Time-optimal Attitude Reorientation Research of a Rigid Spacecraft

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Abstract: The modified switching time optimization algorithm is used to solve minimum-time problem for the rest-to-rest reorientation of a non-inertial asymmetrical rigid-body spacecraft. The essential conditions for solving the problem is inducted with the minimum value principle. Based on the idea of homotopy algorithm, the relaxation time factor introduced into the Genetic algorithm, which is optimized to determine the switch features without the gyroscopic coupling. The improved Simulated Annealing method (Simulated Annealing SA) to determine the optimal trajectory of the switch point. Computer simulation results show its practicability.

1 INTRODUCTION

Space missions have higher request for spacecraft attitude, of which the accurate posture keeping ability is a basic need. In order to perform scheduled tasks, aircraft must have a certain attitude. There are some articles about the shortest time attitude adjustment (Li, and Bainum, 1990). The spacecraft can be classified as three kinds according its structure, which are perfectly rigid body, rigid body with some flexible parts and gyroscopic system. It is meaningful to research the shortest time three dimensional maneuvering of rigid spacecraft (Bilimoria and Wie, 1993). On the one hand, there is potential for space applications. Future spacecraft requires quick reorientation in the limited thrust to solve a variety of tasks, for example the rescue or defense and dodge something. On the other hand it has the academic value, and it is a basic problem in attitude dynamics (Bai and Junkins, 2009). The rigid body is the simplest model of spacecraft, and it is the base of complex multi-body, flexible and charging model of t-he spacecraft.

2 THE BASIC PRINCIPLE OF RIGID BODY MOVEMENT AROUND THE FIXED POINT

Before we discuss the problem of rigid body movement around the fixed point, we must choose attitude parameter, and give the attitude dynamics equations and attitude motion equation. Quaternion overcomes the shortcomings of the singularity of Euler angle method, and the attitude dynamics equations expressed by Quaternion is linear equations. So the Quaternion solution is adopted in this article.

$$\boldsymbol{I}\cdot\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times(\boldsymbol{I}\cdot\boldsymbol{\omega})=\boldsymbol{M} \tag{1}$$

$$\begin{cases}2\dot{q}_0=-\omega_x q_1-\omega_y q_2-\omega_z q_3\\ 2\dot{q}_1=\omega_x q_0+\omega_z q_2-\omega_y q_3\\ 2\dot{q}_2=\omega_y q_0-\omega_z q_1+\omega_x q_3\\ 2\dot{q}_3=\omega_z q_0+\omega_y q_1-\omega_x q_2\end{cases} \tag{2}$$

Where $\boldsymbol{I}$ is central inertia tensor with a 3×3 moment of inertia, and $\boldsymbol{\omega}$ is the rotating angular velocity of the stars in space. $\boldsymbol{M}$ is the resultant moment of force relative to the center of mass. ω_x、ω_y、ω_z are projections of $\boldsymbol{\omega}$ in local coordinate system respectively.

3 THE OPTIMAL CONTROL PROCESS

The least time maneuvering problem can be solved

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based on the minimum theory, which can deduce the necessary conditions of solution. These conditions form two-point boundary value problem, solved by numerical algorithm. The optimal control process can be confirmed after getting the solution meet the necessary conditions.

3.1 The Necessary Conditions of the Least Time Reorientation

When the moment of inertia is zero, the formula (1) can be transformed as

$$\begin{cases} \dot{\boldsymbol{\omega}}_1 - \boldsymbol{D}_1\boldsymbol{\omega}_2\boldsymbol{\omega}_3 = \boldsymbol{u}_1 \\ \dot{\boldsymbol{\omega}}_2 - \boldsymbol{D}_2\boldsymbol{\omega}_1\boldsymbol{\omega}_3 = \boldsymbol{u}_2 \\ \dot{\boldsymbol{\omega}}_3 - \boldsymbol{D}_3\boldsymbol{\omega}_1\boldsymbol{\omega}_2 = \boldsymbol{u}_3 \end{cases} \quad (3)$$

When the original state and the termination state are both stationary, the least time reorientation problem can be described as: search the way to control $u^*{}_1, u^*{}_2, u^*{}_3$, and make the dynamic system decided by formula (2) and (3) change from the original state $(q_{0,}q_{1,}q_{2,}q_3)_{t_0}$ to the termination state $(q_{0,}q_{1,}q_{2,}q_3)_{t_f}$, getting the least value objective function

$$\boldsymbol{J} = \int_{t_0}^{t_f} dt \quad (4)$$

At the same time, the control constraint should be meet

$$-u_{i,\max} \le u_i \le u_{i,\max} \qquad (i = 1,2,3) \quad (5)$$

Where $u_{i,\max}$ is the greatest of u_i.

According the minimum value principle, the select optimum control parameters should make the Hamilton function the least. Firstly we define function Si related with the control process.

$$s_i = \frac{\partial H}{\partial u_i} \quad (i = 1,2,3) \quad (6)$$

The optimal control logic has relations following:

$$u^*{}_i = +u_{i,\max}, \quad if \quad s_i < 0 \quad (7)$$

$$u^*{}_i = -u_{i,\max}, \quad if \quad s_i > 0 \quad (8)$$

$$u^*{}_i = u_{i_s}, \quad if \quad s_i \equiv 0 \quad (9)$$

where $-u_{i,\max} < u_{i_s} < +u_{i,\max}$, and u_{i_s} is the singular controls corresponding with S_i ways being 0.

In the non-singular segment, each component of the control vector uses the boundary value to constitute a maximum torque control, which is named as Bang-Bang control. However, the optimal parameter can't be defined during the singular segment. It must be noted that, the case of non-singular means neither the nonexistence of optimal time control, nor the undefinable of the optimal time control, but it means only that the exact relationship between $u^*{}_i$ and $q^*{}_i$, λ and t can't be deduced using the control equations. Based on the system physical properties, the greater of the control moment mode, the faster the revolution, and the time may be shorter. So during the singular segment maximum torque control is adopted.

3.2 The Solution of the Least Time Reorientation Problem

In the process of the optimal control, the control parameters are in a state of extreme value. So the key is to decide the switching point. And the solution of the primary question is changed to search a time when control torque change from one situation to another.

In this paper, the basic idea is to using modified switching time optimization algorithm to solve static - static shortest time problem of non-inertial asymmetrical aircraft, ignoring singular segment. The exact approach is firstly to use genetic algorithms to determine the switch characteristics of no coupling term, and the Simulated Annealing algorithm is used to determine the switching point, using relaxation time factor.

3.2.1 Relaxation Process

The homotopy method to search the zero point for nonlinear algebraic problem is based on topology theory of continuous. Assuming nonlinear equation $f(x)=0$, f can be decomposed as

$$f = f_s + f_t \quad (10)$$

f_s is the simple solution, while f_t is difficult solution.

Then we can define an auxiliary function $g(\alpha, x)$

$$g(\alpha, x) = f_s(x) + \alpha f_t(x) \quad (11)$$

Where

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$$g(1,x) = f_s(x) + f_t(x) = f(x) \qquad (12)$$

The homotopy method is, searching the known solution from original value when α=0 to 1. When α=1, the solution is the answer off.

The homotopy method needs to using relaxation parameter ε

$$0 \le \varepsilon_n \le \quad (0 \le \varepsilon_1 < \varepsilon_2 < \varepsilon_3 \cdots < \varepsilon_n = 1)$$

Which make formula (3) to

$$\begin{cases} \dot{\omega}_1 - \varepsilon D_1 \omega_2 \omega_3 = u_1 \\ \dot{\omega}_2 - \varepsilon D_2 \omega_1 \omega_3 = u_2 \\ \dot{\omega}_3 - \varepsilon D_3 \omega_1 \omega_2 = u_3 \end{cases} \qquad (13)$$

It is a special situation when $\varepsilon = 0$, and its solution is relatively easy to obtain as for the inertial body symmetry spherical rigid object. Relaxation process begins from the existed boundary conditions and solution, to a series of solution of different dynamic equations, began to linear dynamics equation, finally reached for nonlinear dynamic equations of the problem. By appropriate control of the relaxation ratio, each iteration can get a solution close to the initial iteration. That is to say, in the process of gradual change of relaxation parameters, the objective function must be ensured to achieve a certain requirements.

3.2.2 Seek Switch Features

Multiaxial time optimal control problem is very complicated since the existence of dynamic coupling, so the analytical solution can't be obtained even for the symmetry spherical rigid object. The only way to solve it is using numerical method.

The step-by-step method is used to solve the multiaxial time optimal control problem. Firstly to make clear the switching property using genetic algorithm the when $\varepsilon = 0$. Calculation steps are as follows.

a) Randomly generate initial population. The initial state of the controlling variety is positive in the value when BZ=1, and the state changes once coming across a state switching point; when BZ=0, The initial state of the controlling variety is negative.

b) Calculate the adaptability

$$\begin{aligned} p = {} & c_q \,|\, q_{0f} - q_{00} \,| + c_q \,|\, q_{1f} - q_{10} \,| \\ & + c_q \,|\, q_{2f} - q_{20} \,| + c_q \,|\, q_{3f} - q_{30} \,| \\ & + c_\omega \,|\, \omega_{1f} - \omega_{10} \,| + c_\omega \,|\, \omega_{2f} - \omega_{20} \,| \\ & + c_\omega \,|\, \omega_{3f} - \omega_{30} \,| + c_{tf} \,|\, t_f - t_0 \,| \end{aligned} \qquad (14)$$

Where c_q, c_ω are weight coefficient, t_0 is reference value, q_{i0} and ω_{i0} is optimization value. Given control function u, using fixed step method to integrate the state equation, we can get the value of *p*.

c) Census to choose, exchange and variate to make new population. Calculate the optimal adaptability to make sure if it meets the index requirements. If it is, turn to Step d), or turn to step b) if it is smaller than M. Turn to Step a) if it is greater than M.

d) Export the result of calculation. The program is over.

The final step to make sure the optimal trajectory switching point of every ε_i is calculated using simulated annealing algorithm. During the calculation process, the relaxation parameter is small, so the optimal trajectory change slowly. And the selection of the original temperature T_0 and the persistent time can be a minor value to decrease the calculation.

4 NUMERICAL SIMULATION

Firstly simulate the optimal time control of homotaxial, calculating it using genetic algorithm. The results matches the theatrical value.

Since the complexity of the optimal time control of multiaxial, the spherically symmetric rigid body is discussed firstly using numerical simulation method. When we use the genetic algorithm, firstly setting the original population is 1000, the variation rate and crossing-over rate is 0.04 and 0.2 respectively. There are 3 switching points for every controlling process. When the seed adaptability becomes stable, fix two at a time till everyone is the same. The example is as following.

$$\begin{cases} \dot{\omega}_1 = 1 \\ \dot{\omega}_2 = 1 \\ \dot{\omega}_3 = 1 \end{cases},$$

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$$\omega_1(0)=\omega_2(0)=\omega_3(0)=0\,,\quad q_0(0)=1$$
$$\omega_1(t_f)=\omega_2(t_f)=\omega_3(t_f)=0\,,$$
$$q_1(t_f)=q_2(t_f)=0\,,\quad q_3(t_f)=\sin(\frac{\theta}{2})\,,$$
$$q_0(t_f)=\cos(\frac{\theta}{2})$$

The rotation angle of θ with eigenvector is named as reorientation angle, and it is originally equals to 180 degrees. It can be known that from the boundary conditions, the 3 coordinate axises of local coordinate system are same as reference coordinate system in the beginning time. After the adjustment, it is only same of the O_bZ and O_eZ axises. Consequentially there are 5 switching points for controlling parameters, of which the control moment of the O_bZ axis changes once. The total controlling time is 3.243 seconds, and it is 8.5% less than rotating with the eigenvector axis, being the same as reference (Bilimoria and Wie, 1993). Simulations above are focus on the equal main inertia moments. However, mostly the main inertia moments are unequal and small. Simulations below are for this situation.

Table 1: Time to change the switch condition.

Time(s)	ε= 0	ε= 1
t_1	11.3489	9.30363
t_2	17.1272	11.5518
t_3	6.3945	8.1374
t_4	8.6427	8.8572
t_5	10.7400	7.0771
t_6	17.1435	15.6462
t_f	34.2746	31.5866
Where the rotation time around the characteristic shaft: t_1=17.7439; t_f =35.4878		

The parameters are changed as A=150, B=50, C=100, M=1. In this situation, the results are obvious different. Table 1 and Fig. 1~3 show the calculation results, where $\Delta\varepsilon = 0.05,\quad h = t_f / 4000$. Fig.1 shows the trajectory of angular velocity, and Fig.2 shows the trajectory of quaternion. Fig. 3 shows the trajectory of switch parameter t_f during the relaxation process. At last the optimal control ratio reduce 10.9% compared with rotating eigenvector axis.

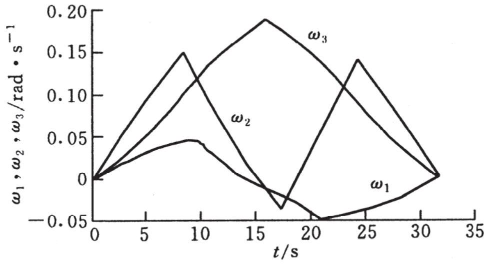

Figure 1: Trajectory of angular velocity.

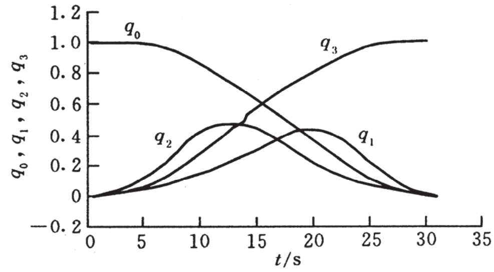

Figure 2: Trajectory of quaternion.

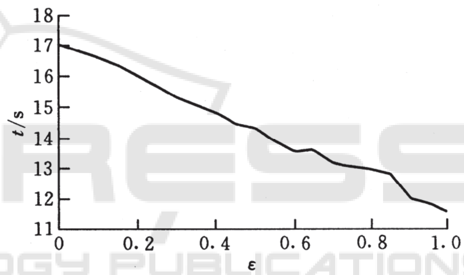

Figure 3: Trajectory of switch parameter t_f .

5 CONCLUSION

This article propose a method to solve the least time reorientation of rigid body. Compared with existed accomplishment, it doesn't need the original value by hand, and it has better astringency. Simulations show that the least time reorientation problem can be solved with different main inertia moment. Conclusions are: the optimal control trajectory is not unique for completely equal main inertia moment; when the dynamics equations are linear, time control for each axis is half positive and half negative; when the main inertia moments are different, the numbers of control switch are uncertain with same rotation angle, and the switch characteristics is the function of reorientation angle; the existence of coupling term make $t_f\big|_{\varepsilon=0}$ > $t_f\big|_{\varepsilon=1}$. Further research needs to be done including

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the asymmetric information i.e. the existence of inertia product and with different switch features.

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