow component for exercise load up to critical power, and Δ is the corresponding decay constant that governs the decay of the steady-state slow component as the load is decreased from the critical power.

The following equations describe the first and second component, $x_1(t)$, $x_2(t)$,

$$\dot{x}_k = \tau_k^{-1}(A_k(P) - x_k), \quad x_k(T_k) = 0, \quad k = 1, 2$$

defined for times $t \geq T_k$ (and setting $x_k(t) = 0$ for $t < T_k$). Here, the power demand is a function of time $P = P(t)$ and $A_k(P)$, $k = 1, 2$, are the steady state amplitudes for the fast and slow components. The total $\dot{V}O_2$ accordingly is given by

$$\dot{V}O_2(t) = \dot{V}O_{2\text{base}} + x_1(t) + x_2(t).$$

The differential equations require the four parameters $\tau_1, \tau_2, T_1,$ and T_2.

In our previous work (Artiga Gonzalez et al., 2015) this dynamic model was applied to oxygen consumption $\dot{V}O_2$. Figure 1 shows that there is a strong relationship between $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate and thus, the model can be expected to work well also for $\dot{V}CO_2$ and heart rate. Corresponding results are given below in Table 1.

Data Collection

The same dataset as in Artiga Gonzalez et al. (2015) is taken to allow a comparison of methods for modeling and prediction. For the sake of completeness we give a short description of data collection and data preprocessing.

Five healthy, recreational to well trained subjects (age 37.8 ± 14.8 yrs, height 180.4 ± 10.1 cm, weight 75.2 ± 7.6 kg) completed four different cycle ergometer (Cylus2, RBM elektronik-automation GmbH, Leipzig, Germany) tests with continuous breath-by-breath gas exchange and ventilation measurements at the mouth (Ergostik, Geratherm Respiratory GmbH, Bad Kissingen, Germany). The tests featured a variety of load profiles in order to comprehensively evaluate the model prediction quality.

The testing procedure commenced with an incremental step test starting at a workload of 80 W with increments of 20 W every 3 minutes. In the initial step the subjects were instructed to choose their preferred cadence between 80–100 rpm and were then instructed to keep the cadence constant at that level in all four test trials. The step test was terminated at volitional exhaustion of the subject. After test termination subjects recovered actively at 80 W and at or
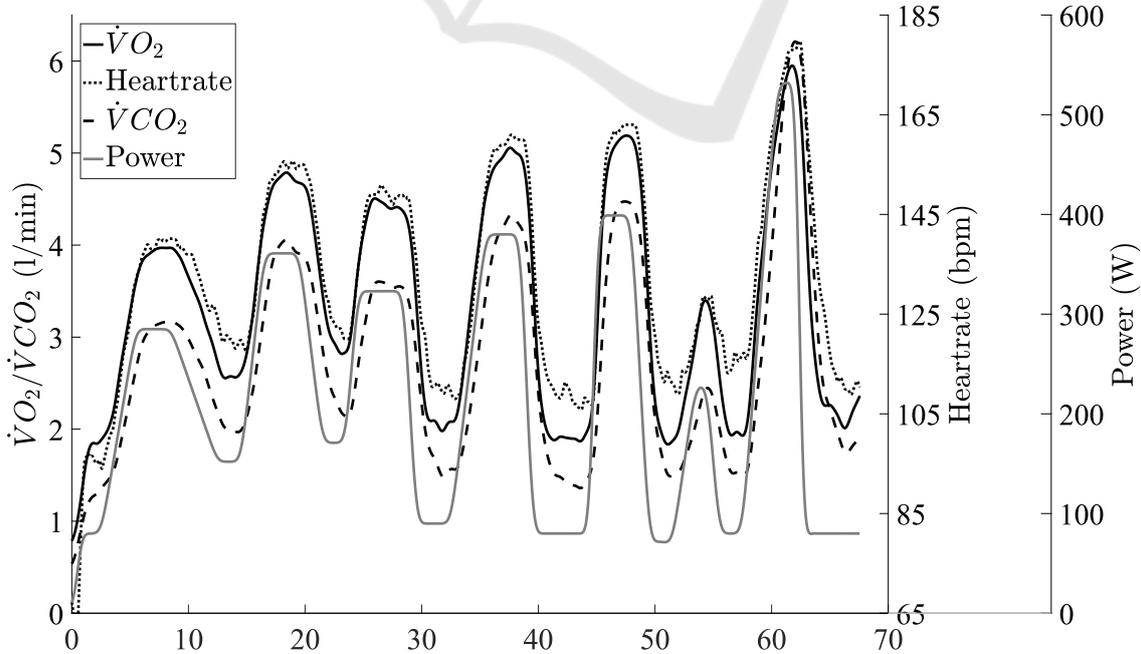

Figure 1: Measured $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate for the power profile of the third test and one subject.

near their self-selected cadence for five minutes.

The second ergometer test consisted of four sprints of 6 s duration each and an incremental ramp test. Two sprints were carried out before and two after the ramp test to obtain the subjects' maximal power output and $\dot{V}O_2$ profiles in a recovered and a fatigued state.

In the third test subjects had to complete a variable step protocol. The steps varied in load and duration and alternated between low and moderate or severe intensity. The linearly in- or decreasing intensity between the steps was also varied in time. The load profile is illustrated in Figure 1.

For the final "synthetic hill climb test" the ergometer was controlled by our simulator software in Dahmen et al. (2011). The load was defined by the mathematical model by Martin et al. (1998) to simulate the resistance on a realistic track. The gradient of that track and the subjects' body weight were the major determinants of the load. While holding the same cadence as before, the subjects were able to choose their exercise intensity by gear shifting. (On the steepest section most subjects were not able to hold the cadence even in the lowest gear.)

Data Preprocessing

In order to validate and compare the models, data series of time-stamped values of produced power and resulting breath-by-breath oxygen consumption are required for exercise intensities ranging from moderate to severe. These time series from ergometer laboratory experiments are typically very noisy, have different sampling rates and the samples may be irregularly spaced.

Therefore, a combined smoothing and resampling operator has to be applied. In this study we have used the standard Gaussian smoothing filter with kernel $(\sigma\sqrt{2\pi})^{-1}\exp(-0.5t/\sigma^2)$ and $\sigma = 20$ s for respiratory gas, heart rate and power measurements.

Dynamic Model

We have extended the dynamic model from Artiga Gonzalez et al. (2015) described above with two more parameters. With these two additional parameters, a much smaller average root-mean-square modeling error was obtained and also the predictive

power of the model was improved (details to be published elsewhere). For a better comparison between the dynamic model and Hammerstein-Wiener models, we also applied this modified dynamic model for $\dot{V}CO_2$ and heart rate modeling and prediction based on power.

3 HAMMERSTEIN-WIENER MODELS

The dynamical model for $\dot{V}O_2$ under variable work rate (Artiga Gonzalez et al., 2015) described in Section 2 is based on physiological evidence collected in many years of research. Thus, in addition to the application for modeling and prediction, the estimated model parameters can be used as indicators for performance capabilities of athletes or enhance the comprehension of physiological processes. For instance, a deeper analysis of the second differential equation might lead to a better understanding of the so called slow component.

Black box models like Hammerstein-Wiener models do not offer the same understanding as physiological models have, but they bring other advantages. Detached from physiological evidence they are more flexible and can adjust better to data and therefore, may deliver better fitting results.

Though not derived by principles of physiology, it is still important to select the right model type and model settings to obtain good results. For this research MATLAB® was used. The System Identification Toolbox™ offers a large selection of models. Different linear (ARX, ARMAX, State-Space) and nonlinear (ARX, Hammerstein-Wiener) models from that toolbox have been tested on selected data sets with the System Identification App. Best results have been achieved with State-Space and Hammerstein-Wiener models. In a direct comparison Hammerstein-Wiener models have shown the best modeling results.

This outcome coincides with the knowledge that there is a strong linear relationship between power and $\dot{V}O_2$ (fast component) and a smaller nonlinear relation (slow component), because Hammerstein-Wiener models consist of nonlinear and linear elements. This holds also for the relationship between power and $\dot{V}CO_2$ or heart rate.
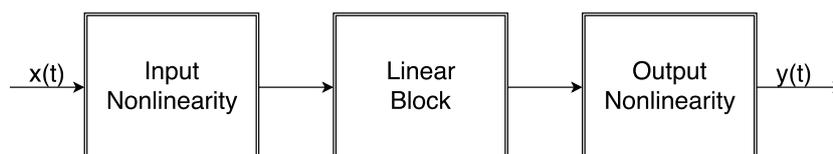

Figure 2: Block diagram of Hammerstein-Wiener model.

In general Hammerstein-Wiener models consist of the three elements shown in Figure 2. The first element is a static nonlinear function transforming the input. MATLAB® offers seven options for the nonlinearity (Piecewise linear function, Sigmoid network, Wavelet network, Saturation, Dead zone, One-dimensional polynomial, Unit gain, Custom network). The input nonlinearity is followed by the second element, a linear block that applies a discrete time linear model (Transfer function model, Input-output polynomial model, State-space model) to the output of the first element. The last element is again a static nonlinear function modifying the result of the second element. The same nonlinear functions as for the first block can be selected.

Different configurations of Hammerstein-Wiener models have been tested and have resulted that for modeling of $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate, we can omit the first element. Thus, our chosen model consists only of a linear block followed by a nonlinear function. This model type is called a Wiener model. Wiener models with a linear transfer model for the linear block and a piecewise linear function for the output nonlinearity performed best.

The Hammerstein-Wiener models were estimated with the MATLAB® function *nlhw*. This function requires, in addition to the input and target output data for training, the orders of the linear transfer function. Orders are the number of zeros, the number of poles and the input delay. To cover a large range of possible combinations for the orders, the Genetic Algorithm

ga from the Global Optimization Toolbox™ was used to search the best combination where all three values have varied in the set $\{2, 3, .., 20\}$.

These models has also been estimated with heart rate as input and with heart rate and power as combined input for modeling and predicting $\dot{V}O_2$ and $\dot{V}CO_2$.

4 RESULTS

The models were estimated for all four tests and five subjects. For prediction, the models estimated for each subject for Test 3 were applied on the other three tests of the subject. The resulting average root-mean-square error (RMSE) and the average mean absolute percentage error (MAPE) for modeling and prediction are given in Table 1 for the dynamic model and in Table 2 for the Wiener models. The results for $\dot{V}O_2$ modeling and prediction are better than those reported in (Artiga Gonzalez et al. (2015), first data row in Table 1) because the improved extended version with two additional parameters was used.

The dynamic model has an average $\dot{V}O_2$ modeling error of 0.09 l/min RMSE respectively 3.1 % MAPE. The Wiener models that are also based on power, perform better with only 0.06 l/min RMSE and 1.8 % MAPE. Figure 3 illustrates $\dot{V}O_2$ and power data and the modeling result for Test 3 of Subject 1.

With heart rate as additional input, performance is even better with 0.04 l/min RMSE and 1.3 % MAPE.
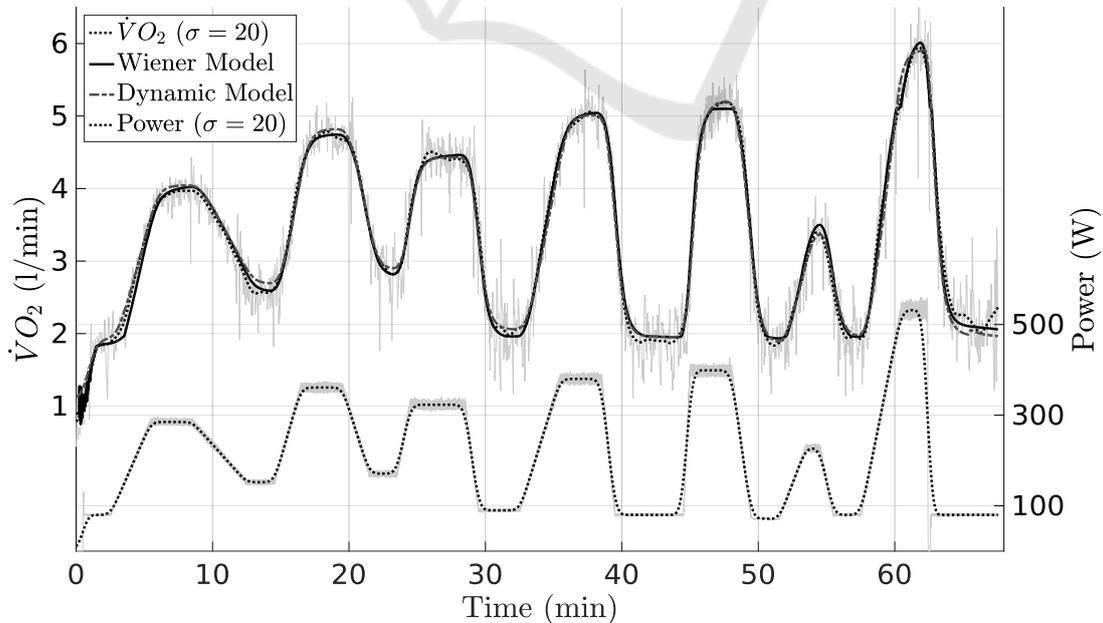

Figure 3: $\dot{V}O_2$ modeling results for Test 3 and Subject 1. Both models are based on power as independent variable. The noisy grey signals are the original (unfiltered) measurements.
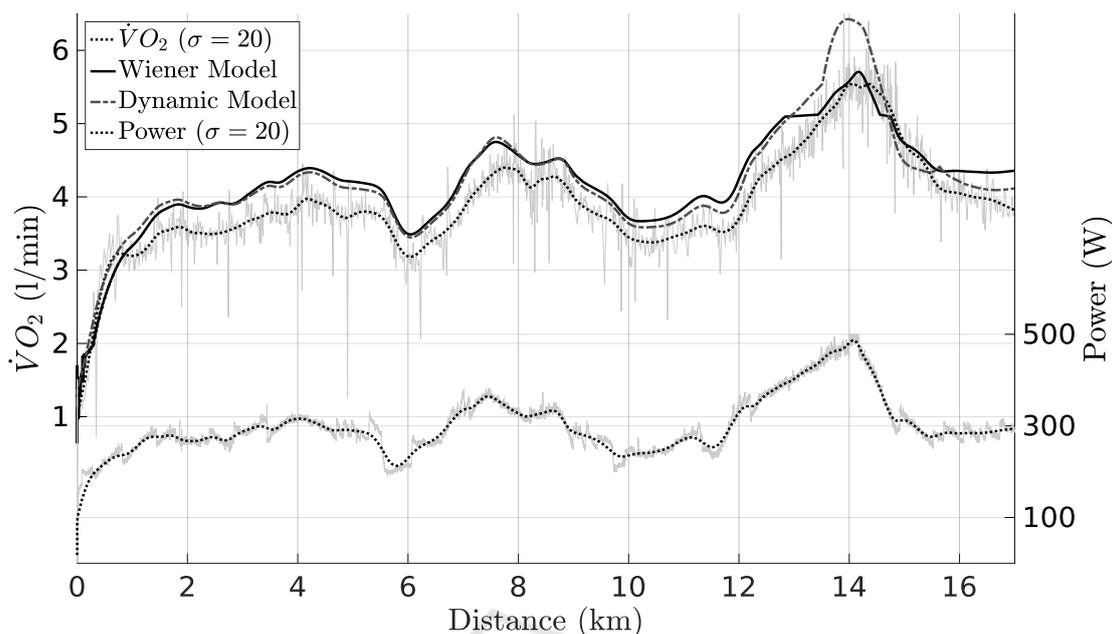
Figure 4: $\dot{V}O_2$ prediction results for Test 4 and Subject 1. Both models are based on power and trained on Test 3.

Table 1: Average modeling and predicting errors for $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate based on power with the dynamic model.

	Modeling		Prediction	
	RMSE	MAPE	RMSE	MAPE
	l/min	%	l/min	%
$\dot{V}O_2$ (2015)	0.23		0.37	
$\dot{V}O_2$	0.09	3.1	0.30	8.8
$\dot{V}CO_2$	0.12	4.7	0.41	12.6
Heart Rate	4.55	2.5	7.46	4.4

The Wiener models with only heart rate as input have similar errors compared to the Wiener models with power or power and heart rate as input.

For the dynamic model, a prediction error of 0.30 l/min RMSE respectively 8.8 % MAPE was observed. With 0.27 l/min RMSE and 7.5 % MAPE the Wiener model performed slightly better. Figure 4 visualizes the prediction results for Subject 1 and Test 4 based on power and models trained on Test 3. Predictive power did not benefit from heart rate as additional input (0.28 l/min RMSE, 8.4 % MAPE).

As expected, estimation of $\dot{V}CO_2$ with Wiener models works as well as estimation of $\dot{V}O_2$, but prediction is worse. Especially with combined input of power and heart rate the Wiener models perform poorly with 0.64 l/min RMSE and 17.6 % MAPE. With the dynamic model similar results were obtained for $\dot{V}CO_2$ based on power. The average modeling root-mean-square error is 0.12 l/min or 4.7 % MAPE while the average prediction error is 0.41 l/min RMSE or 12.6 % MAPE.

Power based heart rate modeling and prediction outperforms the Wiener models for $\dot{V}O_2$ and $\dot{V}CO_2$ with a MAPE of 0.9 % respectively 4.8 %. The dynamic model also produces reliable results with a modeling MAPE of 2.5 % and a MAPE of 4.4 % for prediction.

5 DISCUSSION

Overall, the results show that Hammerstein-Wiener models, respectively Wiener models with a linear

Table 2: Average modeling and predicting errors for $\dot{V}O_2$ and $\dot{V}CO_2$ based on power, heart rate and both with Wiener models.

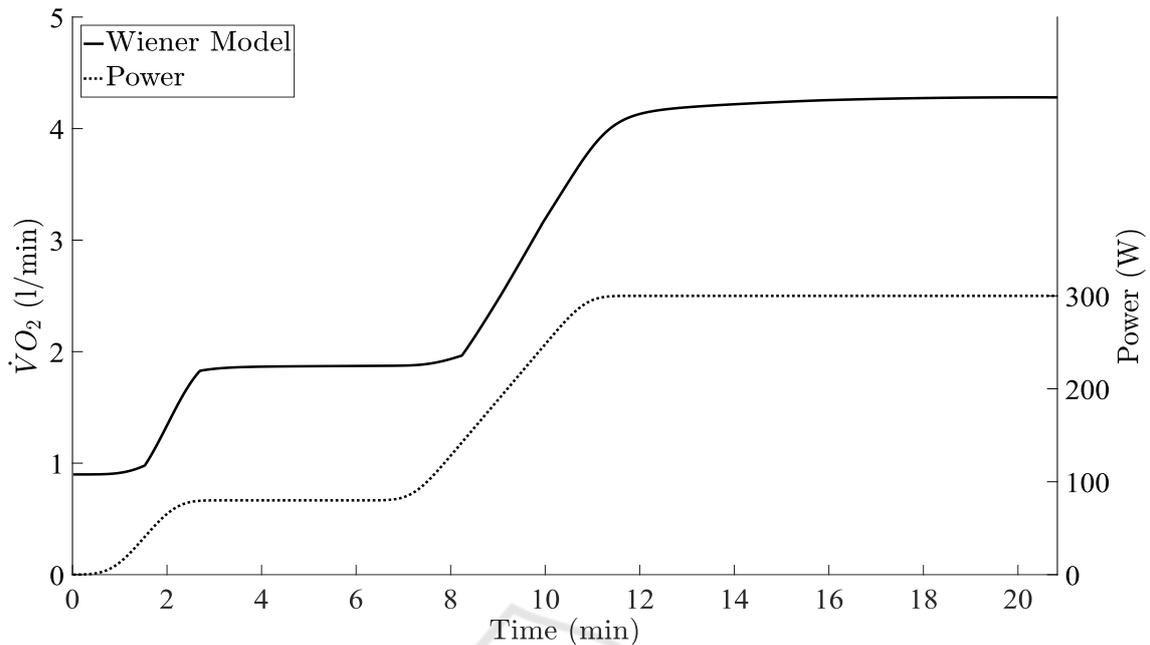|		Power		Heart rate		Power + Heart rate	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
		l/min	%	l/min	%	l/min	%
Modeling	$\dot{V}O_2$	0.06	1.8	0.09	2.8	0.04	1.3
	$\dot{V}CO_2$	0.08	2.9	0.07	2.5	0.04	1.6
Prediction	$\dot{V}O_2$	0.27	7.5	0.34	10.0	0.28	8.4
	$\dot{V}CO_2$	0.44	14.2	0.43	13.3	0.64	17.6

Figure 5: Response of an estimated Wiener model applied on a synthetic power profile with two ramps and two plateaus.

transfer function followed by a static piecewise non-linear function, perform slightly better than the dynamic model. It should be noted that the steady-state function for the dynamic model also consists of a linear component followed by a nonlinear increase. Figure 5 illustrates the linear relationship and the nonlinear influence of an estimated Wiener model applied on a synthetic power profile with two ramps and two plateaus.

In a small study (unpublished work) a grand average root-mean-square difference of 0.09 l/min between $\dot{V}O_2$ measurements of two identical tests of the same subject was obtained. The corresponding mean absolute percentage difference is 2.95 %. The best one can expect from an optimal modeling is that the accuracy is in the range of this natural variability of the modeled quantities. For our results with the Wiener models for $\dot{V}O_2$ consumption we have obtained 0.06 l/min, 0.09 l/min and 0.04 l/min (see first row in Table 2) and with the dynamic model we have obtained an error of 0.09 l/min (see Table 1).

Thus, both models are suitable and the choice of the right model depends on other factors. For example estimation and evaluation of Hammerstein-Wiener models is much faster than parameter estimation for the physiological model. But the latter offers more insights into the physiological processes and estimates parameters like critical power or $\dot{V}O_{2max}$ which can be used for further analysis.

The overall predictive power for both models is not as promising as the modeling results. This is most likely based on the big differences between the four tests and the small data set per subject. Training on only one test may lead to overfitting and therefore weak predictive power. Figure 6 illustrates that both models overestimate $\dot{V}CO_2$ in the severe domain. The models may misbehave there, because Test 3 on which they have been trained does not contain that large parts in the severe intensity domain.

Moreover, there is evidence for an asymmetry between on- and off-transient dynamics (Özyener et al., 2001) but neither model can distinguish between on- and off-transient parts. This leads again towards over-estimation, at least for the dynamic model (unpublished work).

In addition to $\dot{V}O_2$ and $\dot{V}CO_2$, heart rate was successfully modeled with power based Hammerstein-Wiener models and the dynamic model. In general, heart rate prediction is not useful because heart rate can easily be measured directly. But there are some use cases, for example in medical applications where it is important to control and predict heart rate (Cheng et al., 2008). However, forecasting models that know past values collected by a heart rate measurement device are expected to perform much better for controlling issues.

Our results indicate that heart rate based Wiener models also perform well. This could be an interesting alternative to power based prediction in the field as heart rate belts are much cheaper compared to power meters. However there may arise complications because heart rate varies depending on train-
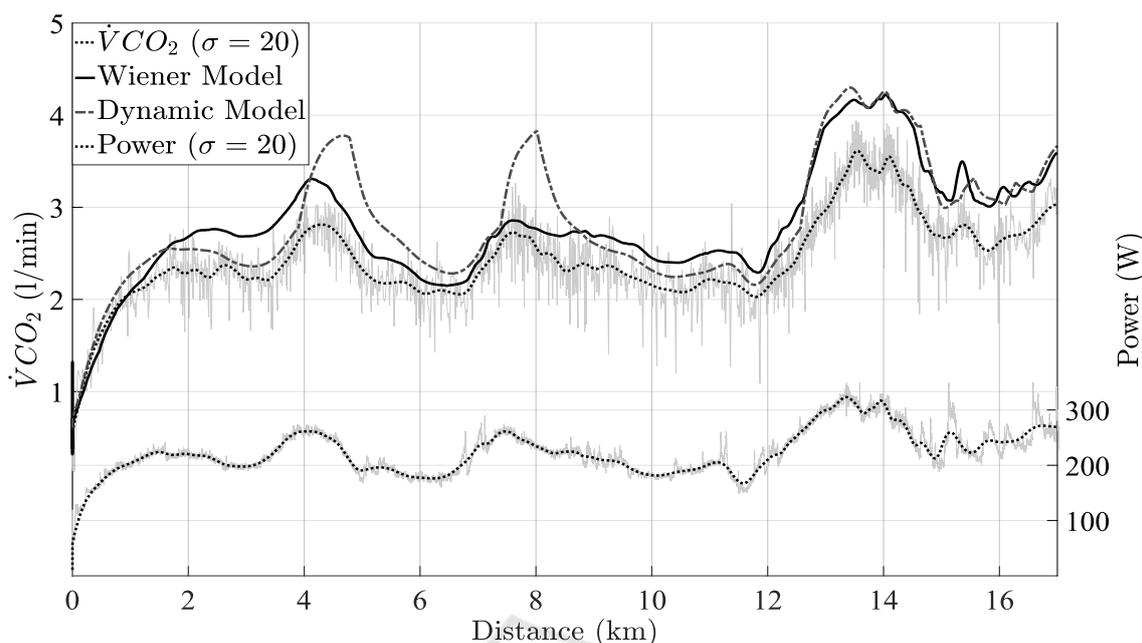
Figure 6: $\dot{V}CO_2$ prediction results for Test 4 Subject 1. Both models are based on power and trained on Test 3.

ing state, daily condition and environment conditions, e. g. temperature.

6 CONCLUSIONS

We showed that Hammerstein-Wiener models are a suitable tool for modeling and predicting $\dot{V}O_2$, $\dot{V}CO_2$ and heart rate. They performed slightly better than the dynamic model for $\dot{V}O_2$ under variable work rate (Artiga Gonzalez et al., 2015), that is based on physiological evidence. Thus, both model types are suitable for modeling and prediction.

We expect a better predictive power for both models when trained on a more suitable or bigger data set.

An alternative modeling approach that was not discussed yet and could perform well or even better are models based on neural networks.

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