

Re-parameterization of a Deformation Model for Non-rigid Registration

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Abstract: In this paper we present a method for non-rigid registration of meshes. The method aligns two surfaces of deformable objects by automatically separating the deformation into a single global transformation and other local deformations. The local deformations are found by applying the deformation model proposed by Sumner (Sumner et al., 2007) that previous methods have used to register two surfaces. However, we specify a rigid transformation for each node of the deformation graph using a rotation matrix and a translation vector. With this model, unit quaternions of rotation matrices are parameterized using an homeomorphic relation between the 4D unit sphere and the 3D projective space. Therefore, the number of unknowns is reduced by half compared to the original models based on affine transformations and the optimization process is less complex. We demonstrate the efficiency of the proposed method by aligning the surfaces of data sets without any prior knowledge and assumptions about the deformation between the two surfaces.

1 INTRODUCTION

With the availability of depth cameras (e.g. Microsoft Kinect and similar RGB-D sensors), obtaining 3D data is simpler and more flexible. Moreover, these cameras capture 3D data at a high frame rate (i.e. about 30 frames/second) and allow the scanned dynamic objects (e.g. human body, animals) to be unconstrained during the scanning process. In contrast, the acquired data is also more complex to process due to noise and outliers as well as object deformation between scans. Non-rigid registration applied on deformable objects is the task of finding a mapping between two scans and transforming the source scan into the target scan. This process is an essential step in the applications such as 3D tracking, 3D model retrieval, 3D reconstruction and 3D animation.

Because the object can deform its shape between different scans, rigid registration cannot be applied to align two scans together. Rigid registration methods find only a single global rigid transformation and cannot solve for other local deformations. Therefore, non-rigid registration is required for dynamic objects since it solves for both global transformation and other local deformations. However, this task is still a difficult and ill-posed problem due to high-dimensional search space of parameters. Various methods impose constraints to transform the problem into a well-posed one. For example, a template can be

used as prior geometry, it can be assumed that the motion between two scans is small, artificial markers can be used as initial correspondences or the deformation can be approximately isometric.

In this paper, we present a method to align two surfaces of scanned dynamic objects without any prior knowledge on the deformation between scans or without simplifying assumptions either. Inspired by the method of (Cao et al., 2015) (sometimes referred to as TSA in this paper), which efficiently solves the non-rigid registration under large deformation between the surfaces, the method first finds a single global transformation and then optimizes other local deformations based on the deformation model proposed by Sumner (Sumner et al., 2007). However, the previous work is only applied on the surfaces for which a single patch of continuously connected triangular mesh is presented in order to compute geodesic distances when generating a graph of deformation model. The method proposed here also performs well on surfaces composed of different disconnected patches (as shown in Fig. 4) because the graph is created using the Euclidean distance. Moreover, the previous work uses twelve parameters for each node's affine transformation in the deformation graph. This causes the cost function to be complex and the solution space to be very large. The current work only specifies six parameters for each node in which three parameters are used for each translation

vector and each unit quaternion associated with each rotation matrix is parameterized completely by three parameters. With this improvement, the number of unknowns is reduced by half while the as-rigid-as-possible regularization for each node is achieved automatically. Consequently, the problem of non-rigid registration is better formulated in order to find the optimal solution.

2 RELATED WORK

Non-rigid registration is an emerging field in computer vision and computer graphics. In this section, we only review the state-of-the-art works that are the most relevant to our method.

In the process of 3D scanning of objects, holes, outliers and noise are inevitable issues of the acquisition process. So, to create a complete model, a template which has the same topology as the object is used as a prior geometry before being merged into the scanned data. The template can be a generic model (Allen et al., 2003; Zhang et al., 2004; Amberg et al., 2007) or it can be obtained during a static acquisition phase (Li et al., 2009; Zollhöfer et al., 2014). Because the templates may not be available for all types of models, the methods in (Chang and Zwicker, 2008; Huang et al., 2008; Sagawa et al., 2009; Cao et al., 2014) assume that local deformations are small to validate local feature descriptors used for finding some initial correspondences between the two surfaces in order to guide the registration process.

Applying the deformation model proposed by Sumner (Sumner et al., 2007), the methods in (Li et al., 2008; Li et al., 2013; Zeng et al., 2013; Zhang et al., 2014; Dou et al., 2015) register the deformation graph of the source surface with the target surface. Each node's displacement is specified by the twelve parameters of an affine transformation. Once the deformation graph aligns with the target surface, the remaining data of the source surface is transformed using linear blending-based interpolation. These methods require that the motion between the two surfaces to be aligned is small to avoid tangential drift during the optimization process. When the motion between two surfaces is large, a rigid transformation between them is required for the above techniques to work.

Instead of using affine transformations for points (nodes) of a sparsely sampled set, the methods in (Bonarrigo et al., 2014; Newcombe et al., 2015) specify only rigid transformations for these points and then use dual-quaternion blending for the interpolation step. The optimization process in (Newcombe et al., 2015) searches for the parameters of a global

rigid transformation and other local rigid transformations at each iteration. This strategy can cause the optimization to converge to a wrong minimum if the motion between the two surfaces is large. The work of (Bonarrigo et al., 2014) assumes that some initial correspondences are available before running the optimization. In approaches based on the signed distance field, the methods in (Rouhani et al., 2014; Zhang et al., 2015) do not need to find any correspondences for the registration process. However, these methods provide misalignment results when the surfaces are not watertight and clean due to the presence of holes, outliers or noise.

Some methods are based on probabilistic approaches for non-rigid registration. In (Myronenko and Song, 2010), Gaussian Mixture Model centroids (source set) are fitted to the data (target set) by maximizing the likelihood function. The registration problem is thus considered as a probability density estimation. In another work, the algorithm in (Jian and Vemuri, 2011) minimizes the statistical discrepancy measure between two Gaussian mixture models in which each model represents a point set. These methods impose a large-complex computational burden to identify the correspondences and local deformations. They are almost only applied to 2D data or small 3D data sets. In (Anguelov et al., 2004), a Markov Random Field optimization is formulated to establish the correspondences and find rigid local transformations between the two surfaces. Assuming that the geodesic distance between any two points is preserved before and after deformation and that the source surface is a subset of the target surface, the method can align the surfaces under large deformation.

3 REGISTRATION METHOD

3.1 Parameterization of Rotation

3D rotations are applied in algorithms in the context of computer vision and computer graphics to describe the transformations of objects under both rigid and non-rigid motion. Although a 3D rotation matrix has nine elements, it only has three degrees of freedom (DOF). When an algorithm uses an optimization process to find the parameters of such a matrix, it is important to use a minimal parameterization that reduces the computational burden and makes the calculation of derivatives relating to rotation matrix simpler.

Compared to other common parameterizations such as Euler angles and axis-angle representation, unit quaternion is preferable because it avoids ambiguous configurations such as gimbal lock. Although

a unit quaternion has four elements, it has only three DOF due to the unit norm constraint. In the optimization process, an efficient parameterization of a unit quaternion should use only three parameters, these three parameters should be allowed to change arbitrarily by the optimization process and the unit norm constraint should be satisfied automatically (Schmidt and Niemann, 2001).

From the above discussion, a unit quaternion parameterization of the rotation matrix proposed in (Terzakis et al., 2012) by exploiting the homeomorphic relation between the 4D unit sphere and the 3D projective space is exploited in our optimization process (section 3.2.2). The unit quaternion parameterization is described as follows.

As shown in Fig. 1, we assume that there is a unit quaternion $\mathbf{q} = (q_0, q_1, q_2, q_3)$ where q_0 is the real part and (q_1, q_2, q_3) are the imaginary parts. This quaternion can be considered as a hypersphere in 4D space and described by:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (1)$$

Let $S = (0, 0, 0, -1)$ be the south pole in this 4D sphere and π be the 3D equatorial hyperplane containing the origin of \mathbb{R}^4 . Let now $r(t)$, parameterized by t , be the ray from S that passes through any point (x, y, z) of the equatorial plane:

$$r(t) = (0, 0, 0, -1) + t[x \ y \ z \ 1]^T \quad (2)$$

The ray intersects the surface of the sphere at P . So, P is back-projected on π through the ray. Because P lies on both the sphere and the ray, its coordinates satisfy the following equation:

$$(tx)^2 + (ty)^2 + (tz)^2 + (t-1)^2 = 1 \quad (3)$$

From Eq. 3, t is obtained in terms of parameters x, y, z : $t = \frac{2}{1+x^2+y^2+z^2}$. Finally, by substituting the value of t into Eq. 2, one obtains the coordinates of a unit quaternion in x, y, z .

$$\mathbf{q} = \left(\frac{2x}{1+\alpha^2}, \frac{2y}{1+\alpha^2}, \frac{2z}{1+\alpha^2}, \frac{1-\alpha^2}{1+\alpha^2} \right) \quad (4)$$

where $\alpha^2 = x^2 + y^2 + z^2$

With this parameterization, a rotation matrix \mathbf{R} is obtained by using only three parameters (x, y, z) when combining Eq. 4 and Eq. 17 (as described in the Appendix). Therefore, in an optimization process, we do not need to impose a unit-norm constraint for a quaternion. This parameterization is specially desired when the number of optimized parameters is large as in non-rigid registration.

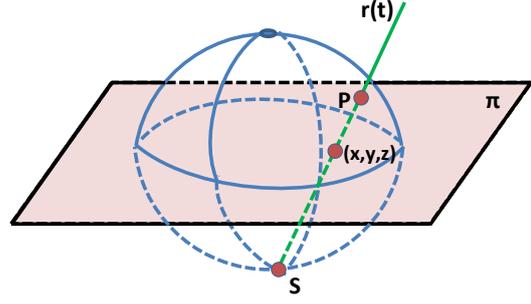


Figure 1: The 4D spherical hypersurface of unit quaternions pictured as a 3D sphere. Point S is the center of projection, (x, y, z) a point on the 3D equatorial hyperplane. The ray $r(t)$ intersects the surface of the sphere at P (Terzakis et al., 2012).

3.2 Non-rigid Registration

3.2.1 Deformation Model

The presented work is derived from the methods in (Sumner et al., 2007) and (Cao et al., 2015). For completeness, in this section, the main idea of the previous work is described and the changes are presented in the new algorithm (for more details, we refer the reader to the work in (Sumner et al., 2007; Cao et al., 2015)).

To register the source surface with the target surface, a graph of the deformation model is generated by sampling the source surface. Then, a 3D point of the graph is called a ‘node’ $\mathbf{g}_j (j \in 1 \dots m)$ and associated with an affine transformation (matrix \mathbf{R}_j and vector \mathbf{t}_j). Under the influence of a set of nodes, a 3D point \mathbf{v}_i is deformed as follows:

$$\tilde{\mathbf{v}}_i = \sum_{k \in \mathcal{N}(i)} \tilde{w}_k(\mathbf{v}_i) [\mathbf{R}_k(\mathbf{v}_i - \mathbf{g}_k) + \mathbf{g}_k + \mathbf{t}_k] \quad (5)$$

where $\mathcal{N}(i)$ is a set of neighbor nodes of \mathbf{v}_i and weight $\tilde{w}_k(\mathbf{v}_i)$ indicates the contribution of each individual node in the blended transformation of $\tilde{\mathbf{v}}_i$.

In the previous work, the goal of the registration process is to minimize the cost function $E = w_{rot}E_{rot} + w_{reg}E_{reg} + w_{pos}E_{pos}$ which is the sum of three energy terms $E_{rot}, E_{reg}, E_{pos}$ with respective weights $w_{rot}, w_{reg}, w_{pos}$. The first energy term, imposing the node’s transformation under a ‘as-rigid-as-possible’ constraint, is described by:

$$E_{rot} = \sum_{j=1}^m [(\mathbf{c}_{1j} \cdot \mathbf{c}_{2j})^2 + (\mathbf{c}_{1j} \cdot \mathbf{c}_{3j})^2 + (\mathbf{c}_{2j} \cdot \mathbf{c}_{3j})^2 + (\mathbf{c}_{1j} \cdot \mathbf{c}_{1j} - 1)^2 + (\mathbf{c}_{2j} \cdot \mathbf{c}_{2j} - 1)^2 + (\mathbf{c}_{3j} \cdot \mathbf{c}_{3j} - 1)^2] \quad (6)$$

where $\mathbf{c}_{1j}, \mathbf{c}_{2j}, \mathbf{c}_{3j}$ are column vectors of \mathbf{R}_j .

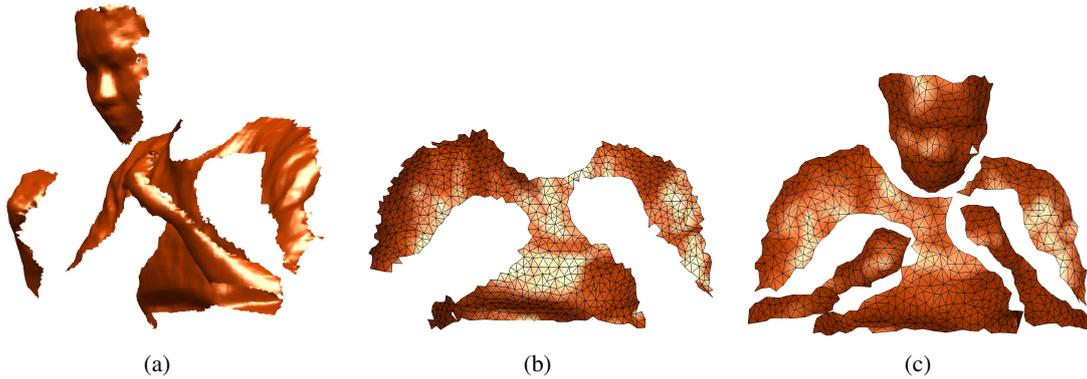


Figure 2: Generating a graph (sub-mesh) from original data: (a) original surface, (b) generated sub-mesh with some missing patches with TSA, (c) sub-mesh generated in our method.

The second energy term imposes the deformation graph to deform smoothly:

$$E_{reg} = \sum_{j=1}^m \sum_{k \in \mathcal{N}(j)} \bar{w}_k(j) \|\mathbf{R}_j(\mathbf{g}_k - \mathbf{g}_j) + \mathbf{g}_j + \mathbf{t}_j - (\mathbf{g}_k + \mathbf{t}_k)\|_2^2 \quad (7)$$

where $\mathcal{N}(j)$ is a set of neighbors of node j and weight $\bar{w}_k(j)$ is proportional to the degree of overlap between node j and node k .

The third energy term minimizes the deviation of a deformed point $\tilde{\mathbf{v}}_i$ on the source surface to its corresponding point \mathbf{u}_i on the target surface:

$$E_{pos} = \sum_{i=1}^{\mathcal{M}} \|\tilde{\mathbf{v}}_i - \mathbf{u}_i\|_2^2 \quad (8)$$

where \mathcal{M} is the number of points selected on the source surface.

The goal of the optimization process is to find $12m$ parameters to move the deformation graph toward the target surface. TSA divides the registration process into two stages. The first stage aims to find a single global transformation between the source surface and the target surface. This stage also determines *potential regions* in which a corresponding point \mathbf{u}_i of a point \mathbf{v}_i should lie. After bringing the two surfaces closer, in a second stage, the optimization process is implemented in three cycles to gradually change the deformation graph and align it with the target surface by updating new corresponding points, smoothing the correspondence field and measuring deformation distortion at each node and each enhancing point.

3.2.2 Re-parameterized Deformation Model

The graph of the deformation graph \mathcal{G} in TSA is generated using the re-meshing method proposed by (Peyré and Cohen, 2006) which keeps the topology

of the obtained graph the same as the source surface and also results in a small error at the interpolation step (Eq. 5). However, the graph can miss some patches if the source surface consists of some disjointed patches, as shown in Fig. 2(a), 2(b). This is because the sample distribution is based on the geodesic distance map in which the fast marching algorithm (FMA) is applied on meshes having continuous connectivity when calculating the geodesic distance. Hence, if FMA starts at a point on one patch, it may not reach another point on other patches due to discontinuity or gaps between patches. Consequently, samples cannot distribute on these patches.

The graph of the deformation graph \mathcal{G} in this paper is generated in a different way and is comprised of two steps. First, the source surface is sampled using the method proposed by (Corsini et al., 2012) which is based on a pre-defined radius r_s for sample distribution. The sample distribution can be chosen using the geodesic distance or the Euclidean distance. Compared with the use of the Euclidean distance, the use of the geodesic distance preserves the features better but is not robust in the presence of noise. In the second step, we apply the method of (Bernardini et al., 1999) to generate the graph. The maximum radius to connect two nodes in this step is 30% larger than r_s . If a node is not connected to any other node, a maximum of four nearest nodes is selected in the range of $2r_s$ as its neighbors. By doing so, each node may have a different number of neighbors depending on the local geometry of the surface. Normal vectors at nodes are calculated by the method proposed in (Chen and Wu, 2004). In our experiments, using the Euclidean distance radius in the sampling process is proved to be sufficient enough for alignment (Fig. 2(c)). Moreover, with this procedure, our method can be applied to point cloud data instead of meshes. Note that the way to generate the graph is also applied to produce a sub-mesh from the target surface.

Because TSA assigns the affine transformations to nodes, the number of unknowns ($12m$, m is the number of nodes) becomes very large. This can cause the optimization process to become trapped in a local minimum because non-rigid registration is an ill-posed problem. In (Sagawa et al., 2009; Bonarrigo et al., 2014; Zhang et al., 2015; Newcombe et al., 2015), rigid transformations ($6m$) are specified for sampled points before they are used to interpolate the entire data. Such deformation models can describe variable deformations such as those involving the human body, elastic objects, animals. Inspired from the aforementioned model in section 3.2.1, our method associates a rigid transformation for each node to reduce the computational burden for the optimization. Moreover, by doing so, the as-rigid-as-possible constraint (Eq. 6) is automatically satisfied.

Algorithm 1: Non-rigid registration algorithm.

Input: *source surface S , target surface \mathcal{T}*
 Output: *optimized parameters \mathbf{z}^* , deformed surface \tilde{S}*

I. First stage

- 1: Generate a deformation graph \mathcal{G} and a submesh
- 2: Determine initial correspondences
- 3: Find a single global rigid transformation \mathbf{T}

II. Second stage

- 1: Initialization: $w_{reg}^0 = 10; w_{fit} = 10; \mathbf{z}^0 = \mathbf{0}$
- 2: Optimization using a hierarchical scheme:
 Set $k=1; w_{reg}^k = w_{reg}^0; \mathbf{z}^{0k} = \mathbf{z}^0$

while ($w_{reg}^k > 0.01$) **do**

while (not converged) **do** (use LM algorithm)

- Convert each (x_j^n, y_j^n, z_j^n) to \mathbf{q}_j^n (Eq. 4)
- Convert each \mathbf{q}_j^n to \mathbf{R}_j^n (Eq. 17)
- Calculate deformed nodes \mathbf{g}_j^n
- Update corresponding points \mathbf{u}_j^n
- Smooth correspondence field E_s^n (Eq. 11)
- Measure distortion $M_{d_j}^n$ (Eq. 12)
- Remove wrong correspondences
- Update new value of E^n (Eq. 10)

end
 Set $w_{reg}^{k+1} = w_{reg}^k / 2; \mathbf{z}^{0(k+1)} = \mathbf{z}^{*k}$

end

3: Do interpolation (Eq. 5)

Although there are many parameterizations for a rotation matrix, using unit quaternion becomes a prominent approach due to its robustness and the use of a minimum number of DOF (i.e. 3). Unit quaternion parameterization from an axis-angle representation (Grassia, 1998) is widely used to parameterize a rotation matrix. However, when applied in non-linear optimization, derivative calculations of irrational functions (cosines, sines) are only approximated and may deteriorate the result of the optimization process. With this in mind, our method applies

the parameterization described in section 3.1 due to the robustness it brings to the stability of the optimization process (for more details see the Appendix). So, the proposed optimization process solves only for $6m$ unknowns (stored in a vector \mathbf{z}) consisting of 3D coordinates (x, y, z) and translation vectors (t_x, t_y, t_z) . The framework of our method is described in Algorithm.1.

In the work of (Cao et al., 2015), the deformation graph moves toward to the target surface by minimizing the Euclidean distance between the enhancing points and their corresponding points (Eq. 8) after the two surfaces are brought closely by a single rigid transformation \mathbf{T} . The corresponding points are found by searching for points in *potential regions*. In this paper, we do not use the enhancing points but directly use nodes of the deformation graph in a so-called fitting term:

$$E_{fit} = \sum_{j=1}^m \|\tilde{\mathbf{g}}_j - \mathbf{u}_j\|_2^2 \quad (9)$$

where $\tilde{\mathbf{g}}_j = \mathbf{g}_j + \mathbf{t}_j$ is the deformed node j at each iteration.

If a node has an initial corresponding point in the first stage (Cao et al., 2015), then this corresponding point is updated during optimization by searching a new point \mathbf{u}_j located in a sphere whose center is at the initial corresponding point and radius $r_N = 1.5h_{max}$ (h_{max} is the maximum distance between two points on the sub-mesh of the target surface). If a node does not have the initial corresponding point, we find a closest point on the target surface as a corresponding point at the current iteration.

The cost function for the re-parameterized deformation model is now expressed as:

$$E = w_{reg}E_{reg} + w_{fit}E_{fit} \quad (10)$$

Because each node's transformation is specified by a rigid transformation, the influence of a node on a point and the mutual influence between two nodes described by the respective weights $\bar{w}_k(\mathbf{v}_i)$ and $\bar{w}_k(j)$ (Eq. 5 and Eq. 7) is different from the case of an affine transformation for a node (Cao et al., 2015). In the current work, the weights are defined as $w_k(\mathbf{v}_i) = \exp(-d_i^2/2d_{max}^2)$, $w_k(j) = \exp(-d_j^2/2d_{max}^2)$ where d_i is the distance from \mathbf{v}_i to node k , d_j is the distance between node j and node k , d_{max} is the maximum distance between two nodes in the deformation graph. These weights are then normalized to sum to one according to the number of neighbor nodes (denoted as $\bar{w}_k(\mathbf{v}_i)$, $\bar{w}_k(j)$).

As in the work of (Cao et al., 2015), we use the Levenberge-Marquardt algorithm for the optimization process. However, we do not use the 3-cycle optimization process to find the unknowns, we rather use a hierarchical scheme (Algorithm.1). Initially,

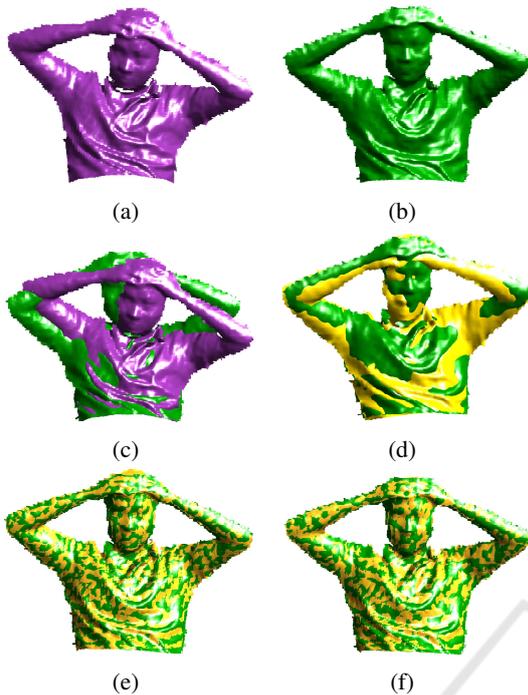


Figure 3: Alignment results on low resolution data: (a) source surface, (b) target surface, (c) before alignment, (d) alignment result of GMM, (e) alignment result of TSA, (f) our alignment result.

we set $w_{reg} = 10, w_{fit} = 10$. The optimization starts with these values until convergence conditions are satisfied: $|E^n - E^{n-1}| < 10^{-7}$ or $\|\mathbf{z}^n - \mathbf{z}^{n-1}\| < 10^{-6}$. Then, the value of w_{reg} is reduced by half and the optimization is initiated again. This procedure is repeated until the value of w_{reg} is less than 0.01. For each value of w_{reg} , the smoothing procedure is also applied for corresponding points of nodes at each iteration:

$$E_s(f) = \sum_{j=1}^m \|\mathbf{f}(\mathbf{g}_j^n) - \bar{\mathbf{f}}(\mathbf{g}_j^n)\|_2^2 \quad (11)$$

where $\mathbf{f}(\mathbf{g}_j^n) = \mathbf{u}_j - \mathbf{g}_j^n$ and $\bar{\mathbf{f}}(\mathbf{g}_j^n) = \frac{1}{|C_j|} \sum_{k \in C_j} \mathbf{f}(\mathbf{g}_k^n)$ is the mean of \mathbf{f} over C_j neighbors of \mathbf{u}_j (C_j consists of the points on the target surface in the sphere of radius r_N) and $\mathbf{g}_j^n, \mathbf{g}_k^n$ are node positions at the n^{th} iteration.

We also measure the distortion at each node:

$$M_d = \frac{1}{\mathcal{N}(j)} \sum_{k \in \mathcal{N}(j)} \frac{|d_{jk}^n - d_{jk}^0|}{d_{jk}^0} \quad (12)$$

where d_{jk}^0, d_{jk}^n are distances between nodes before deformation and at iteration n . If M_d is larger than the threshold \mathcal{T}_d (in our experiments, $\mathcal{T}_d = 0.2$), the node's transformation is imposed only by the regularization (Eq. 7).

Moreover, if a corresponding point lies on the boundary of the target surface or the angle between

Table 1: Statistics for comparison between the GMM method and our method.

		Fig. 3	Fig. 4	Fig. 5
Ours	# points on \mathcal{S}	69710	45942	113837
	# points on \mathcal{T}	70636	37761	113408
	# nodes	1090	902	1016
	Times(s)	452	307	487
GMM	# points on \mathcal{S}	17353	11677	25519
	# points on \mathcal{T}	17590	9651	27461
	Times(s)	2485	1476	5506

its normal and the node's normal is greater than 45 degrees, the corresponding point is not used in the fitting term at the current iteration.

4 EXPERIMENTAL RESULTS

Our method was implemented in MATLAB on a 3.2 GHz Intel Core i7 platform and its robustness was compared with the method proposed in (Cao et al., 2015) and another method (referred to as GMM in this paper) proposed in (Jian and Vemuri, 2011). For the implementation of GMM, its input data is obtained by decimating the original size to 25% using the method proposed in (Corsini et al., 2012). The statistics for our method and GMM is shown in Table 1.

In the first experiment, shown in Fig. 3, we asked a subject to sit in front of a Kinect camera. Because the camera provides low resolution data at each frame, the data at each pose is obtained by merging ten consecutive frames to collect more details. The data at the second pose is a deformed version of that at the first pose when the user twists the upper part of his body. The data is presented in triangular meshes and each mesh is comprised of a single patch. Using almost the same number of nodes for the deformation graph in our method and TSA (1020 of TSA and 1090 for ours), the optimization processes of the two methods achieve optimal values to correctly align the two surfaces (Fig. 3(e), 3(f)). Although the alignment result of TSA is slightly better than ours, our method using a re-parameterized deformation model is approximately four times faster (1658s of TSA and 452s for ours). GMM deforms the source surface (second pose) by twisting this surface toward the target surface (first pose). However, when the head of the source surface reaches that of the target surface, the optimization process stops at a local minimum and causes a misalignment between the two surfaces (Fig. 3(d)). Moreover, the computation time of GMM is five times higher than for our method although it runs on a reduced-size data set.

The data in the next experiment was also acquired by the Kinect camera (Fig. 4(a), 4(b)). In this exper-

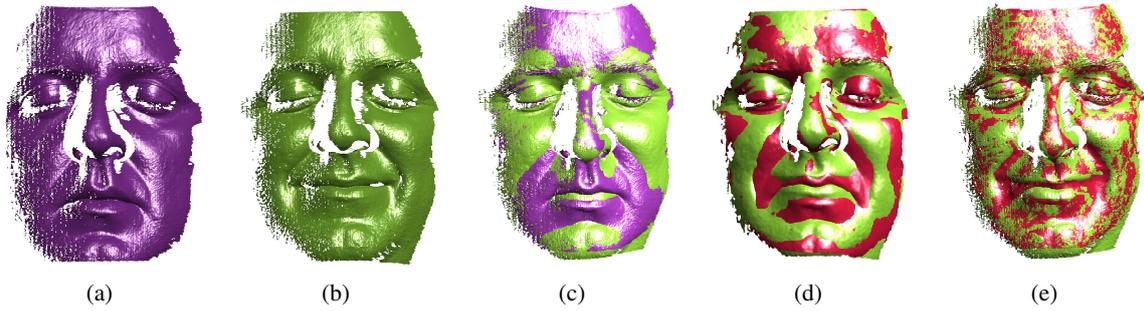


Figure 5: Alignment results between two surfaces of the disjoint connectivity: (a) source face, (b) target face, (c) before alignment, (d) alignment result of GMM, (e) our result.

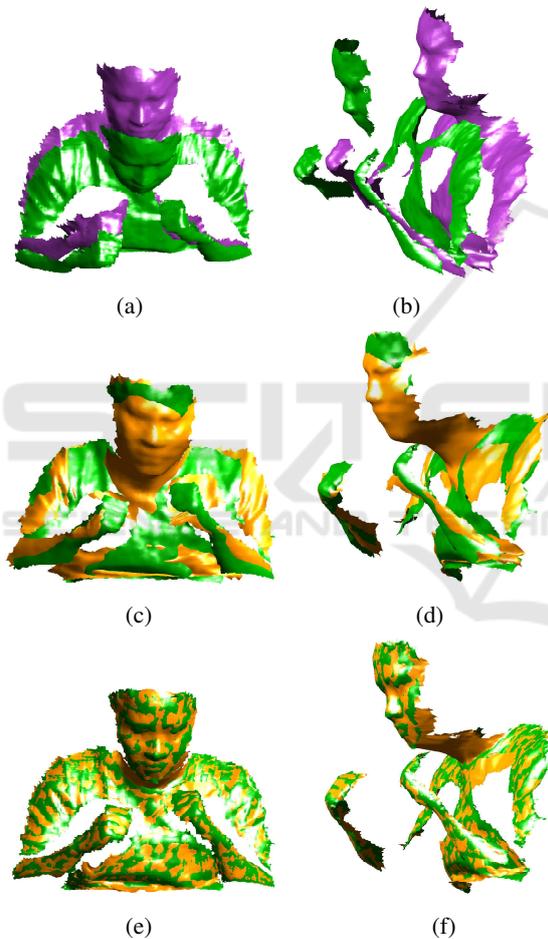


Figure 4: Alignment results for the data consisting of the separated patches: face, torso, arms (view 1, view 2): (a, b) before alignment, (c, d) alignment result of GMM, (e, f) our alignment result.

iment, the surfaces consist of different patches due to occlusion. In the TSA method, initial correspondences and potential regions in the first stage are not determined because the source surface (target surface) cannot be used to generate a graph of the deformation model (a sub-mesh), as shown in Fig. 2(b). TSA can-

not thus be applied in this case. GMM can be applied because it is used to register two point sets. However, GMM also provides a misalignment in the final result because the face is stretched and the left arm is shrunk when the source surface deforms toward the target surface (Fig. 4(c), 4(d)). In contrast, our method creates a graph of 902 nodes and a sub-mesh of 890 points and then aligns them together in the first stage. Based on the results of this first stage, in the second stage, the re-parameterized deformation model is applied to obtain a good alignment between the two surfaces (Fig. 4(e), 4(f)). As seen in the region around the neck of the subject on the source surface, our deformation process behaves elastically as the neck is elongated to align the two faces.

In the work of (Cao et al., 2015), the alignment between the two surfaces, which captures different expressions of a person’s face, is performed. It can be used in applications such as 3D games, dynamic face reconstruction or 3D face recognition. However, it is required that the mesh of each surface be a continuous mesh to create the sub-meshes. When one of the two meshes is a disjoint mesh, as shown in Fig. 5(a), 5(b), this method cannot be applied. The deformation between the two surfaces mostly occurs around the mouth and cheek region. GMM deforms incorrectly the mouth region of the source surface when aligning with one of the target surface (Fig. 5(d)). We apply our method and obtain the alignment result shown in Fig. 5(e). Around the mouth rim, the source surface does not align well with the target surface but this issue can be solved by increasing the number of nodes while allowing more time for the optimization process to converge. From the experiments, we also note that the computation time of GMM increases exponentially with the number of data points (Table 1) while that of our method depends mainly on the number of nodes which can be specified by the user when generating the graph of the deformation model.

In another comparison with TSA for aligning small details between two surfaces, we register the

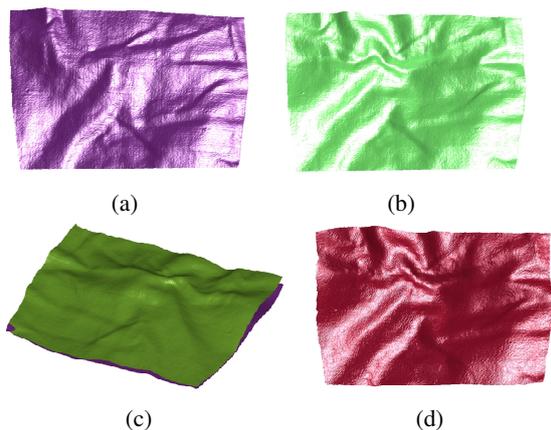


Figure 6: Our method deforms correctly to align small details on the pillow surfaces: (a) source surface, (b) target surface, (c) before alignment, (d) deformed version of source surface.

two surfaces of a deformable pillow, as shown in Fig. 6. This is high resolution data, the noise level is low but the surfaces have many small creases. We measure the root mean square error (*RMS*) based on the squared distance function (Cao et al., 2015)

$$RMS(\mathcal{S}, \mathcal{T}) = \sqrt{\frac{1}{|\mathcal{S}|} \sum_{i=1}^{|\mathcal{S}|} F_i^+} \quad (13)$$

where $|\mathcal{S}|$ is the number of points on the source surface and F_i^+ is value of squared distance function at point $\tilde{\mathbf{v}}_i$.

The deformed surface obtained with our method is shown in Fig. 6(d) and the comparison with the TSA method is shown in Fig. 7. In both methods, the source surface is deformed to align correctly with the target surface at small details (creases in this case). Both methods use about 1000 nodes in the deformation graph, our *RMS* error (mm) is greater than that of TSA (0.306 vs 0.164). However, this experiment shows that our method can obtain a good alignment with the presence of small details. We can accept a slightly larger error while the alignment process can be accelerated four times (597s vs 2212s).

In the last experiment, we register two high resolution partial models which are acquired at the eighth frame and the twenty-fourth frame by the system proposed in (Vlasic et al., 2009). For this data, we use a deformation graph with 1932 nodes and align it with the target surface. As shown in Fig. 8, our method with a re-parameterized deformation model is robust and can align two models without any prior and assumptions for non-rigid registration. This experiment demonstrates that our method can be a promising solution for 3D applications such as 3D dynamic reconstruction, 3D animation or 3D retrieval for dynam-

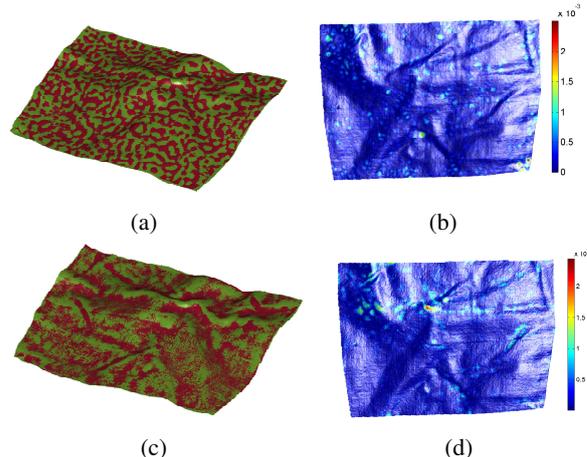


Figure 7: Registration of pillow data: (a,b) alignment result and error map of TSA, (c, d) alignment result and error map of our method.

cally deformable objects.

5 CONCLUSION

In this paper, we present an improvement to a robust non-rigid registration method. The main contribution of this work is to provide an alignment method that can be applied to the data captured by different types of 3D cameras with high frame rates such as low or high resolution data, partial data, data with small details and data under elastic deformation. The Euclidean distance is used in graph generation such that the method can be applied for meshes with disconnected patches. We also develop a registration framework in a re-parameterized deformation model in which the computation time is reduced and the optimization process is less complex. The proposed method outperforms the GMM method in alignment quality as well as computation time. Compared with the TSA method, the alignment quality implemented by the proposed method is almost as good and achieves acceleration of around four times for the alignment process. The presented method can be applied to different data types without any prior knowledge of the geometry of the objects or assumptions.

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Figure 8: Our method is applied on the data of dynamic-deformable objects: (a) source surface, (d) target surface, (b, e) two models before alignment (view 1, view 2), (c, f) our alignment results (view 1, view 2).

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APPENDIX

In iterative non-linear optimization process of our method, calculation of Jacobian matrix plays an important role for correctly updating the parameters at each iteration. This process relates to computing the derivatives of each rotation matrix \mathbf{R}_j with respect to a given value (x_j, y_j, z_j) of a point ψ_j on the equatorial plane.

The derivatives of a rotation matrix \mathbf{R} with respect to a point ψ are calculated using the chain rule as follows (for simplicity, we remove the index j):

$$\frac{\partial \mathbf{R}(\mathbf{q}(\psi))}{\partial x} = \sum_{n=0}^3 \frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_n} \frac{\partial q_n(\psi)}{\partial x} \quad (14)$$

$$\frac{\partial \mathbf{R}(\mathbf{q}(\psi))}{\partial y} = \sum_{n=0}^3 \frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_n} \frac{\partial q_n(\psi)}{\partial y} \quad (15)$$

$$\frac{\partial \mathbf{R}(\mathbf{q}(\psi))}{\partial z} = \sum_{n=0}^3 \frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_n} \frac{\partial q_n(\psi)}{\partial z} \quad (16)$$

A rotation matrix parameterized by a unit-quaternion is given by the following formula:

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad (17)$$

The partial derivatives of the rotation matrix with respect to the unit-quaternion are:

$$\frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_0} = 2 \begin{bmatrix} q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \quad (18)$$

$$\frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_1} = 2 \begin{bmatrix} q_1 & q_2 & q_3 \\ q_2 & -q_1 & -q_0 \\ q_3 & q_0 & -q_1 \end{bmatrix} \quad (19)$$

$$\frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_2} = 2 \begin{bmatrix} -q_2 & q_1 & q_0 \\ q_1 & q_2 & q_3 \\ -q_0 & q_3 & -q_2 \end{bmatrix} \quad (20)$$

$$\frac{\partial \mathbf{R}(\mathbf{q})}{\partial q_3} = 2 \begin{bmatrix} -q_3 & -q_0 & q_1 \\ q_0 & -q_3 & q_2 \\ q_1 & q_2 & q_3 \end{bmatrix} \quad (21)$$

The partial derivatives of unit quaternion with respect to the point $\psi = (x, y, z)$ are:

$$\frac{\partial \mathbf{q}(\psi)}{\partial x} = \begin{bmatrix} \frac{2}{1+\alpha^2} - \frac{4x^2}{(1+\alpha^2)^2} \\ \frac{4xy}{(1+\alpha^2)^2} \\ -\frac{4xz}{(1+\alpha^2)^2} \\ \frac{4x}{(1+\alpha^2)^2} \end{bmatrix} \quad (22)$$

$$\frac{\partial \mathbf{q}(\psi)}{\partial y} = \begin{bmatrix} -\frac{4xy}{(1+\alpha^2)^2} \\ \frac{2}{1+\alpha^2} - \frac{4y^2}{(1+\alpha^2)^2} \\ -\frac{4yz}{(1+\alpha^2)^2} \\ \frac{4y}{(1+\alpha^2)^2} \end{bmatrix} \quad (23)$$

$$\frac{\partial \mathbf{q}(\psi)}{\partial z} = \begin{bmatrix} -\frac{4xz}{(1+\alpha^2)^2} \\ -\frac{4yz}{(1+\alpha^2)^2} \\ \frac{2}{1+\alpha^2} - \frac{4z^2}{(1+\alpha^2)^2} \\ -\frac{4z}{(1+\alpha^2)^2} \end{bmatrix} \quad (24)$$