Maritime Traffic Models for Vessel-to-Vessel Distances

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Abstract: The maritime traffic is significantly increasing in the recent decades due to its advantageous features related to costs, delivery rate and environmental compatibility. The Vessel Traffic System (VTS), mainly using radar and AIS (Automatic Identification System) data, provides ship's information (identity, location, intention and so on) but is not able to provide any direct information about the way in which ships are globally positioned, i.e. randomly distributed or grouped/organized in some way, e.g. following routes. This knowledge can be useful to estimate the mutual distances among ships and the mean number of surroundings vessels, that is the number of marine radars in visibility. The AIS data provided by the Italian Coast Guard show a Gamma-like distribution for the mutual distances whose parameters can be estimated through the Maximum-Likelihood method. The truncation of the Gamma model is a useful tool to take into account only ships in a relatively small region. The result is a simple one-parameter distribution able to provide indications about the traffic topology. The empirical study is confirmed by a theoretical distribution coming from the bi-dimensional Poisson process with ships being randomly distributed points on the sea surface.

1 INTRODUCTION

Maritime traffic is strictly connected to economic growth: the international shipping industry is responsible for delivering about 90% of all trade worldwide (with 7 to 9 billion of tons loaded per year), and it is vital for bulk transport of raw material, oil and gas. The linear regression between the economic growth of the nations in the Organisation for Economic Cooperation and Development (OECD) shows a 4% increase of imports and exports for a 1% increase in the Gross Domestic Product (GDP). So, marine transportation is an integral, although sometimes less visible, part of the global economy.

The marine transportation system includes a network of specialized vessels, as well as the ports they visit and transportation infrastructure from factories to terminals to distribution centres to markets. Maritime transportation is a necessary complement to other modes of freight transportation, and it has the peculiar advantage of lower damaging emissions. In fact, shipping is emitting about 2.7% of the global greenhouse gases (GHG) (versus 93.7 % of road) and its energy consumption is about 1.4% (versus 2.6% of rail, 13.5% of air, 82.5% of road transport). For many commodities and trade routes, there is no direct substitute for waterborne commerce. On other routes, such as some coastwise or short-sea shipping or within inland river systems, marine transportation may provide a substitute for roads and rail, depending upon cost, time and infrastructure constraints. Other important marine transportation activities include passenger transportation (ferries and cruise ships), national defence, fishing and resource extraction as well as navigational service, including tugs.

The number of vessels in the world commercial fleet is about 110000 (for comparison, the number of operating commercial planes are is about 19% of this figure: roughly one commercial plane for five commercial vessels), 41% are cargo (general cargo, tankers, bulk/combined vessels, containers vessels), 42% "non-cargo" (fishing, passengers, tug boats etc.) and 17% military, for a global gross tonnage of the order of 650 millions (Bosch, et al., 2010). A much larger number of leisure (or pleasure, recreational) boats is sailing near the shores: only in the USA, this fleet is about 70000 vessels between 12 and 20 m and 11000 over 20 m. If we consider also these pleasure boats, even forgetting the billions of smaller leisure boats worldwide, the spatial distribution of marine traffic increases significantly in the areas close to one or more ports.

Since the marine navigation is a potentially

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dangerous activity for the people involved as well as for the environment, a more efficient and a more controlled navigation is required to lower the risks and to increase the overall maritime safety.

To get these achievements, the Vessel Traffic Service (VTS) has been introduced by the International Maritime Organization (IMO) in 1985 and then updated in 1997 with the Resolution A.857(20). The VTS is a service implemented by a Competent Authority, designed to improve the safety and efficiency of vessel traffic and to protect the environment (IMO, 1997).

Unlike the Air Traffic Control (ATC) which directs aircrafts through controlled airspace (ICAO, 2001), VTS only provides guidelines for procedures and manoeuvres in a crowded marine area, as well as information requested by the crew. Hence, outside the harbour waters the VTS has no any authority to impose speed and route to follow which are demanded to the captain’s decision.

In addition to being a “VTS target”, all ships of 300 gross tonnage (or more) engaged on international voyages and all cargo ships of 500 gross tonnage (and upwards) even if not engaged on international voyages, and finally all passenger ships, are required to carry on an Automatic Identification System (AIS) transponder (SOLAS, 2002), (IMO, 2001) capable of automatically exchange relevant information about the ship (radio call sign, IMO identification number, vessel name and type, position, heading, course, speed, destination, navigational status and more) with other ships and with coastal stations, providing a kind of Automatic Dependent Surveillance. The primary use of AIS is to permit each equipped ship to "see and be seen" by other ships. Concerning the related radio link, AIS uses the VHF region: Channel A 161.975 MHz, Channel B 162.025 MHz, with a particular self-organized time-division multiple access to the radio channel, for short, SO-TDMA. The maximum distance in this ship-to-ship radio communication is limited by propagation over sea of the used waves and, depending on the environment and VHF antenna height, it is about 20 nautical miles (one nautical mile - N.M. or *nm* or *n mi* - equals 1852 m), while marine radars, operating in the microwave region, are generally propagation-limited to about half this figure. The aforementioned autonomous operation of vessels, however, does not help to achieve a well-organized marine traffic and, based on raw AIS or radar data, little can be said – in general – about the overall way in which ships are positioned in a given area and about the distribution of their mutual distances. The type of ship, and its destination, are only available for AIS-equipped vessels, the model proposed in this paper is aimed to infer some characteristics of all marine traffic for every type of vessels, including non-cooperating ones whether they are VTS or coastal radar targets.

The knowledge of the mutual distances, for example, can be useful to evaluate the minimum safety separation as well as, more important from the scientific point of view, the mean numbers of marine radars (Briggs, 2004) in visibility that can interfere with the on-board radar of a given ship (Galati, et al., 2015). Such visibility results can also be useful to evaluate the load of the AIS radio channels for applications such as performance analysis and installation planning of coastal AIS stations.

In this paper we build up a statistical model of the mutual distances between pairs of ships focusing on six areas of the Mediterranean sea, see Figure 1. The model has been derived from real-world AIS data provided by the Italian Coast Guard for the week Feb 23th – Mar 1st, 2015. The data analysis has shown that the mutual distance among ships follows a Gamma-like statistical distribution. In order to make the model more general and not AIS-data dependent, we have estimated the parameters for the empirical Gamma distribution through the Maximum-Likelihood estimation. Finally we have considered a conditioned, i.e. truncated, distribution in order to take into account the horizon for radar and VHF visibility.

In Chapter 2 the AIS data provided by the Italian Coast Guard are presented, with the related statistical analysis in which the parameters of the Gamma and Generalized Gamma models are estimated.

Chapter 3 considers the truncation of the distribution of the mutual distances in order to evaluate the mean number of ships in a given region, for example for radar applications. A simplified truncated model with only one parameter has been developed for the mutual distances. The relationship between the model parameters and the topology of the traffic has been investigated. To confirm the empirical work, a more general theoretical Poisson-like model has been treated.

2 THE MARINE TRAFFIC MODEL

In this section the statistical model for the mutual distances is derived from the AIS data.

2.1 AIS Data and their Distribution

The General Command of the Italian Coast Guard kindly provided the AIS data for the week Feb 23th – Mar 1st, 2015 related to six areas: (1) Central Adriatic, (2) Otranto Canal, (3) Central Tyrrhenian, (4) Messina Strait, (5) Canal of Sicily and (6) Dardanelles/Bosporus (see Figure 1).

See Table 1a for more details. Each area was sampled at regular intervals of four hours from midnight (Galati, et al., 2015), (Galati and Pavan, 2015).

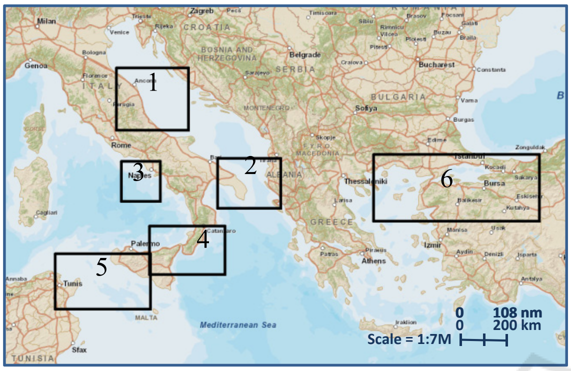

Figure 1: View of the six Mediterranean areas.

Table 1a: Main characteristics of the six areas.

Area	Point N-E (DMS)	Point S-O (DMS)	Total Surface $[nm^2]$	Sea Surface $[nm^2]$	Sea [%]
(1) Central Adriatic	44°10'18.40''N 15°55'16.71''E	42°09'26.58''N 12°43'13.25''E	22632	13600	60
(2) Otranto Canal	41°12'57.47''N 20°01'18.74''E	39°31'42.97''N 17°12'28.32''E	17712	12300	69
(3) Central Tyrrhenian	41°07'27.98''N 14°40'34.17''E	39°46''07.02''N 12°55'19.09''E	8455	6700	79
(4)Messina Strait	38°55'08.47''N 17°33'00.99''E	37°13'27.60''N 14°10'22.01''E	20384	13700	67
(5) Canal of Sicily	37°56'26.98''N 14°14'01.89''E	35°59'03.12''N 09°56'44.44''E	30186	22800	75
(6) Dardenelles Bosporus	41°21'26.79''N 31°32'03.49''E	39°05'16.24''N 24°09'53.99''E	60112	21700	36

From the first analysis of the AIS data, we derived the time slot with maximum number of ships in each area, as shown in Table 1b.

In the following we refer to the area with the highest traffic as the area with the highest number of ships.

The density z of en-route ships is calculated as the number of ships over the percentage of sea in the highest traffic condition.

We extrapolated ships' positioning information from the AIS data related to Table 1b (i.e. highest traffic condition) for each area. We used the flat earth approximation for distance due to the small-sized areas (max distance in area (6) is about 370 nm).

Table 1b: Maximum number of ships per each area and their density z. Data for the week Feb 23th – Mar 1st, 2015.

Area	Day and Time (in May, 2015)	Max number of ships, N	Ships' density $z\left[\frac{ships}{nm^2}\right]\times 10^{-3}$
(1) Central Adriatic	Tue 24th 04:00	285	20.88
(2) Otranto Canal	Tue 24th 08:00	46	3.74
(3) Central Tyrrhenian	Fri 27th 08:00	45	6.72
(4) Messina Strait	Fri 27th 16:00	74	5.40
(5) Canal of Sicily	Fri 27th 08:00	104	4.56
(6) Dardenelles Bosporus	Thu 26th 12:00	53	2.44

The number of mutual distances is:

$$N = \frac{n \cdot (n-1)}{2} \qquad (1)$$

in which n is the total number of ships in the area in a specific time slot (e.g. the highest traffic condition). It is worth to note that the N distances are *not statistically independent* because they are "mutual"

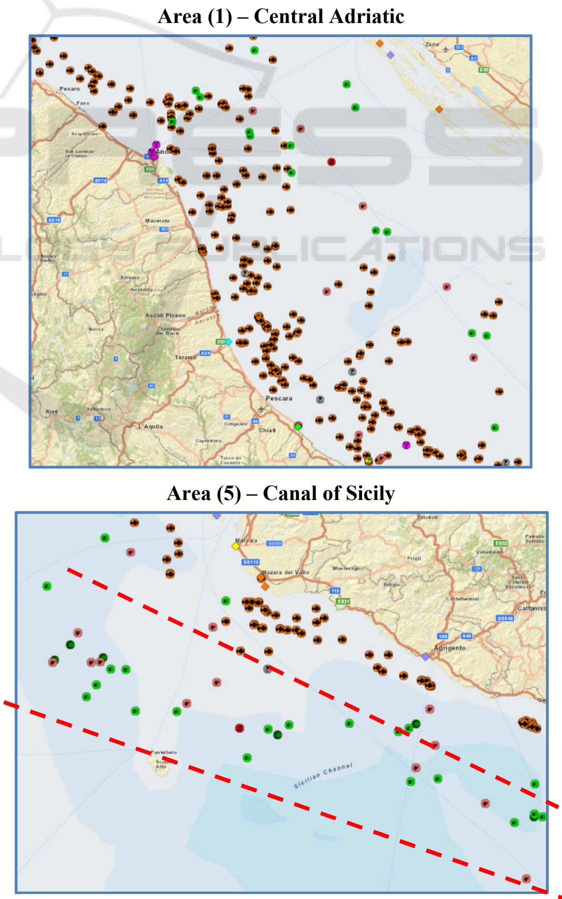

Figure 2: Distributed traffic of Area (1) Central Adriatic and in-line traffic of Area (5) Canal of Sicily. The dashed lines highlight a possible route.

among ships: given n ships, if only one of them is moved, $n-1$ distances do change.

Figure 2 shows the AIS positions of the vessels for Central Adriatic and Canal of Sicily.

It is known that the traffic in Central Adriatic is mainly made of fishing boats (88%) whose positions are someway randomly distributed, while in the Canal of Sicily are present cargos (20%) following some well defined (non random) routes.

2.2 Statistical Analysis of Inter-Ship Distances

The ship-to-ship distance R can be fitted with a probability density function $f_R(r)$ having the following properties:

- $f_R(r) = 0, \ r \leq 0$
- $\lim_{r\to\infty} f_R(r) = 0$

A suitable candidate for this positive random variable is the Gamma model whose parameters may be related to the density of ships. According to the performed "Goodness of Fit" analysis the Rayleigh distribution (or "one parameter" Gamma) does not provide the best fitting because of the very different traffic conditions difficult to be modelled with one parameter. On the other hand, the Gamma density function (Papoulis, 1990):

$$f_R(r) = \frac{\lambda^b}{\Gamma(b)} r^{b-1} e^{-\lambda r} \quad r \geq 0 \tag{2}$$

where $\Gamma(b) = \int_0^{+\infty} y^{b-1} e^{-y} dy$ is the Gamma function, having two-parameters (i.e. the scale parameter λ and the shape parameter b), can be better matched to the empirical data.

In order to improve the model of the AIS data, a third parameter μ can be added in Eq. (2) obtaining a Generalized Gamma model (Stacy, 1962):

$$f_R^{GEN}(r) = \frac{\mu \cdot \lambda^{b\mu}}{\Gamma(b)} r^{b\mu-1} e^{-(\lambda r)^{\mu}} \quad r > 0 \tag{3}$$

The quantities b, μ are shape parameters. When $\mu = 1$ the Generalized Gamma density function coincides with the Gamma model.

These parameters can be estimated by the Maximum Likelihood (ML) method, which leads to a system of non-linear equations whose solutions are the values shown in Table 2 where the last column (right side) reports the estimated mean values $\hat{m}_R$ (in *nm*).

The sample size for each area is varying from 990 distances with average value of 32.8 *nm* (area 3) to 40470 distances with average value of 55.6 *nm* (area 1); day and time are listed in the above Table 1b.

Table 2: Estimated parameters of the Gamma model (a) and of the Generalized Gamma model (b) for the six areas.

(a)

Area	Gamma Model		$\hat{m}_R = \frac{\hat{b}_{ML}}{\hat{\lambda}_{ML}}$ [nm]	$\hat{m}_R/z$
	$\hat{b}_{ML}$	$\hat{\lambda}_{ML}$ $[nm^{-1}] \times 10^{-3}$		
(1)	2.1542	38.7	55.66	2.66
(2)	1.9371	47.2	41.04	11.0
(3)	2.0472	62.4	32.80	4.88
(4)	2.4059	42.9	56.08	10.4
(5)	1.8674	34.7	53.81	11.8
(6)	1.5753	27.4	57.50	23.5

(b)

Area	Generalized Gamma Model			$\hat{m}_R = \frac{\Gamma\left(\hat{b}_{ML} + \frac{1}{\hat{\mu}_{ML}}\right)}{\hat{\lambda}_{ML}\Gamma(\hat{b}_{ML})}$
	$\hat{b}_{ML}$	$\hat{\mu}_{ML}$	$\hat{\lambda}_{ML}$ $[nm^{-1}] \times 10^{-3}$	
(1)	0.6061	2.287	11.9	55.63
(2)	0.8303	1.709	19.1	41.01
(3)	0.3334	3.576	16.5	33.04
(4)	0.3939	3.525	10.6	55.89
(5)	0.3848	3.03	10.3	53.63
(6)	0.7918	1.559	13.1	57.63

For the Gamma model the ratio $\frac{\hat{m}_R}{z} \left[\frac{nm^3}{ships}\right]$ gives an idea about the topology of the traffic on the considered sea surface (e.g. en-route or randomly distributed): a low ratio values correspond to a distributed, or random, topology (i.e. Central Adriatic, Area (1)), while higher values are related to a route, more regular topology (for example, in Otranto Canal (2), Messina Strait (4) and Canal of Sicily (5)).

Moreover we observe that the ML estimation of μ leads to a system of three non linear equations where the μ-th power of the sample values (i.e. the measured distances) is present. Therefore it is necessary to find that value of μ whereby the derivative of the Likelihood function, $f(\mu)$, is equal to zero (see Figure 3). However, as shown in Figure 3, the values of $f(\mu)$ in the field of practical interest, i.e. $0 < \mu < 3$, are close to zero, i.e. there are sub-optimal solutions (values of $\hat{\mu}$) that can be considered, including $\hat{\mu} = 1$.

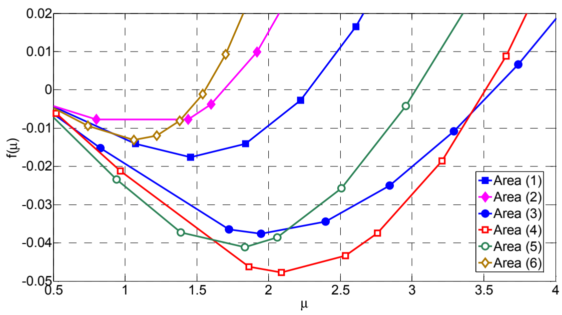

Figure 3: The derivate of the Likelihood function for the estimation of the Generalized Gamma parameter μ. The $\hat{\mu}_{ML}$is obtained posing $f(\mu) = 0$.

The use of $\hat{\mu} = 1$ simplifies the model leading back to the Gamma model that looks more convenient than its generalization (see also in the following).

In order to validate the estimated parameters $\hat{b}_{ML}$, $\hat{\mu}_{ML}$, $\hat{\lambda}_{ML}$ the Kolmogorov-Smirnov test and the χ^2 test (Papoulis, 1990) should be applied with the null hypothesis being (resp. for the Gamma and the Generalized Gamma distribution):

$$H_0: F(r) = F_R(r) \quad \text{or} \quad H_0: F(r) = F_R^{GEN}(r)$$

However, since the N distances are not independent, the tests reject too often the null hypothesis H_0 (Gleser & Moore, 1983), and cannot be effectively applied in the present case. However, a visual inspection gives a fairly good idea of the goodness of fit of the measures mutual distances with these distribution. In fact, in Figure 4a-f the histograms of distances for all areas are presented with the overlapped Gamma and Generalized Gamma estimated models.

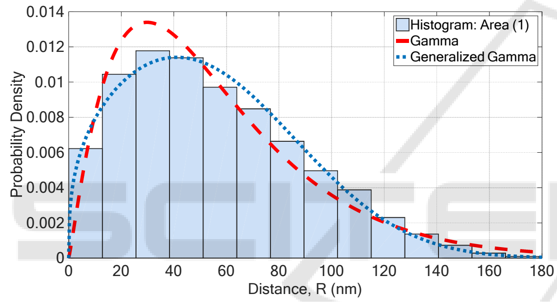

Figure 4a: Histogram and densities of R for Areas (1).

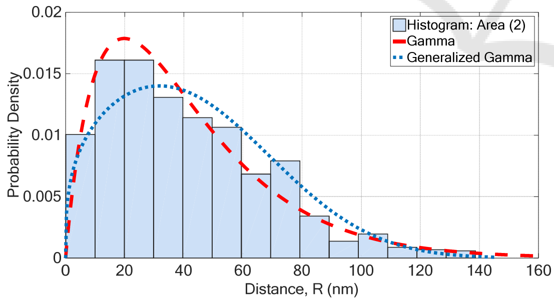

Figure 4b: Histogram and densities of R for Areas (2).

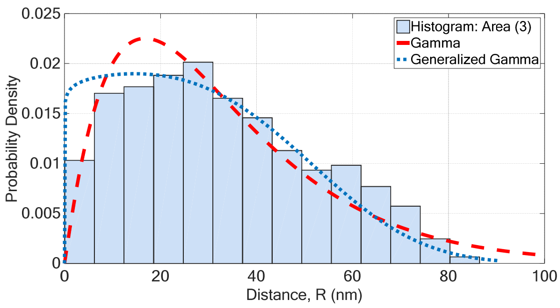

Figure 4c: Histogram and densities of R for Areas (3).

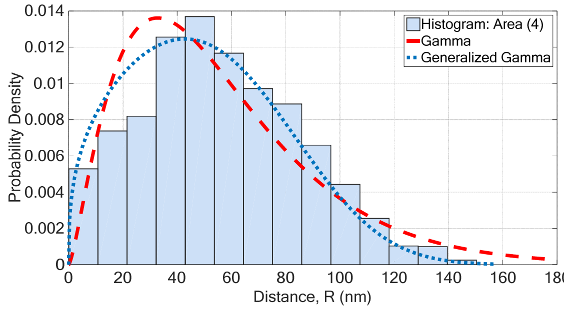

Figure 4d: Histogram and densities of R for Areas (4).

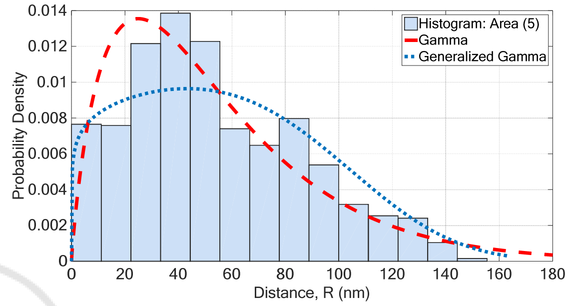

Figure 4e: Histogram and densities of R for Areas (5).

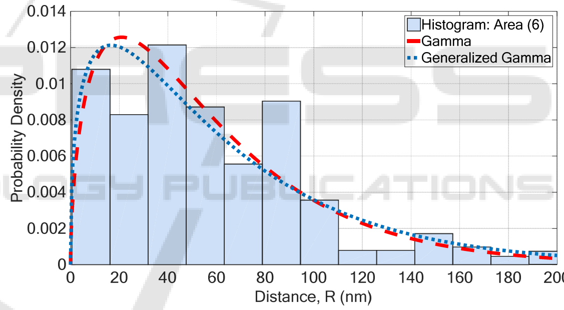

Figure 4f: Histogram and densities of R for Areas (6).

In some cases, e.g. Areas (3) and (5), the Generalized Gamma model is not the best fit because the third parameter μ improves the fitting only locally. Hence, the Gamma model with parameters λ and b will be used in the remaining part of this paper.

3 VISIBILITY

In the previous section we have shown that the distances between pairs of ships can be modelled with a random variable R distributed according to a Gamma model with parameters b and λ.

It can be useful to consider, for a generic ship, the mean number of vessels in the surroundings within a specific area. This need refers to the VHF communications as well as to the radar interferences due to solid-state marine radars on board nearby other vessels (Galati, et al., 2015). In the radar case the optical horizon – with the 4/3 earth propagation model – and the heights of ships must be considered in order to compute the maximum distance at which two on-board radars may interfere. This radar horizon is related to the heights of on-board radars h_k and h_i as shown in Figure 5.

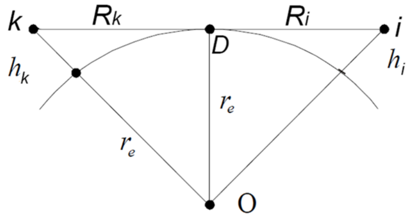

Figure 5: Radar visibility between ships $k$and i.

In standard atmosphere, making use of the equivalent earth radius $r_e = \frac{4}{3} r_{earth} \cong 8500\ km$, the horizon R_{ki} results:

$$R_{ki} = R_k + R_i \cong \sqrt{2r_e} \cdot \left(\sqrt{h_k} + \sqrt{h_i}\right) \quad (4)$$

The antenna height is not included in AIS data, hence we had empirically estimated the relation between the length (as provided by AIS) of the ship and the radar antenna height (Galati, et al., 2015). For example, if we consider two cargos with their radar antenna at 30 m above sea level, the optical horizon is about $r_{MAX} = 35\ nm$, while it becomes $r_{MAX} = 10\ nm$ for small and pleasure boats, with antenna heights of the order of 4 m. In this section we focus only on the latter case ($r_{MAX} = 10\ nm$).

Let's consider an all-sea circular section with diameter r_{MAX}. It is possible to calculate the average number of ships randomly distributed in this circular sea surface through the probability that the mutual distances among them should not exceed r_{MAX}:

$$P\{R \le r_{MAX}\} = \frac{1}{\Gamma(b)} \int_0^x e^{-t} t^{b-1} dt = \gamma(b, x) \quad (5)$$

where $\gamma(b, x)$ is the Incomplete Gamma Function (Abramowitz & Stegun, 1964) with $x = \lambda \cdot r_{MAX}$. The parameters b and λ have been estimated with the Maximum Likelihood method for each area (Table 2). Multiplying the probability in Eq. (5) by the total number of ship in the area (n_{TOT}) we obtain the expected number of ships inside the related area.

$$n_{ships} = P\{R \le r_{MAX}\} \cdot n_{TOT} \quad (6)$$

The probability density of the random variable R, i.e. the mutual distances among the n_{ships} vessels in the area (with $0 \le R \le r_{MAX}$) is given by the conditional density function of Eq. (2):

$$f(r|R \le r_{MAX}) = \begin{cases} \dfrac{f_R(r)}{F_R(r_{MAX})} & 0 < r < r_{MAX} \\ 0 & r \ge r_{MAX} \end{cases} \quad (7)$$

This conditional density function can be computed using the already described and evaluated Gamma model. This approach uses, for the conditioned model, the same parameters estimated for the original model and therefore might be not fully reliable.

Using Eq. (7) to compute the conditional density model from the Gamma model with parameters b, λ it is readily obtained:

$$f(r|R \le r_{MAX}) = \begin{cases} \dfrac{\lambda^b x^{b-1} e^{-\lambda x}}{\gamma(b, \lambda r_{MAX})} & 0 < r < r_{MAX} \\ 0 & r \ge r_{MAX} \end{cases} \quad (8)$$

In Eq. (8) we have added the third parameter r_{MAX} named *truncation* parameter which takes into account the maximum distance at which the model should be considered (e.g. the optical horizon).

To estimate b and λ in Eq. (8), having fixed the value of r_{MAX}, a closed-form solution such as the well-known one for the Gamma and Generalized Gamma distribution does not exist. The problem of finding the maximum for the Likelihood function has to be solved by a non-linear optimization method. In particular, we have used the Nelder-Mead algorithm (Nelder & Mead, 1965).

This estimation often gives very low values for λ, as shown in Table 3 for Areas (1) – (4).

Table 3: Estimation of b, μ, $\lambda$for $r_{MAX} = 10\ nm$.

Area	**Truncated Gamma**		**Truncated Generalized Gamma**		
	$\hat{b}_{ML}$	$\hat{\lambda}_{ML}\ [nm^{-1}]$	$\hat{b}_{ML}$	$\hat{\mu}_{ML}$	$\hat{\lambda}_{ML}\ [nm^{-1}]$
(1)	1.46	9.3×10^{-12}	1.46	1	9.5×10^{-14}
(2)	1.58	2.7×10^{-12}	1.59	1	2.7×10^{-12}
(3)	1.25	3.6×10^{-12}	1.26	1	3.7×10^{-12}
(4)	1.02	0.012	0.19	5.25	6.9×10^{-4}
(5)	0.99	0.017	1.21	0.82	0.015
(6)	1.74	0.078	0.22	7.10	0.09

Therefore, a different model with $\lambda \to 0$ has been considered for the *"short range"* (i.e. $r < r_{MAX}$, having set $r_{MAX} = 10\ nm$) distance between a pair of vessels.

If $\lambda \to 0$ in Eq. (8), the only remaining term is x^{b-1} multiplied by a constant c depending on b. Posing $\beta = b - 1$ we obtain:

$$f(r|R \le r_{MAX}) = c \cdot x^{\beta} \quad (9)$$

The unity area condition for Eq. (9) leads to:

$$\int_0^{r_{MAX}} c \cdot x^{\beta} dx = 1 \quad \Rightarrow \quad c = \frac{\beta + 1}{r_{MAX}^{\beta+1}} \tag{10}$$

Therefore, the conditional density function for truncated distances with a single parameter β is:

$$f(r|R \leq r_{MAX}) = \begin{cases} \frac{\beta + 1}{r_{MAX}^{\beta+1}} \cdot r^{\beta} & 0 < r < r_{MAX} \\ 0 & r \geq r_{MAX} \end{cases} \tag{11}$$

Figure 6 shows Eq. (11) for different values of β ($\beta = 0, \beta = 1,\ \beta < 1$ and $\beta > 1$) with $r_{MAX} = 10\ nm$.

If $\lambda \to 0$ (cfr. Table 3) the Gamma model leads to Eq. (11) and, if $\beta \cong 0$ (cfr. Table 4), the model converges to the uniform distribution in $(0, r_{MAX})$ as shown in Figure 6.

For the six marine areas the ML estimation of the parameter β is shown in Table 4 with $r_{MAX} = 10\ nm$.

Table 4: Estimation of β for $r_{MAX} = 10\ nm$.

Area	$\hat{\beta}_{ML}$
(1)	0.461
(2)	0.589
(3)	0.257
(4)	1.95×10^{-6}
(5)	6.1×10^{-8}
(6)	0.455

From Table 4 we can find very low values for β in areas (4) and (5), those where the traffic is strongly channelized. This suggests that strongly channelized areas generally correspond to low β. In fact if the ships are placed in line, the mean value of the mutual distances increases making less sharp the slope of the density function for low values of R.

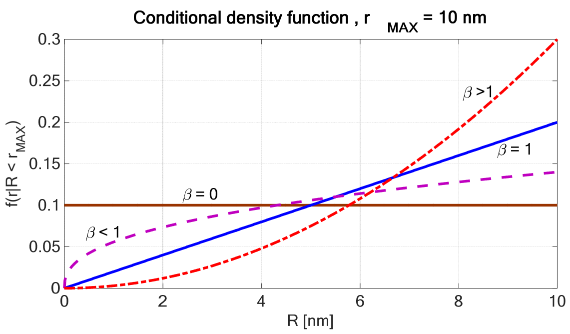

Figure 6: Conditional density model $f(r|R \leq r_{MAX})$ for $\beta = 0, \beta = 1, \beta < 1$ and $\beta > 1$ with $r_{MAX} = 10\ nm$.

Figure 2 shows the traffic condition for the Central Adriatic and the Canal of Sicily, the former with β greater than the latter because of the more randomly distributed vessels in Central Adriatic, as previously noticed.

It is worth to note that in area (6) β is comparable with the one in Central Adriatic although the area provides a main route. This effect is due to the presence in area (6) of two different seas (Aegen and Sea of Marmara) as well as of Dardanelles, one of the world's narrowest strait used for international navigation, with the likely effect of strongly distorting the behaviour of ships' distances with respect to the open sea. In general, the sea percentage in Table 1 also gives an idea about the reliability of the β values.

3.1 General Poisson's Model

To corroborate the results we considered another theoretical model for marine traffic starting from the bi-dimensional Poisson distribution in which the ships are placed uniformly in a square with side L. Conditioning the maximum distance to r_{MAX} (with $r_{MAX} \ll L$) we obtained the conditional density of R for $0 < R < r_{MAX}$ whose limit for $L \to \infty$ is shown in Eq. (12) (details are not shown here for the sake of brevity):

$$\lim_{L \to \infty} f_R(r|R < r_{MAX}) = \frac{2r}{r_{MAX}^2} \tag{12}$$

Such a limit represents the condition for which the range of distances we are interested is much less than L, as in the previous paragraph where $r_{MAX} = 10\ nm \ll L \approx 200\ nm$.

The limit found in Eq. (12) represents the traffic uniformly distributed in a interval with edge $r_{MAX} \ll L$, that is the case of $\beta = 1$ in Eq. (11).

If $\beta \to 1$ the traffic is Poisson distributed, if $\beta \to 0$ there is a kind of traffic regularity possibly due to one or more routes. In Table 4 the values of β do not reach the unity because the land imposes a constraint to the positions, hence on the distances.

The case of $\beta > 1$ is not realistic for marine traffic since it imposes a mandatory minimal distance among ships as shown in Figure 6 where the density for low value is almost zero. This case may be useful to model other situations such as, possibly, the air traffic.

4 CONCLUSIONS

The empirical analysis of the AIS data has led to a Gamma model for the mutual distances among ships, with the Generalized Gamma model being not the best solution to fit the data. We estimated the parameters of the models through the ML method.

Considering the application related to the interferences between marine (navigation) solid-state radars, we have truncated the Gamma model to a maximum distance r_{MAX} in order to take into account only the ships inside the horizon.

The truncation has led to a more convenient one-parameter distribution whose parameter β is related to the topology of the traffic in the area of interest.

The relation between β and the traffic topology has been confirmed, at a very first extent, by the study of a general Poisson model with only one-parameter, that is β, in addition to the truncation limit r_{MAX}.

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