An Interval Type-2 Fuzzy Logic System for Assessment of Students' Answer Scripts under High Levels of Uncertainty

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Keywords: Students' Evaluation, Uncertainty, Interval Type-2 (IT2) Fuzzy Sets, Type Reduction, Footprint of Uncertainty (FOU).

Abstract: The proper system for evaluating the learning achievement of students is the key to realizing the purpose of education and learning. Traditional grading methods are largely based on human judgments, which tend to be subjective. In addition, it is based on sharp criteria instead of fuzzy criteria and suffers from erroneous scores assigned by indifferent or inexperienced examiners, which represent a rich source of uncertainties, which might impair the credibility of the system. In an attempt to reduce uncertainties and provide more objective, reliable, and precise grading, a sophisticated assessment approach based on type-2 fuzzy set theory is developed. In this paper, interval type-2 (IT2) fuzzy sets, which are a special case of the general T2 fuzzy sets, are used. The transparency and capabilities of type-2 fuzzy sets in handling uncertainties is expected to provide an evaluation system able to justify and raise the quality and consistency of assessment judgments.

1 INTRODUCTION

As highlighted by Boud (1988), assessment methods and requirements probably have a greater influence on how and what students learn than any other factor. This influence may become of greater importance than the impact of teaching materials itself. A high quality, reliable and transparent assessment system supports and improves student lifelong learning and ensures that all students receive fair treatment in order not to limit students' present and future opportunities. The evaluation of a students' learning achievement is done over years and provides the basis for certification of individual achievement, therefore, it should regularly reviewed and improved to ensure that the systems are educationally beneficial to all students (Saleh and Kim, 2009; Hameed, 2011). Students' evaluation and scoring are largely based on human judgments, which tend to be subjective, and hence represents a rich source of uncertainties. Assessment process, as well, is suffering from uncertainty due to assigning erroneous grades and indifferent and inexperienced practices. Uncertainty is an attribute of information (Zadeh, 2005). More often than not, the decision-relevant information is subjected to uncertainty arising from different sources. Consequently, decisions involve an undeniable amount of risk (Daradkeh et al., 2013).

In an attempt to reduce the uncertainty in the students' assessment process, several attempts have been made in the last decade to use fuzzy set theory in educational evaluation. Biswas (1995) presented two methods for students' answerscripts evaluation using fuzzy sets; a fuzzy evaluation method and a generalized fuzzy evaluation method and a matching function. Echauz and Vachtsevanos (1995) proposed a fuzzy logic system for translating traditional scores into letter-grades. Law (1996) built a fuzzy structure model for education grading system with its algorithm to aggregate different test scores in order to produce a single score for individual student. Wilson, Karr and Freeman (1998) presented an automatic grading system based on fuzzy rules and genetic algorithms. Chen and Lee (1999) presented two methods for applying fuzzy sets to overcome the problem of rewarding two different fuzzy marks the same total score that could arise from Biswas'

Hameed, I., Elhoushy, M., Zalam, B. and Osen, O.
An Interval Type-2 Fuzzy Logic System for Assessment of Students' Answer Scripts under High Levels of Uncertainty.
In *Proceedings of the 8th International Conference on Computer Supported Education (CSEDU 2016) - Volume 2*, pages 40-48
ISBN: 978-989-758-179-3

method. Ma and Zhou (2000) proposed a fuzzy set approach to assess the outcomes of student-centered learning using the evaluation of their peers and lecturer.

Weon and Kim (2001) presented an evaluation strategy based on fuzzy MFs. They pointed out that the system for students' achievement evaluation should consider the three important factors of the questions that the students answer: the difficulty, the importance, and the complexity. Weon and Kim used singleton functions to describe the factors of each question reflecting the effect of the three factors individually, but not collectively. Wang and Chen (2008) presented a method for evaluating students' answerscripts using fuzzy numbers associated with degrees of confidence of the evaluator. Bai and Chen (2008b) pointed out that the difficulty factor is a very subjective parameter and may cause an argument about fairness in evaluation.

Bai and Chen (2008a) proposed a method to automatically construct the grade MFs of fuzzy rules for evaluating student's learning achievement. Bai and Chen (2008b) proposed a method for applying fuzzy MFs and fuzzy rules for the same purpose. To solve the subjectivity of the difficulty factor of Weon and Kim's method (2001), they obtained the difficulty as a function of accuracy of the student's answer script and time consumed to answer. However, their method still has the subjectivity problem, since the results in scores and ranks are heavily depend on the values of several weights that are determined by the subjective knowledge of domain experts.

Saleh and Kim (2009) proposed three nodes fuzzy logic approach based on Mamdani's fuzzy inference engine and the center of gravity (COG) defuzzification technique as an alternative to Bai and Chen's method (2008b). The transparency and objective nature of the fuzzy system makes their method easy to understand and enables teachers to explain the results of evaluation to persuade skeptic students. Hameed (2011) proposed using Gaussian MFs as an alternative of the triangle MFs used in Saleh and Kim (2009). A sensitivity study showed that using Gaussian MFs with standard deviation higher than 0.4 provide more reliable and robust evaluation system which is able to provide new ranking orders without changing students' scores.

In this paper, a type-2 fuzzy logic (T2FL) system is proposed. The general framework of T2 fuzzy reasoning allows handling much of the uncertainty inherited in students' evaluations systems. T2FL has better capabilities in reducing the amount of uncertainty in a system due to its ability in handling linguistic uncertainties by modeling vagueness and unreliability of information (Liang and Mendel, 2000). In this paper, a new implementation of the three-nodes fuzzy evaluation system presented in Saleh and Kim (2009) and Hameed (2011) using T2FSs will be presented. An example will be given to highlight the differences between traditional, T1FSs- and T2FSs-based approaches.

The paper is organized as follows: a review of some existing evluation approaches is presented in Section 2. The proposed inteleval type-2 fuzzy logic based evalaution system is presented in Section 3. In Section 4, results of the appraoches presented in Sections 2 and 3 applied to a real world example are presnted. Comaprsons between different approahces, concluding remarks and future work are presnted in Section 5.

2 REVIEW OF EVALUATION METHODS

2.1 Classical Approach

Assume that there are *n* students to answer *m* questions. Accuracy rates of students' answerscripts (student's scores in each question divided by the maximum score assigned to this question) are the basis for evaluation. We get an accuracy rate matrix of dimension *m* x *n*,

$$A = [a_{ij}],\ m \times n,$$

where $a_{ij} \in [0, 1]$ denotes the accuracy rate of student *j* on question *i*. Time rates of students (the time consumed by a student to solve a question divided by the maximum time allowed to solve this question) is another basis to be considered in evaluation. We get a time rate matrix of dimension *m* x *n*,

$$T = [t_{ij}],\ m \times n,$$

where $t_{ij} \in [0, 1]$ denotes the time rate of student *j* on question *i*. We are given a grade vector

$$G = [g_i],\ m \times 1,$$

where $g_i \in [1, 100]$ denotes the assigned maximum score of question *i* satisfying $\sum_{i=1}^{m} g_i = 100$.

Based on the accuracy rate matrix *A* and the grade vector *G*, we obtain the total score vector of dimension *n* x 1,

$$S = A^T G = [s_j],\ n \text{ x } 1, \quad (1)$$

where $s_j \in [0, 100]$ is the total score of student j which is obtained by

$$s_j = \sum_{i=1}^{m} a_{ij} \cdot g_i \quad (2)$$

The classical rank of students is then obtained by sorting values of S in a descending order. In this approach, the time used in solving each question is not considered

2.2 Three-nodes Fuzzy Evaluation Approach

The system consists of three nodes, the difficulty node, the cost node, and the adjustment node, as it is shown in Figure 1 (Saleh and Kim, 2009).

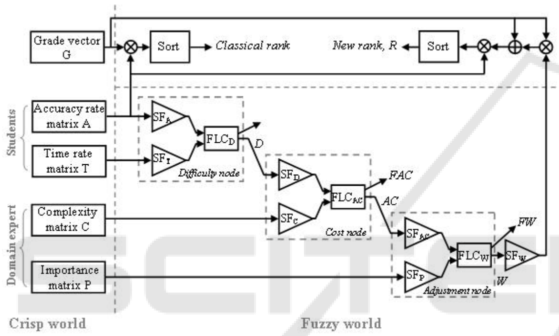


Figure 1: Block diagram of the three nodes fuzzy evaluation system.

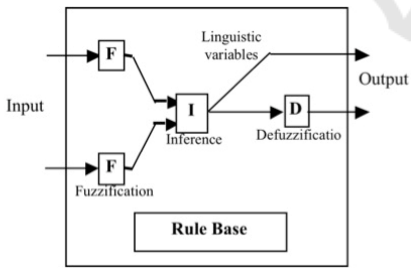


Figure 2: Node representation as a fuzzy logic controller.

Input to the system, in the left part of the figure, is given either by exam results or domain expert. Each node of the system behaves like a fuzzy logic controller (FLC) with two scalable inputs and one output, as it is shown in Figure 2. It maps a two-to-one fuzzy relation by inference through a given rule bases, shown in Tables 1 & 2 where 1, 2, 3, 4 and 5 stands for the five linguistic labels or levels low, more or less low, medium, more or less high and high. Fuzzy sets (FSs) are sets whose elements have degrees of membership, and were first introduced by Zadeh in 1965 as an extension of the classical notion of set (Zadeh, 1965). The inputs are fuzzified based on the predefined defined levels (fuzzy sets) shown in Figure 3.

Table 1: A fuzzy rule base to infer the difficulty.

Accuracy	Time rate				
	1	2	3	4	5
1	3	4	4	5	5
2	2	3	4	4	5
3	2	2	3	4	4
4	1	2	2	3	4
5	1	1	2	2	3

Table 2: A fuzzy rule base to infer the cost and adjustment.

Difficulty/ Cost	Complexity/ Importance				
	1	2	3	4	5
1	1	1	2	2	3
2	1	2	2	3	4
3	2	2	3	4	4
4	2	3	4	4	5
5	3	4	4	5	5

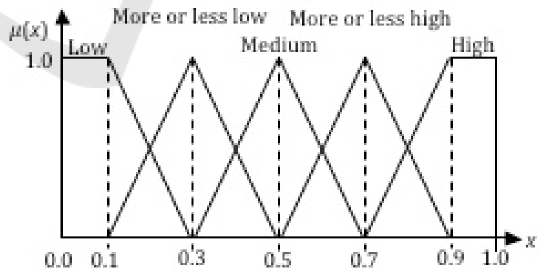


Figure 3: Triangular membership functions of the five levels (i.e., five MFs).

In the first node, both inputs are given by exam result, whereas in the later nodes, one input is the output of its previous node while a domain expert gives the other input. The output of each node can be in the form of a crisp value (defuzzified) or in the form of linguistic variables (MFs). Each node has two scale factors (SFs) that can be chosen in a manner to reflect the degree of importance of each input. Here, SFs are chosen to be equal to 1 to reflect the equal influence of each input on the output. In this method, each fuzzy node proceeds in following four steps.

Step. 1: Fuzzification step in which inputs, if given in crisp values, the degree to which these inputs belong to each of the appropriate fuzzy sets is determined. Triangular MF is the commonly used due to its simplicity and easy computation. We note that the same five fuzzy sets, shown in Figure 3, are applied to represent the accuracy, the time rate, the difficulty, the complexity, and the adjustment of questions in the fuzzy domain.

Step. 2: Rule evaluation where the fuzzified inputs are applied to the antecedents of the fuzzy rules to obtain a single number that represents the result of the antecedent evaluation (i.e., rule or firing strength). The result of the antecedent evaluation is then applied to the membership function of the consequent (i.e., rule implication). Two implication methods are commonly used; clipping where the consequent membership function is sliced at the level of the level of the rule firing strength. The clipped function set loses some information, however, it is preferred because it involves less complex computations and generates an aggregated output surface that is easier to defuzzify (Iancu, 2012). Another method, named scaling, offers a better approach for preserving the original shape of the fuzzy set: the original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent

Step. 3: Aggregation of rule outputs where the membership functions of all rule consequents previously clipped or scaled are combined into a single fuzzy set. Implication is modeled by means of minimum operator, and the resulting output MFs are combined using maximum operator (i.e. aggregation).

Step. 4: defuzzification in which the aggregated fuzzy sets are converted into a single crisp output. The most popular method is the centroid technique where a point representing the center of gravity (COG) of the aggregated fuzzy set is found. In this paper, the center of gravity (COG) method is applied. The crisp value of question i is then obtained by

$$y_i = \int_x x.\mu(x)dx \Big/ \int_x \mu(x)dx \tag{3}$$

where integrals are taken over the entire range of the output and $\mu(x)$, and $\mu(x)$ is the membership degree of x. By taking the center of gravity, conflicting rules essentially are cancelled and a fair weighting is obtained.

Each of the three nodes follows the above scheme. The difficulty node has two inputs, the accuracy rate and the time rate, and one output of the difficulty. The cost node has two inputs, the difficulty and complexity, and one output of the cost. The adjustment node has two inputs, the cost and the importance, and one output of the adjustment.

The adjustment vector, W, is then used to obtain the adjusted grade vector of dimension m x 1, $\widetilde{G} = [\widetilde{g}_i]$, m x 1, where $\widetilde{g}_i$ is the adjusted grade of question i, and is obtained using the formula:

$$\widetilde{g}_i = g_i \cdot (1 + w_i), \tag{4}$$

It is then scaled to its total grade by using the formula:

$$\widetilde{g}_i = \widetilde{g}_i \cdot \sum_j^m g_j \Big/ \sum_j^m \widetilde{g}_j \tag{5}$$

Then we obtain the adjusted total scores of students by,

$$\widetilde{S} = A^T \widetilde{G} \tag{6}$$

The new rank of students is then obtained by sorting values of $\widetilde{S}$ in a descending order.

2.3 Gaussian based Three Nodes Fuzzy Evaluation Approach

The three nodes fuzzy evaluation system described in Section 2.2 is based on the simple triangular-shaped MF formed using straight lines. Triangular MFs are defined by three scalar parameters *a*, *b* and c. The parameters *a* and *c* locate the feet of the triangle MF while *b* locates its peak. There is no way to get its optimum values, however, they should be chosen in a manner to provide a satisfying overlap between different MFs. The simplicity of this function makes it ideal for control applications where computational power and resources are crucial (Zhao and Bose, 2002). However, it was noted that when theses parameters are changed slightly, different ranking orders are obtained which could impair the system's reliability.

In order to avoid losing reliability and having a robust evaluation system, it should be able to give the same ranking orders without changing students' scores and for various values of these parameters. In this connection, Gaussian MFs are proposed (Hameed, 2011). Gaussian MFs are suitable for problems that require continuously differentiable curves and smooth transitions between levels, whereas triangular MFs do not have. Gaussian MFs are defined by two parameters; c which locates the distance from the origin to the center of each MF and σ which determines its width. Gaussian MFs is one parameter less than that of the triangular MFs which will lead to an evaluation system with 15 less Degrees Of Freedom (DOF) and hence a more robust performance (Zhao and Bose, 2002). Gaussian MFs is defined as

$$\mu_{A_i}(x) = e^{-\frac{1}{2}(x-c_i/\sigma_i)^2}, \tag{7}$$

where c_i is the center (i.e., mean) and σ_i is the width (i.e., standard deviation) of the i^{th} fuzzy set, which has by nature, infinite support. Therefore, for Gaussian MFs with wide widths it is possible to obtain a membership degree to each fuzzy set greater than 0 and hence every rule in the rule base fires. Consequently, the relationship between input and output can be described accurate enough. Here, the centers of the five Gaussian MFs are chosen to be the same as that of the triangular MFs shown in Figure 3 (i.e. [0.1 0.3 0.5 0.7 0.9]). Gaussian MFs of the five levels for σ=0.1 are shown in Figure 4.

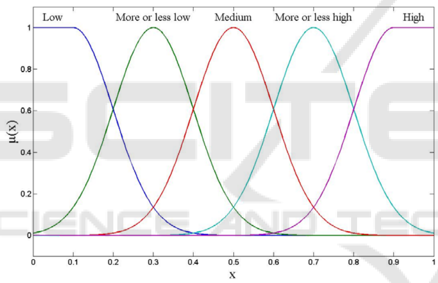


Figure 4: Gaussian membership functions of the five levels for $\sigma = 0.1$.

From Figure 4 it is obvious that Gaussian MFs provide more continuous transition from one interval to another and hence provides smoother control surface from the fuzzy rules. The Gaussian based fuzzy evaluation system was able to provide correct ranking order of students with equal total scores without changing the total mean scores of all students and the score of each student for $\sigma \geq 4.0$ (Hameed, 2011; Hameed and Sørensen, 2010).

3 INTERVAL T2FL SYSTEM BASED EVALUATION SYSTEM

Interval type-2 fuzzy logic systems (IT2 FLSs) have demonstrated better abilities to handle uncertainties than their type-1 (T1) counterparts in many applications (Wu, 2013). The concept of T2FSs was first introduced by Zadeh in 1975 (Zadeh, 1975) as an extension of the concept of an ordinary type-1 fuzzy set. Such sets are fuzzy sets whose membership grades themselves are T1FSs instead of crisp numbers in T1 FS. Interval type-2 (IT2) FSs are T2 FSs whose memberships are intervals instead of T1FSs in a general T2FS (Zadeh, 2005).

T2FSs are useful in such cases when it becomes difficult to determine exact membership function for a fuzzy set and hence are useful for incorporating linguistic uncertainties. Figure 5 shows the schematic diagram of an IT2 FLS. It is similar to its T1 counterpart, shown in Figure 2, the major difference being that at least one of the FSs in the rule base is an IT2 FS. Hence, the outputs of the inference engine are IT2FSs. A type-reducer is needed to convert them into T1FSs before defuzzification can be carried out (Wu, 2013).

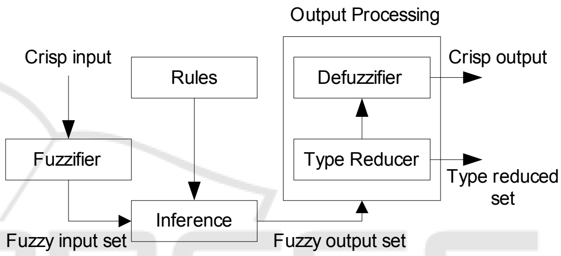
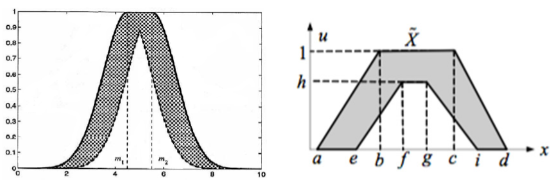


Figure 5: Schematic diagram of IT2 FLS.

Mendel and Liang (1999) demonstrated the first type-2 fuzzy framework where the information about the linguistic/numerical uncertainty can be incorporated. They introduced the concept of concept of *footprint-of-uncertainty* (FOU) where the an interval type-2 membership function (MF) is characterized by an upper and lower type-1 MFs bounding the region called FOU, as it is shown in Figure 6. The internal structure of T2FLS is shown in Figure 6. A fuzzy logic system can be considered as T2 when at least one of the antecedents or consequents of its rule-base's FSs is T2. As the outputs of the inference engine are IT2 FSs, a type-reducer is required to convert its T2FSs into T1FSs to be defuzzified. A detailed description can be found in (Mendel, 2001).



(a) (b)

Figure 6: FOU for (a) T2 Gaussian MF, and (b) Triangle MF.

In this paper, the three-nodes fuzzy evaluation framework shown in Figures 1 and 2 is implemented using triangle T2FSs shown in Figure 6(b). Five T2 triangle MFs are used to represent the five levels used to describe each variable, as it is shown in Figure 7(a). The FOU (i.e., thickness of the MFs) is provided as an external input by the domain expert as an estimate of the amount of uncertainty in his/her knowledge. In this paper, FOU is chosen to be a number in the range of 0 to 0.3 where 0 refers to zero uncertainty, 0.1 refers to low uncertainty, 0.2 refers to medium uncertainty and 0.3 refers to high uncertainty. It is worth noting that the T2 fuzzy system will converge to its T1 counterpart when uncertainty measure is set to zero (Hameed, 2009), as it is shown in Figure 7(a). In this paper, it is assumed that the domain expert has a medium degree of uncertainty in his knowledge (i.e., FOU≈0.2).

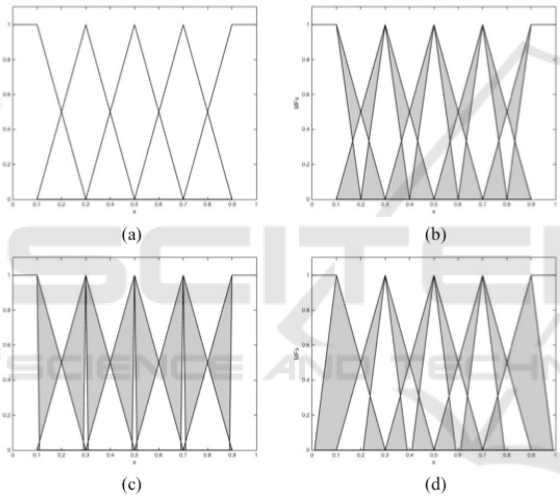


Figure 7: FOU: (a) zero uncertainty (FOU=0), (b) low uncertainty (0<FOU≤0.1), (c) medium uncertainty (0.1<FOU≤0.2), and (d) high uncertainty (0.2<FOU≤0.3).

4 RESULTS

In this section, a comparison between the different evaluation approaches presented in Sections 2 and 3 will be introduced using an example.

4.1 Example

Assume that we have n students laid to an exam of m questions where n=10 and m=5. The accuracy rate matrix, A, the time rate matrix, T, and the grade vector, G, are given as follows (Bai and Chen, 2008b; Saleh and Kim, 2009; Hameed, 2011):

$$A=\begin{bmatrix} 0.59 & 0.35 & 1 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix},$$

$$T=\begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0 & 0.1 & 0 & 0.1 & 0.9 & 1 & 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix},$$

$$G^T=\begin{bmatrix} 10 & 15 & 20 & 25 & 30 \end{bmatrix}$$

Here, $A=[a_{ij}]$ and $T=[t_{ij}]$ are of $n\times m$ dimensions, where $a_{ij}\in[0, 1]$ denotes the accuracy rate of student j on question i, $t_{ji}\in[0, 1]$ denotes the time rate of student j on question i. G^T denotes the transpose of G, where G is of $m\times 1$ dimension, G= $[g_i]$, $g_i\in[1, 100]$ denotes the assigned maximum score to question i. Importance and complexity of each question, I and C, are determined by the domain expert as follows:

$$I=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix}\begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix},$$

$$C=\begin{bmatrix} 0 & 0.85 & 0.15 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.69 & 0.31 \\ 0.56 & 0.44 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 \end{bmatrix}\begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix}.$$

Matrices $I=[i_{ik}]$ and $C=[c_{ik}]$ are of dimension $m\times l$ where $i_{ik}\in[0, 1]$ denotes the membership value of question i belonging to the importance level k, and $c_{ik}\in[0, 1]$ denotes the membership value of question i belonging to the complexity level k.

4.2 Classical Approach

In this approach, total score can be obtained using formula (1) as follows:

$$S^T=\begin{bmatrix} \overset{S_1}{67.60} & \overset{S_2}{54.05} & \overset{S_3}{38.40} & \overset{S_4}{49.70} & \overset{S_5}{49.70} & \overset{S_6}{48.80} & \overset{S_7}{46.10} & \overset{S_8}{52.30} & \overset{S_9}{85.95} & \overset{S_{10}}{49.70} \end{bmatrix}$$

Thus the "classical" ranks of students can then be obtained by simply sorting S in a descending order to get:

$S_9 > S_1 > S_2 > S_8 > S_4 = S_5 = S_{10} > S_6 > S_7 > S_3$,

where $S_a > S_b$ means score of student a is higher than score of student b while $S_a = S_b$ means that their scores are equal.

4.3 Triangle MFs based Fuzzy Approach

The process starts by averaging the accuracy rate and answer-time rate matrices A and T, respectively, for each student to get:

$$\bar{A}^T = \begin{bmatrix} 0.45 & 0.31 & 0.711 & 0.47 & 0.637 \end{bmatrix},$$

$$\bar{T}^T = \begin{bmatrix} 0.57 & 0.48 & 0.31 & 0.50 & 0.57 \end{bmatrix}.$$

Based on the fuzzy MFs shown in Figure 3 we obtain the fuzzy accuracy rate matrix and the fuzzy time rate matrix as follows:

$$FA = \begin{bmatrix} 0 & 0.25 & 0.75 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.945 & 0.055 \\ 0 & 0.15 & 0.85 & 0 & 0 \\ 0 & 0 & 0.315 & 0.685 & 0 \end{bmatrix},$$

$$FT = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.1 & 0.9 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.65 & 0.35 & 0 \end{bmatrix}.$$

In the first node, both inputs are given by examination results, whereas in later nodes, one input will be the output of its previous node and while a domain expert will provide the other. The output of each node can be in the form of a crisp value (defuzzified) or in the form of fuzzy numbers (i.e., degrees of membership (MFs) of each variable in the five linguistic levels). Each node has two scale factors (SFs shown in Figure 1). Here, we let both scaling factors have the same value of unity assuming equal influence of each input on the output.

Each fuzzy node performs Mamdani fuzzy inference to compute its output given the inputs and the fuzzy rules (described in Tables 1 and 2). Each fuzzy node proceeds in a number of steps described in Section 2.2. By applying FA and FT to the first node, the difficulty vector, D, and its fuzzy counterpart, FD. In the same way, the cost vector will be obtained by applying the difficulty and complexity to the second node. Finally, the adjustment vector, W, will be obtained by applying the cost and importance to the third node as follows:

$$W^T = \begin{bmatrix} 0.7 & 0.552 & 0.749 & 0.177 & 0.5 \end{bmatrix}$$

The new scores of students S_1 to S_{10} are then obtained using Equations 3-5 to be 67.151, 53.168, 42.096, 52.190, 48.307, 51.814, 48.474, 49.272, 85.253, 51.493, respectively. The new rank of students is then obtained by sorting values of $\hat{S}$ in a descending order

$$S_9 > S_1 > S_2 > S_4 > S_6 > S_{10} > S_8 > S_7 > S_5 > S_3.$$

4.4 Gaussian MFs based Fuzzy Approach

By replacing triangle MFs, used in Section 4.3, which are formed simply using straight lines with Gaussian MFs with the same center points (i.e., mean) as the triangle MFs, as it is shown in Figures 3 and 4 with stand deviation (i.e., width) σ≥4.0, new scores for the 10 students are obtained where the mean score is still equal to that of their original scores obtained using the classical approach. The new scores of students are then obtained using Equations 3-5 as:

$$\hat{S}^T = \begin{bmatrix} \overset{S_1}{64.60} & \overset{S_2}{54.05} & \overset{S_3}{38.40} & \overset{S_4}{49.70} & \overset{S_5}{49.70} & \overset{S_6}{48.80} & \overset{S_7}{46.10} & \overset{S_8}{52.30} & \overset{S_9}{84.95} & \overset{S_{10}}{49.70} \end{bmatrix}$$

The new rank of students is then obtained by sorting values of S in a descending

$$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_5 > S_6 > S_7 > S_3.$$

Table 3: Ranking order for different FOU values.

FOU	Rank									
	1>	2>	3>	4>	5>	6>	7>	8>	9>	10
0	9	1	2	4	6	10	8	5	7	3
0.1	9	1	2	4	6	10	5	8	7	3
0.2	9	1	2	4	6	10	8	7	5	3
0.3	9	1	2	8	5	10	4	6	7	3

4.5 IT2 MFs based Fuzzy Approach

In this Section, IT2 MFs with different value of FOU (i.e., zero, low, medium and high uncertainty) as it is shown in Figure 7. The new ranking orders for different FOU values are shown in Table 3. A comparing between the ranking orders of the four types is shown in Table 4.

Table 4: Ranking order for different approaches: class for classical, T1T for type-1 triangle MFs, T1G for type-1 Gaussian MFs, and T2 for type-2 MFs.

Method	Rank									
	>	2>	3>	4>	5>	6>	7>	8>	9>	10
Class	9	1	2	8	4=	5=	10=	6	7	3
T1T	9	1	2	4	6	10	8	7	5	3
T1G	9	1	2	8	4	10	5	6	7	3

T2	9	1	2	8	5	10	4	6	7	3

5 CONCLUSIONS

As it is shown in Table 1, classical assessment approach resulted in students of equal scores that make it difficult to determine a distinguished order of each student. T1 Triangular FSs overcome the problem of students of equal scores but at the same time it changed scores of other students who does not fall in that category which might spark questions and make students skeptic about the evaluation process. On the other hand, T1 Gaussian FSs based system influenced only that category of students with equal scores while other students of different scores are left intact. Similarly, T2 FSs changed only the scores and hence the rank order of students with equal scores while the others are left intact. A major difference between T2 and T1G FSs is that T2 system gave preferences to complexity of questions over importance and that is clear from GIVING A higher rank for student S5 who given a higher rank (rank#5) on account of student S4 who is given a lower rank (rank#7). On the other side, T1G gave preferences to importance of questions over its complexity and that explains why S4 is given higher rank (rank#5) on account of S5 who has given a lower rank (rank#7).

The transparency and the human logic nature of fuzzy logic system make it easy to interpret and explain why certain scores have changed. The system inherently has a kind a feedback system to correct erroneous scores assigned by indifferent or inexperienced examiners. Easy of implementation of the proposed system recommended it to spread out and to be broadly used in other decisions support systems. In this paper, a collective FOU for all the fuzzy variables is used to represent a collective uncertainty in the knowledge of the domain expert. As a future work, the effect of using various FOU values for each fuzzy variable such as importance, complexity, etc. will be investigated. The evaluation systems proposed in this paper hav been implemented using the Fuzzy Logic Toolbox™ for building a fuzzy inference system from MathWorks™ (Fuzzy Logic Toolbox, 2016).

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