# A Fuzzy System to Automatically Evaluate and Improve Fairness of Multiple-Choice Questions (MCQs) based Exams 

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#### Abstract

Examination is one of the common assessment methods to assess the level of knowledge of students. Assessment methods probably have a greater influence on how and what students learn than any other factor. Assessment is used to discriminate not only between different students but also between different levels of thinking. Due to the increasing trends in class sizes and limited resources for teaching, the need arises for exploring other assessment methods. Multiple-Choice Questions (MCQs) have been highlighted as the main way of coping with the large group teaching, ease of use, testing large number of students on a wide range of course material, in a short time and with low grading costs. MCQs have been criticised for encouraging surface learning and its unfairness. MCQs have a variety of scoring options; the most widely used method is to compute the score by only focusing on the responses that the student made. In this case, the number of correct responses is counted, the number of incorrect answers is counted and a final score is reported as either the number of the correct answers or the number of correct answers minus the number of incorrect answers. The disadvantages of this approach are that other dimensions such as importance and complexity of questions are not considered, and in addition, it cannot discriminate between students with equal total score. In this paper, a method to automatically evaluate MCQs considering importance and complexity of each question and providing a fairer way to discriminating between students with equal total scores is presented.


## 1 INTRODUCTION

Assessment is defined as 'the multi-dimensional process' in which learning is appraised and feedback is used to improve teaching (Angelo and Cross, 1993). Assessment methods have a greatest influence on how and what students learn than any other factors. Students are usually preoccupied with what constitutes the assessment in their chosen field and therefore assessment usually drives student learning. Assessment determines student approaches to learning (Boud, 1988). Assessment method sends messages to students to define and priorities what is important to learn and ultimately how they spend their time leaning it. Assessment can be used to, as far as possible, create positive incentives for teachers to teach well, and for students to study well (Wiliam, 2011). However, despite its importance, 'assessment remains the aspect of the curriculum teaching and learning practices that is least
amenable to change' (Scarino, 2013). Despite the challenges of making changes to assessment, there has been a need for 'change' due to the increasing trends in class sizes and limited resources for teaching (Donnelly, 2014).

MCQs based examinations are utilised as a result primarily of limited resources, and are used in the majority of cases to address the need to assess a large class of students in a short time (Donnelly, 2014). MCQs are popular for evaluating medical students given the logistical advantages of being able to test large numbers of candidates with minimal human intervention, their ease of use, low grading cost and testing efficiency comprise the sole rational for their continued use (McCoubrie, 2004). Due to the weakness and the criticism that MCQs cannot assess the foundational knowledge or core concepts and encourage superficial learning (Pamplett and Farnill, 1995), MCQs would not be the ideal form of assessment for lecturers if
resources and time allowed (Donnelly, 2014). MCQs based exams are reliable only because they are timeefficient (McCoubrie, 2004). Brady (2005) suggested when deciding on assessments, lectures are carrying out an ethical activity, and that they must be confident and justified in the assessment that they are have chosen.

MCQs based exams have a variety of scoring options. The most widely used method is to compute the score by only focusing on the responses that the student made. In this case, the number of correct responses is counted, the number of incorrect answers is counted and a final score is reported as either the number of the correct answers or the number of correct answers minus the number of incorrect answers. The practicality of MCQs is to evaluate large groups of students in short time and it might be difficult or time consuming to set different grades for each question. Another aspect is the socalled 'assessment by ambush' where the choice of questions is determined by the desire to discriminate as clearly as possible between high and low achievers (Brown, 1992). This may lead to omissions of questions on essential or fundamental parts of the curriculum because they are 'too easy' and insufficiently discriminatory which may drive examiners to skip over potentially important topics (McCoubrie, 2004). This might lead to an assessment approach that is unable to discriminate between students with equal total scores. A student who answered a set of more significant questions to the curriculum and more complex questions that might require more time and thinking may be rewarded a score equal to that of another student who answered a set of less significant and easy questions (Hameed, 2010; 2011).

Importance is based on how much a question is essential for the curriculum. Difficulty of a question is based upon the amount of effort needed to answer a question, solve a question, or complete task. Such questions, problems, or tasks are defined as easy or hard and are determined by how many people can answer the question, address the problem, or accomplish the task correctly or successfully. Complexity, on the other hand, defined as easy and hard and relates to the kind of thinking, action, and knowledge needed in order to answer a question, solve a problem, or complete a task and how many ways are there to do this. Complex questions, problems, and tasks are often challenge and engage students to demonstrate thinking (Francis, 2014). Fair assessment should not just consider plain grades but should also consider the aforementioned dimensions as well (Saleh and Kim, 2009; Hameed,
2011). Improving the fairness of MCQ is an increasingly important strategic concept to improve the validity of their use (McCoubrie, 2004) and to ensure that all students receive fair grading so as not to limit students' present and future opportunities (Saleh and Kim, 2009; Hameed, 2011).

In this paper, a fuzzy system based evaluation approach for MCQs based exams considering importance, complexity, and difficulty of each question is proposed. The main purpose is to provide a fairer way to discriminate between students with equal total scores and to reflect the aforementioned dimensions for fairer evaluation. The paper is organized as follows: the proposed evaluation system is presented in Section 2. In Section 3, an example and results are presented. Concluding remarks and future work are presented in Section 4.

## 2 EVALUATION SYSTEM DESIGN

The proposed evaluation system will consist of some modules as follows:

### 2.1 Difficulty Ratio

For other forms of written exams, difficulty ratio of a question can be calculated as a function of the accuracy rate a student has achieved and the time used to answer a question (i.e., answer-time) (Saleh and Kim, 2009). So if a student has obtained a higher accuracy rate in less time, it means that the question is easy, and vice versa. In case of MCQs based exam where answers are either true or false, a student will get either the full mark of the question or nothing at all (Omari, 2013). Therefore and for the sake of simplicity, difficulty in this paper will be defined as 'the percentage of the number of students who answered the question correctly'. Difficulty ration or coefficient can be calculated using the formula:

$$
\begin{equation*}
D_{i}=1-T_{i} / N \tag{1}
\end{equation*}
$$

where $D_{i}$ is the difficulty ratio or coefficient of question $i, T_{i}$ is number of students who answered question $i$ correctly, and $N$ is the total number of students who answered the question or attended the exam. As an example, assume that (4) students from (10) answered the first question correctly, so the difficulty coefficient for this question is given by (1$4 / 10)=0.6$. Since the difficulty coefficient is a ratio, so its value is between zero and one, and when the coefficient of difficulty is zero or close to zero it is a
sign that the question is very easy, and if its value is 1 or close then that means that the question is very difficult. This means that the difficulty factor of a question is inversely proportional to its easiness. It is recommended that the difficulty value to be in the range of 0.50 to 0.75 . Exam designers are recommend to put some easy questions at the beginning of the exam to encourage students, but some hard questions that determine strong students are posted at the end of the exam.

### 2.2 Score Adjustment

In this paper and as a proof of concept (PoC) to realize the proposed approach, only difficulty will be used in evaluation to adjust students' grades. The developed approach is shown in Figure 1.


Figure 1: Schematic diagram of the proposed evaluation system.

Assume that there are $n$ students laid to an exam of $m$ questions. Here, the accuracy rate matrix, $A=\left[a_{i j}\right]$ is of $m \times n$ dimensions, where $a_{i j}$ denotes the accuracy rate of student $j$ on question $i$. In case of written form exams, $a_{i j} \in[0,1]$ and in case of true/false MCQs based exams, $a_{i j} \in\{0,1\}$. $G^{T}$ denotes the transpose of $G$, where G is of $m \times 1$ dimension, $\mathrm{G}=\left[g_{i}\right], g_{i} \in[1,100]$, denotes the assigned maximum score to question $i$, where:

$$
\begin{equation*}
\sum_{i=1}^{m} g_{i}=100 \tag{2}
\end{equation*}
$$

Classical ranking approach relies merely on accuracy rate of each student in his/her exam questions and therefore it can be considered as a quantitative approach that is unable to differentiate between students with equal total scores and cannot reflect other dimensions such as importance, complexity, and difficulty of each question. The classical ranking is then obtained as:

$$
\begin{equation*}
S=G^{T} A \tag{3}
\end{equation*}
$$

The fuzzy evaluation system, on the other hand, incorporates difficulty of each question and
produces a new grading vector, $W$, as it is shown in Figure 1. Inputs to the system, on the left hand side of the figure, are the difficulty vector calculated in Section 2.1, and the accuracy rate matrix, $A$, given by exam results. A Mamdani type fuzzy inference system (FIS) with two scalable inputs and one output is used, as it is shown in Figure 2.


Figure 2: Mamdani FIS to map difficulty and accuracy into adjustment (Saleh and Kim, 2009).

Table 1: Fuzzy rule base to infer Adjustment.

| Accuracy | Difficulty |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 1 | 2 | 2 | 3 |
| 2 | 1 | 2 | 2 | 3 | 4 |
| 3 | 2 | 2 | 3 | 4 | 4 |
| 4 | 2 | 3 | 4 | 4 | 5 |
| 5 | 3 | 4 | 4 | 5 | 5 |

Mamdani's fuzzy inference method is the most commonly seen fuzzy. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators methodology (Mamdani and Assilian, 1975). Mamdani's effort was based on Zadeh's 1973 paper on fuzzy algorithms for complex systems and decision processes (Zadeh, 1973). The proposed FIS maps a two-to-one fuzzy relation by inference through a given rule base, shown in Table 1 where 1, 2, 3, 4 and 5 stands for the five linguistic labels of the fuzzy sets shown in Figure 3; low, more or less low, medium, more or less high and high, respectively.

In this paper, five Gaussian membership functions (GMFs) with fixed mean and variable variance or standard deviation are used to fuzzify each input into a linguistic variable with a degree of membership. Variable variance value is used to reflect the degree of uncertainty chosen by the domain expert or examiner to reflect his/her degree of uncertainty in the grades assigned to each question. The FIS has two input scale factors and
one output scale factor; difficulty scale factor, $S F_{D i f}$, accuracy scale factor, $S F_{A c c}$, and adjustment scale factor, $S F_{\text {Adj }}$. Input scale factors are chosen in a manner to emulate the degree of importance of each input. In this paper, SFs are chosen to be unity to consider the equal influence of each input on the output. In total 25 fuzzy rules, shown in Table 1, are used to infer adjustment in terms of accuracy and difficulty. As an example:

IF Accuracy is low (1) AND Difficulty is medium
(3) THEN Adjustment is more or less low (2).


Figure 3: Five Gaussian Mfs with $\sigma=0.2$; low, more or less low, medium, more or less high, and high.

The surface view of the fuzzy relation to infer adjustment in terms of difficulty and accuracy is shown in Figure 4.


Figure 4: Input/output mapping to infer adjustment.

## 3 RESULTS

In this Section, an example is tailored to test the proposed MCQs based fuzzy evaluation system.

### 3.1 Example

Assume that we have $n$ students laid to an exam of $m$
questions where $n=10$ and $m=5$. The accuracy rate matrix, $A$, and the grade vector, $G$, are given as follows:
$A=\left\lfloor\begin{array}{llllllllll}1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1\end{array}\right\rfloor$
$G^{T}=\left[\begin{array}{lllll}10 & 15 & 20 & 25 & 30\end{array}\right]$.

### 3.2 Classical Grading Approach

The classical ranking is obtained using Equation (3) as follows:

$$
\left.\begin{array}{rl}
S & =G^{T} A \\
& =\left[\begin{array}{lllllllll}
90_{9} & 70_{5} & 65_{10} & 65_{1} & 60_{4} & 55_{8} & 50_{6} & 45_{2} & 30_{7}
\end{array} 30_{3}\right.
\end{array}\right]
$$

From which we can find that student number 9 has got 90 and therefore he/she has occupied the first rank. Student number 5 has got 70 and therefore he/she occupies the second place, etc. Classical ranking method relies only on accuracy rate of each student and therefore there are four students with equal total final scores. As an example, both students 1 and 10 occupy the $3^{\text {rd }}$ and $4^{\text {th }}$ highest ranks with an equal total score of 65 while students 3 and 7 occupy the last $9^{\text {th }}$ and $10^{\text {th }}$ ranks with an equal total final score of 30 . Students 1 and 10 have correctly solved two different sets of questions; student 1 has solved the set $\left\{q_{1}, q_{4}, q_{5}\right\}$ while student 10 has solved the set $\left\{q_{2}, q_{3}, q_{5}\right\}$, while students 3 and 7 have correctly solved the same set of questions, i.e., $\left\{q_{1}, q_{3}\right\}$. From results, it is obvious that this approach is unable to differentiate between students with equal total final grades even though they have solved different sets of questions with different difficulty ratios.

### 3.3 Fuzzy Grading Approach

In this approach, the difficulty ratio of the exam questions is first calculated using Equation (1) to be: $D^{T}=\left[\begin{array}{lllll}0.6 & 0.7 & 0.3 & 0.5 & 0.3\end{array}\right]$

The difficulty ratio is recommended to be in the range of 0.50 to 0.75 and exam should start with easy questions (i.e., less difficulty ratio) to encourage students (Omari, 2013). Sorting questions according to its difficulty ratio gives $q_{2} \gg q_{1} \gg q_{4} \gg q_{3}=q_{5}$. The average grade of each question is then obtained as:

$$
\begin{aligned}
\bar{A}^{T} & =\sum_{i=1}^{n} \text { aij } \forall j=1: m \\
& =\left[\begin{array}{lllll}
0.4 & 0.3 & 0.7 & 0.5 & 0.7
\end{array}\right] .
\end{aligned}
$$

The difficulty ratio and the average grade of each question are then fuzzified using the five Gaussian MFs shown in Figure 3 and used to infer adjustment, $W$, though the rule base given in Table 1 to be:
$W=\left[\begin{array}{lllll}0.29 & 0.30 & 0.30 & 0.29 & 0.30\end{array}\right]$
where $w_{i}$ is the adjustment factor (\%) required for modifying the grade of question $i$ in order to compensate for its difficulty. As a result, the adjusted grade vector is slightly modified to be:

$$
G_{F u z}^{T}=\left[\begin{array}{lllll}
9.9 & 15.1 & 20.1 & 24.8 & 30.1
\end{array}\right] \text {. }
$$

The modified grade vector is then used to recalculate the final grades and the new ranks of each student using Equation (3) as follows:

$$
\begin{aligned}
S_{F U Z Z} & =G_{F U Z}^{T} A \\
& =\left[\begin{array}{llllllllll}
90.1_{9} & 70.0_{5} & 65.3_{10} & 64.9_{1} & 60.1_{4} & 55.0_{8} & 50.2_{6} & 44.9_{2} & 30.0_{7} & 30.0_{3}
\end{array}\right]
\end{aligned}
$$

By comparing the original grade vector, $G$, and $G_{f u z}$, it becomes obvious that the grades are slightly changed. This change could be increased or decreased further by tuning the scale factors $S F_{A c c}$, $S F_{D i f}$, and/or $S F_{A d j}$ in a manner to reflect the effectiveness and importance of each variable. The modified grades did not make any dramatic changes in students' final grades, however, it provided distinct ranking especially of students with equal final grades. Students 1 and 10 in the classical grading approach have obtained equal final score of 65 and therefore occupied the same ranking order but with the new fuzzy approach, where difficulty is considered, the final score of student 10 has slightly increased and that of student 1 has slightly decreased so student number 10 now clearly occupies the highest $3^{\text {rd }}$ rank while student 1 occupies the $4^{\text {th }}$ highest rank. The proposed approach can provide distinct ranks and consider other qualitative dimension in the evaluation process such as complexity and importance. In this paper, difficulty ratio has been calculated using formula (1), however, it could be obtained directly from the domain expert or examiner .

## 4 CONCLUSIONS

In this paper, a fuzzy based evaluation approach for MCQs based exams is presented. The proposed system can automatically grade students considering
difficulty of each question. It can discriminate between students of equal final total grades and hence can provide fairer grading in a manner that foster motivation and learning. Other qualitative dimensions such as complexity and importance can also be considered. The proposed system can be used or can be extended to various areas of decision support system. As a future work, complexity and importance will be considered and the real exam data will be used to validate it. The evaluation systems proposed in this paper have been implemented using the Fuzzy Logic Toolbox ${ }^{\text {TM }}$ for building a fuzzy inference system from MathWorks ${ }^{\text {TM }}$ (Fuzzy Logic Toolbox, 2016).

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