Post Flash Crash Recovery: An Agent-based Analysis

Iryna Veryzhenko¹ and Nathalie Oriol²

¹LIRSA, Conservatoir National des Arts et Métiers, 40 rue des Jeûneurs, 75002, Paris, France ²GREDEG-CNRS, University of Nice Sophia-Antipolis, 250 rue Albert Einstein, 06560 Valbonne, France

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Abstract: In this paper we focus on the traders that purely rely on algorithms in their decision making and their impact on market quality during moments of instability. We describe an agent-based framework that successfully reproduces main aspects of flash crash. We simulate the effect of a large liquidity shock generated by a very aggressive market order. We show that, despite the absence of market makers, the electronic order-book architecture favors market resiliency and recovery.

1 INTRODUCTION

Artificial intelligence-based applications for assets trading has completely reshaped financial markets design and end-users behaviors. The development of information systems trading over the centuries stimulated the emergence of financial clusters and located geographic proximity at the heart of recent trading strategies. At a microscopic level, the development of algorithms able to compute past charts set the basic tenets of automated trading systems. Human fundamentalists traders has been partly replaced by mechanical trading strategies based on various types of indicators designated as technical analysis strategies. At a macroscopic level, the 1970s saw the introduction of the first exclusive electronic stock exchanges (such as the NASDAQ in 1971), while the traditional open outcry call market's organization began to wane. Taking advantage of technological innovation, the double auction system based on an electronic order book (a dematerialized version of the open outcry blackboard hosted by an algorithm) started to be widely adopted by official stock exchanges. The shift from a physical artifact, such as a blackboard managed by humans, to a digital artifact, such as an algorithm managed by computers, led to a complete reorganization of the price discovery process and the coordination between traders. Interestingly, this recent process of digital transformation of trading desks and microstructure coincides with appearance of extreme events initiating the so-called "flash crashes era" for financial markets (Giles, 2012). We propose to study the role of agents' behaviors and market microstructure derived from artificial-based applications on flash crash emergence.

Agent-based models, which provide the conditions for a controlled experiment and allow to illustrate cause and effect relationships between initial scenario settings and model behavior, is a proper tool to study extreme events on the stock market, like flash crash phenomena. Moreover, the flexible character of simulation program allows to study separately the influence of electronic order book and automated chartists strategies, to test different regulator policies in order to prevent such market dynamics. To the best of our knowledge there is only few papers that study flash crash in an artificial market framework. A wide variety of traders are designed in the paper of (Vuorenmaa and Wang, 2014) to simulate the key aspects of the flash crash of May 6, 2010. The authors define stylized traders (fundamentalists, noise and opportunistic traders), institutional algorithmic trader and high-frequency traders (HFT). It is reported that HFT agents provide higher liquidity in normal trading conditions, but they become aggressive liquidity takers during the crash. As the number of HFT agents increases, so does the probability of flash crash. (Paddrik et al., 2012b), propose a similar agent-based model of the E-Mini S&P 500 futures market including fundamental traders, market makers, HFT, small traders and opportunistic traders. They conclude that a "hot potato" effect was generated by HFT. Alternatively, (Lee et al., 2011) or (Brewer et al., 2013) distinguish themselves from previous papers by their conclusions. (Lee et al., 2011) claim that extreme event problem might be less about

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HFT but rather about a dominant population of traders that are responding to a given set of market variables in similar ways. They conclude that any regulatory attempt to "slow down" trading may cause more problems than it solves. (Brewer et al., 2013) focus on the effect of large-volume order and change in fundamentals on the market quality. In this paper, market is populated only by zero-intelligence agents. They conclude that the nature and impact of liquidity erosion is sensitive to the market structure and that the exact nature of crash and book erosion depends on the structure of the order flow. Authors test different mechanisms to reduce the negative effect of flash crashes: introducing minimum resting time; shutting off trading for a certain period; switching to call auction market session, that is particularly effective.

Our paper is most closely related to those two lasts. We study the impact of market participants relying only on algorithm decision making on market quality during a liquidity shock. We argue that both microstructure and agents' behavior are part of the crash and the market recovery.

2 FLASH CRASH IN ARTIFICIAL MULTI-AGENT MARKET

To study flash crash phenomenon we use ArTifcial Open Market (ATOM) (Brandouy et al., 2013), which is highly flexible simulations platform and allows different parametrization of microstructure and traders behavior for different scenarios. This platform contains three main modules. The first one is market microstructure, that provides a mechanism for orders routing. ATOM clones the main features of the Euronext-NYSE stock exchange microstructure. Such market is ruled by a negotiation system between traders on a double auction machanism structured in an order book. The next system's component is economic world that provides exogenous information about corporate development, dividends and coupons changes. Finally, agents component encompasses multiple types of agents with different utility functions, believes and strategies. Trading strategies take into account exogenous information, functionality and rules of market microstructure, and endogenous information (post transaction information, generated by agents interaction).

2.1 Continuous Double Auction Market

For current experiments, we choose to set a continuous double auction (CDA) microstructure which is a replication of the widely used electronic order book. The CDA is a market protocol where agents place orders on separate buying (bid) and selling (ask) books. There are two main types of orders: limit and market orders. *Limit order* is an order to buy or sell a given quantity of stocks with a specific price or better. These orders are stored in the order book until their complete execution or end of validity. The highest bid is called best bid, the lowest ask is the best ask. A transaction occurs when best bid is higher than best ask. *Market order* is an order to immediately buy or sell a given quantity at the best available prices. A transaction occurs when market order hits the limit orders stocked on the opposite side.

A major aspect of the quality of market place is its *liquidity*. Liquidity is a possibility to realize immediately a big-size transaction without affecting a price in significant way. A market order for more shares than the size of best bid (ask) will execute at worse price until it is fully executed and will provoke significant price variations. This type of orders is considered as an aggressive one. Liquidity is directly related to market *resilience*, which is a speed of recovery of the market after a large shock, like big-size aggressive market order. We test these properties in the market populated by strategic agents and zero-intelligence traders.

2.2 Traders' Strategies

One of the goals of our paper is to determine the impact of strategic agents driven only by the technical signals on market quality (liquidity and volatility) during a liquidity shock. We wish to keep our model as simple as possible, to be able to catch such effects. For this reason we focus only on two types of traders, fundamentalists and chartists (technical traders).

Fundamentalists are driven by the true (fundamental) asset's value. The fundamental value of each stock evolves according to the jump process $V_t = V_{t-1} + \delta_t$, where $\delta_t \sim N(0, \sigma)$. As the agents are bounded rational (or noisily informed), the fundamental value is biased by ε_i , which determines the accuracy of the agent *i* to interpret the fundamental information $W_t = V_t + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma_W)$. Agents are heterogeneous with respect to their parameter ε_i . To make a buy/sell decision an agent compares the stock's current price P_t with fundamental value W_t . The price fixing mechanism is inspired from the paper (Chan et al., 1999).

The fundamentalists submit their order according to procedure described in Table 1. To summarize, agents buy undervalued stocks and sell overvalued stocks according to their beliefs. They stop trading when they are out of cash or stocks, if short selling is not allowed.

Table 1: The order-submission procedure. P_{ask} denotes a best ask price, P_{bid} best bid price, $alph_t$ ask tick size, β_t bid tick size, Q_t is a volume of the order issued at the moment t, S_{t-1} is a number of stocks hold by an agent at moment t - 1, C_{t-1} is available cash hold by an agent at moment t - 1, $U(x_1, x_2)$ the uniform distribution in the interval $[x_1, x_2]$.

Conditions	Order type			
Existing bid, existing ask				
$W_t > P_{ask}$	bid market order			
$W_t < P_{bid}$	ask market order			
$P_{bid} < W_t < P_{ask}$	bid/ask order with probability 50%/50% at price $\sim U(P_{bid}, P_{ask})$			
Order book is empty				
with probability 1/2	² limit ask order at $W_t + \alpha_t$, $Q_t \sim U(1, S_{t-1})$			
with probability 1/2	limit bid order at $W_t - \beta_t$, $Q_t \sim U(1, C_t/(W_t - \beta_t))$			
Empty bid side, existing ask				
$W_t > P_{ask}$	bid market order, $Q_t \sim U(1, C_t/P_{ask})$			
$W_t \leq P_{ask}$	limit bid order at $W_t - \beta_t$, $Q_t \sim U(1, C_t/(W_t - \beta_t))$			
Existing bid side, empty ask side				
$W_t < P_{bid}$	ask market order, $Q_t \sim U(1, S_{t-1})$			
$W_t \ge P_{bid}$	limit ask order at $W_t + \alpha_t$, $Q_t \sim U(1, S_{t-1})$			

Technical traders or chartists rely on algorithms to generate trading signals using historical price series or charts as a main source of information. The basic hypothesis of technical trading are as follows:

- Market is not efficient, price is impacted not only by a fundamental information but also by past trends.
- Price series follows the trends.
- History is repeating itself.
- Price reflects predictions and a common mood of traders.

A graphic is considered as a synthesis of market behavior. There exist upward and downward trends. Upward and downward confirmed trends determine buy and sell trades. Technical traders use the algorithms to determine a trend and to estimate whether the stock is underpriced of overprice compared to this trend. In other terms, they identify tradings signals based on past price movements (Murphy, 1999). Technical analysts focus on generating trading signals that provide a higher investment return. For these simulations we use three strategies of technical analysis, widely used by practicians and largely studied in theoretical and empirical literature and quoted as profitable on intraday data (Aldridge, 2013).

Momentum indicator compares the current price with the price in the past $D(t,n) = \frac{P_t}{P_{t-n}} \times 100$, where *n* is a historical window length, which is uniformly driven from an interval [50,500]. If D > 100, an agent sends a bid order, and if D < 100 the agent sends an ask order.

Simple Moving Average strategy determines a general tendency on the market. A price series is replaced by $Y_t = \frac{1}{n} \sum_{i=t-n}^{t} P_t \quad \forall t = n, \overline{N}$. They issue bid/ask orders when price P_t crosses either above or below

a long-period trend Y_t . The lower the length of the moving average *n* the more closely it tracks the price moving and more often it generates buy/sell signals. Longer moving average indicates overall direction of the market and reacts slower to market fluctuations.

Relative Strength Index strategy measures the strength of a trend.

$$RSI = \frac{U}{U+D} \times 100$$

U – an upward change, D – a downward change. The RSI indicator is plotted on a scale of 0 to 100. 0 represents the most oversold conditions and 100 the most overbought.

The population of chartist is heterogeneous with respect with their trading signals and their individual periods of position recalculation. Then, at each tick, you can have either trend followers or contrarian traders.

To decide a volume of new bid limit order an agent computes first of all a potential quantity $Q_t^{potential} = \lfloor \frac{C_{t-1}}{P_t} \rfloor$, the order volume is determined as $Q_t \sim U(1, \Delta \times Q_t^{potential})$, where Δ is borrowing rate. $\Delta > 1$ if borrowing is allowed. The volume of ask order is determined as follow $Q_t \sim U(1, \Delta \times S_{t-1})$, where Δ is a short selling rate. $\Delta > 1$ if short selling is allowed. Q_t is a volume of the order issued at the moment t, S_{t-1} is a number of stocks hold by an agent at moment $t - 1, C_{t-1}$ is available cash hold by an agent at moment $t - 1, U(x_1, x_2)$ the uniform distribution in the interval $[x_1, x_2]$

However, we recognize the importance of quantity as a choice variable and that our volume submission is a simplification of a real one, which depends on risk aversion.

Another important issue is how the limit price of chartists is determined, as it impacts market liquidity

and volatility. In each trading session, chartist agents trade near best bid and best ask. For this type of agents, we use the procedure inspired by price setting rules described in (Jacobs et al., 2004).

1. Bid price

$$P_{Bid_t} = P_{Bid_{t-1}} + \beta_t$$

where $P_{Bid_{t-1}}$ is the best bid price in the order book in t-1; β_t is a random value in the range [1; 5]: it means that best bid price at the moment twill be increased by value from 1 to 5 cents. P_{Bid_0} is equal to the previous day closing price. If agent gets an intention to buy stocks, she should check our the best bid order and increase it by a certain value β_t in order to set up her order on the top of the order book, decrease bid-ask spread and increase own chances to realize transaction. In other words, bidder is ready to pay up to 5 cents more than the rest of traders.

2. Ask price

$$P_{Ask_t} = P_{Ask_{t-1}} - \alpha_t$$

where $P_{Ask_{t-1}}$ is the best ask price in the order book in t - 1; α_t is a random value with the range [1; 5]: it means that best ask price at the time t will be decreased by value from 1 to 5 cents. P_{Ask_0} is previous day closing price.

This rule provides liquidity and reduce the *bid-ask spread* (difference between buy/sell prices).

In the condition of double auction market, a profitoriented buyer sets up the price lower his limit price because there would be a seller willing to accept this low bid price. Similarly, a seller sets a price higher his limit price, expecting that there would be a bidder ready to accept a high ask price. In condition of competitive market, the price comes closer to the market equilibrium price. As long as the buyer can undercut a competitor and still make a profit, he will add some insignificant amount to the last best bid price, similarly, seller will decrease the last best ask price by insignificant value, if it does not exceed his limit price.

2.3 Simulations and Results

Here we describe the model of market mechanism and agents interaction parameters we use in our experiments.

As in real market, trading occur asynchronously at discrete-time interval t = 1, 2...510, that represents a trading day of 8.5 hours at a minute granularity. At each round a trader is picked by the system to make a decision. A trader can have only one open position at the time and, therefore, before issuing a new order he should cancel an old one pending in the order book. The agents send limit, market and cancel orders. They also have a possibility to send a null order. In such a way we model different trading frequencies, and hence model realistic patterns of activity throughout the day.

According to study realized by (Paddrik et al., 2012a) there are around 2500 fundamentalists (buyers and sellers). Due to the large number of traders, we scale our simulations to 1/10 of the market, and populate our artificial market by 250 fundamentalists.

In the first experiment we run two scenarios: i) operation shock (without fundamental reasons) in the market populated by 250 fundamentalists only (these simulation results serve as a benchmark to study an impact of technical traders) ii) operation shock in the market populated by 100 fundamentalists, 50 momentum agents, 50 RSI agents, and 50 SMA agents. Each of these experiments we conduct consists of 100 runs, where each run begins with the same initial conditions (initial wealth, hold stocks, etc). In such a way, all statistics are averaged by over 100 repetitions. The main parameters of these experiments are detailed in Table 2.

Flash crash can be initiated by events and practices destroying liquidity. In two scenarios we cause a flash crash by introducing an aggressive market order like in the paper of (Brewer et al., 2013). In such a way we get an immediate effect on the market dynamics. Flash crash is produced by submitting a 20-time higher volume market order compared to average order size, that can be considered as operational error produced by a trader. Liquidity measures take about 20 best limit updates to return to their initial level (Degryse et al., 2005). For this reason an agent submits a 20-times higher volume market order in the expectation of matching about 20 best limits. This error is introduced in the middle of trading day, at 255th round.

We study the impact of this operational shock on the market liquidity and price dynamic. In both scenarios, ask market order destroys bid side liquidity and price falls rapidly. Just after this crash, bid side contains few orders, hence the market is at its most vulnerable and sensitive stage. High volatility period follows the crash. Figures 1(b) and 1(d) report an increasing of bid/ask spread, and consequently, a high volatility.

In the presence of intraday technical traders, which ignore the true value of the stock, the market crash is deeper (for the same shock, the speculative market loses on average 26.5%, while fundamentalists market declines on 12.3%) because speculative strategies bring down the prices. Downward trend is quickly explored by chartist, which exacerbate the

Parameter	Value	Description
N _{fund}	i) 250 ii) 100	Number of fundamentalists
N _{tech}	i) 0 ii) 150	Number of technical traders
$C_{0,i}$	[1 000 000; 2 000 000]	Initial cash attributed at moment 0 to the agent <i>i</i>
$S_{0,i}$	[100; 10 000]	Number of stocks attributed at moment 0 to the agent <i>i</i>
Nrounds	510	Number of rounds per day
\bigtriangleup	2	Short selling and borrowing are allowed with respect to
		the portfolio constraints: $\overline{\omega}_s \in [-1, 1]$ and $\overline{\omega}_c \in [-1, 1]$,
		where $\overline{\omega}_s$ and $\overline{\omega}_c$ are respectively stock and cash weights
		in total portfolio

Table 2: Parameters and their initialization used in simulations.

already declining prices. As far as downward trend is registered, some part of speculative traders cancel their old bid orders to submit the new ones with lower limit price, the other part takes short position on this stock.

In the market of fundamentalists, the crash depth is determined by the initial state of the order book. The crash is finished as far as market order is over. If a large majority of agents (60%) follows speculative strategy bid side of order book is negatively impacted by orders canceling and short sales. It provokes liquidity problems and makes the correction more complicated. As far as fundamentalists start actively buying the stocks (as it is undervalued) the stock price goes back up but high volatility is registered due to high bid/ask spread.

Next, we study the statistical properties of stock market before and after flash crash in the two scenarios. We particularly focus on volatility as risk measure (Grouard et al., 2003). A price series is divided in two subsequences before and after the crash. The first series is represented by $P_1, P_2, ..., P_k$, where k is a moment of big-volume ask order arrival. We calculate returns based on this price series $r_1, r_2, ..., r_{k-1}$, where $r_i = ln(P_{i+1}) - ln(P_i)$. This series represents a period before the crash. The second series is based on the prices $P_{k+m+1}, ..., P_n$, where $P_{k+1} > P_{k+2} > ... > P_{k+m}$ and $P_{k+m} < P_{k+m+1}, m$ is the time length of the flash crash. Next we calculate the series $r_{k+m+1}, r_{k+m+2}, ..., r_{n-1}$ which represents a series after the flash crash.

In table 3, we show the mean and standard deviation of each subsets along with higher order statistics such as skewness and kurtosis for historical intraday returns. We report increased volatility just after the crash: in the market populated by fundamentalists increases on average by 20% (from 0.003777883 to 0.004609726), in the mixed market standard deviation jumps (from 0.003969339 to 0.005433639). Next, we estimate skewness and kurtosis as well. In the market populated by fundamentalists only, the coefficient of asymmetry is negative (from -6.138734 to -2.496489, that indicates a high probability of extreme loss, which decreases after crash), this coefficient is positive in the mixed market (it varies from 1.807211 to 3.125376, that means a high probability for extreme loss, which increases after crash).

Table 3: Summary statistics of log returns before and after flash crash. All statistics are averaged by over 100 repetitions.

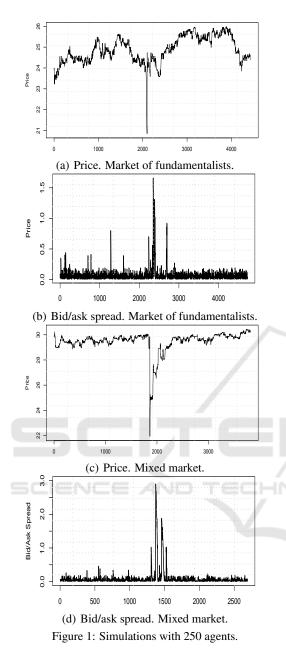
	Market of fundamentalists		Mixed market	
	Before	After	Before	After
Mean	-0.0001	4.5420e-05	-1.9091e-06	0.0002
Sd	0.0038	0.0046	0.0039	0.0054
Skewness	-6.1387	-2.4965	1.8072	3.1254
Kurtosis	131.1714	38.6699	38.76373	77.4977

The results provide the evidence that in presence of chartists the operational shock affects a price dynamic in a more significant manner. In addition, this practice reduces resiliency properties of market and makes the price rapid convergence towards its fair value more difficult. In the next section we focus on the market microsctructure mechanisms and their effect on the price dynamic during an operational shock.

3 THE ROLE OF MARKET MICROSTRUCTURE

The literature indicates that many stylized facts and price patterns are due to the market microstructure and not to some sophisticated traders strategies. Good market performance should not be automatically attributed to traders rationality and intelligence, it can be also explained by market mechanisms (auction) (Gode and Sunders, 1997). In this section, we follow the spirit of Gode and Sunder's (1993) "zerointelligence" traders (ZIT) to study the role of market micro-structure in market recovery after flash crash. To address this question formally, we use artificial market populated only by zero-intelligence agents.

For traceability reasons, the market is populated



only by 15 zero-intelligence traders, who negotiate only one asset. The fixed price is a result of orders submitted to the order book by traders. The price of an order is randomly selected from the interval $[P_{min}, P_{max}]$. That are settled to 43 and 46 respectively. This interval initially communicated to all agents and stay constant over simulations. We define two subgroups of agents which differ with respect to their traded volume: "Small fishes" and "Big fishes" (send 10-times higher volume orders). *Big fishes* "feed" *Small fishes*. In other words, *Small fishes* can easily buy or sell stocks with a price close to the current market price, until a big order is completely exe-

Table 4: Zero intelligence trader's decision making. P_{min} , P_{max} , V_{min} , V_{max} minimum and maximum prices and volumes, which are input parameters for algorithm.

Conditions	Order type	
with probability 1/2	limit ASK order at	
	price $\sim U(P_{min}, P_{max})$	
	volume $Q \sim U(V_{min}, V_{max})$	
with probability 1/2	limit BID order at	
	price $\sim U(P_{min}, P_{max})$	
	volume $Q \sim U(V_{min}, V_{max})$	

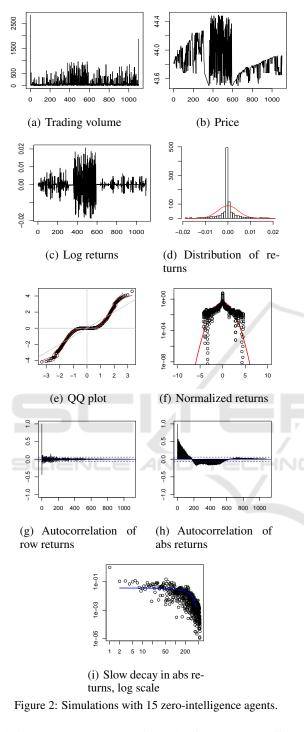
cuted. That generates patters of price stability. "Small fishes" and "big fishes" represent 33% (1/3) and 67% (2/3) of the total population respectively. This proportion is explained in (Brandouy et al., 2012). Small fishes determine the trading volume arbitrary from the interval $[V_{min}, V_{max}]$. The order's volume of big fishes is uniformly driven from the interval $[V_{max}, 10 \times V_{max}]$. V_{min} and V_{max} are defined as 5 and 50 respectively. The ZIT decision making is presented in Table 4

There are two possible order types: limit, market. Market order is used only once by an agent to generate a market crash and to get an immediate effect on the market dynamics. The rest of the agents submits only limit orders, with no expiry.

Each agent can buy or sell the asset with the same probability. The short selling is allowed, so the agents can conclude the transaction even if they don't have enough cash to do so.

Figure 2(b) reports a typical picture of the price series generated by the population of zerointelligence agents. The crash is followed by the highvolatility regime over about 200 ticks. The upward trend comes just after the high-volatility period. We also report a significant reduction of volatility. The market mechanism explains this phenomenon. Over first 200 ticks big fishes provide liquidity and permit the market to stay quite stable. Due to operational error a new big-size market order appears in the ask side. This order immediately matches all Bid orders pending in the order book as well as new incoming orders. While the big-size ask order is not totally fulfilled we observe the high-volatility regime. As bid/ask orders arrive with the same probability and only "ex-market" order takes part in transaction, ask side becomes deep and dense. As far as market order is completed, other ask orders pending in the order book can be proceed. According to the timestamps priority, the concluded transaction price corresponds to the ask order which generates a transaction. As a result, market goes up that can be considered as the market "autocorrection".

Figure 2(h) reports *unusual behavior* of slow decay of autocorrelation in absolute returns $|r_t|$ over 174 lags that is followed by a cluster of negative corre-



lations. As one can see from the figure, the coefficients are significant over long period which supports the so called mean-reversion behavior of stock market returns. These results are in agreement with observations reported in (Cont, 2007). This non trivial behavior of autocorrelation is also reported in the absolute returns of the real individual stocks during the flash crash on April 23, 2013. This observation is remarkably stable across S&P 500 stocks. This phenomenon can be explained by the fact that positive effects of past order flows on current prices are reinforced during periods of high stress (Cohen and Shin, 2002). These results show that the electronic order book and the market microstructure's role is determinant in the post-flash crash recovery.

4 CONCLUSIONS

This paper aims at studying the flash crash caused by an operational shock with different market participants. In particular, by including automated trading strategies and electronic order-book microstructure, simulations give some insights about artificialintelligence applications role in extreme disruptions. We reproduce this shock in artificial market framework to study market quality in different scenarios, with or without strategic traders. Trading interactions within a group of technical traders make a shock reflected in the price dynamic deeper compared to the similar shock in the market populated only by fundamentalists. We find that flash crash is much more a feedback trading issue than a single population of trader problem. Additionally, we study the role of market microstructure on the prices collapse and recovery. We show that, despite the absence of market makers, the order-driven market is resilient. The intraday continuous trading and the electronic doubleauction mechanism favor price recovery. Finally, flash crash anatomy is an outcome of complex interaction between three key-ingredients: agents strategies, liquidity withdrawal, and pre-existing microstructure features.

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