A New Approach to Aggregation of Inconsistent Expert Opinions

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Abstract: The aim of this paper is to present a new way of aggregation two expert opinions. These opinions are disjoint and inconsistent, thus it is difficult to find a common solution using currently known methods. The authors suggest using horizontal membership function and RDM (Relative Distance Measure) method to get complete and unambiguous result. A general outline of this approach is presented and its equations are shown. The numerical example is given to illustrate the efficiency of the proposed method to practical issues in decision-making problems.

1 INTRODUCTION

Aggregation of data items delivered by various sources, of expert opinions, of measurements from various sensors and measuring instruments is at present intensively investigated because of the tendency to automate decision-making. However, this task is very difficult because aggregated data items usually are uncertain (expressed as distributions of possibility or probability density) and they are more or less inconsistent. Aggregation of e.g. few expert opinions expressed in forms of distributions consists on determining of one distribution which in the best way represents the experts' opinions. If the distributions are at least partly consistent (their supports have common part but models are not identical) then some methods in the subject literature can be found, which allow for aggregation (Dubois, 2004). If the opinions are considerably inconsistent and have no common range then the standard aggregation methods are e.g. AND, OR-operations, linear opinion pooling (O'Hagan, 2006). Unfortunately they give strongly disputable results, which rather cannot be applied in practice. Aggregation of inconsistent expert opinions, when the quality of the experts is unknown, can be understood as: a) a possibility distribution derived from experts distributions with certain required conditions imposed by an expert; b) a possibility distribution derived from experts distributions representing their opinions according to the accepted criterion of optimality (sum of absolute errors, sum of squared errors, etc).

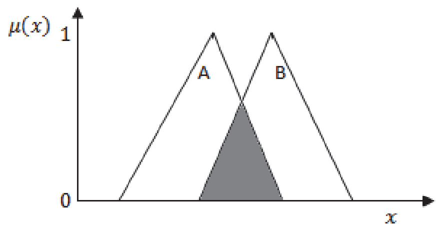

Figure 1a: AND (MIN) operation for joint opinions.

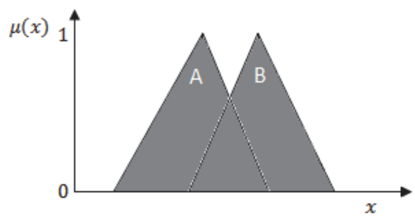

Figure 1b: OR (MAX) operation for joint opinions.

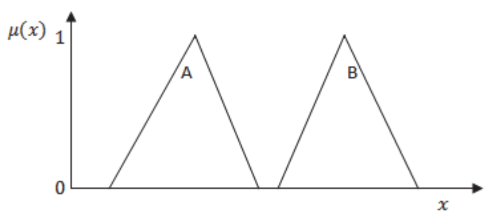

Figure 1c: Disjoint opinions.

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However, inconsistency of expert opinions occurs frequently (Beg, 2013; Herrera-Viedma, 2004; Son, 2014) and has to be solved because it concerns not only decision-making by people but also by technical devices as automatic alarms, automatic airplane defence devices, controllers (Gegov, 2015). There are several aggregation strategies for combining different estimates, including: null aggregation, intersection, envelope, Dempster's rule and its modifications, Bayes' rule and logarithmic pool but they are Type-1 methods and used only when the borders of fuzzy numbers/intervals are certain (Ferson, 2003). A new and interesting possibility of inconsistent opinions aggregation opens combination of fuzzy sets Type-2 theory (FST2) developed mainly by J. Mendel and co-workers (Mendel, 2002), and the concept of horizontal membership functions (Piegat, 2015; Tomaszewska, 2015). In this paper an aggregation method of two inconsistent expert opinions will be shown. Aggregation of three or more opinions and mathematical properties of this operation will be presented in next papers of authors.

2 METHODOLOGY

In this paper a horizontal membership model will be used (Piegat, 2015). Constructing horizontal MFs requires using multidimensional RDM interval-arithmetic based on relative-distance-measure variables. This method is a new approach to interval arithmetic. In this method an information granule is given as a variable x, which has a value contained in interval $\epsilon[\underline{x},\bar{x}]$, where $\underline{x}$ is the lower limit and $\bar{x}$ is the upper limit of the interval. Thus variable x can be described with formula (Tomaszewska, 2015):

$$x\epsilon[\underline{x},\bar{x}]: x=\underline{x}+\alpha_x(\bar{x}-\underline{x}), \alpha_x\epsilon[0,1]$$

These distributions can have a great meaning in the case of complex mathematical formulas or schemes. RDM-variable is used in horizontal MFs in the way that the function of fuzzy number assigns two values of x: $x_L(\mu)$ and $x_R(\mu)$ for one value of μ and the relative distance between these two values of x is α_x. On the left border $\alpha_x=0$ and on the right border $\alpha_x=1$. The transitional segment $x(\mu)$ can be defined by function (Piegat, 2015):

$$x=x_L+(x_R-x_L)\alpha_x\ , \alpha_x\in[0;1]$$

Inconsistent opinions can be interpreted as follows: the experts in different way evaluate position of the minimal (left) x_L and of maximal (right) border x_R of their evaluations. Thus, the left border x_{LE} and the right border x_{RE} of the aggregated evaluation AgB is uncertain (g means operation of aggregation). A possible left border x_{LE} of aggregated MF $\mu_{AgB}(x)$ in terms of interval FSs Type-2 is called left embedded border and a possible right border x_{RE} is called right embedded border. Fig.2. shows denotations used in further formula derivations.

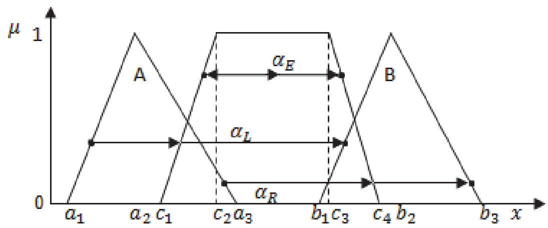

Figure 2: Membership functions $\mu_A(x), \mu_B(x)$and one possible aggregated function $\mu_{AgB}(x)$of Type-1.

In Fig.2. α_L and $\alpha_R$variables are left/right border transformation and α_E is inner RDM variable of embedded MF Type-1. Formulas (1)-(4) give values x for points c_1, c_2, c_3, c_4 which characterize embedded MFs- Type-1:

$$c_1=a_1+\alpha_L(b_1-a_1) \tag{1}$$

where $\alpha_L\epsilon[0,1]$ and $\alpha_L\le\alpha_R$

$$c_2=a_2+\alpha_L(b_2-a_2) \tag{2}$$

$$c_3=a_2+\alpha_R(b_2-a_2) \tag{3}$$

where $\alpha_R\epsilon[0,1]$ and $\alpha_R\ge\alpha_L$

$$c_4=a_3+\alpha_R(b_3-a_3) \tag{4}$$

On the basis of formulas (1)-(4) a horizontal model of the left uncertain border of the embedded aggregated MF is achieved.

$$x_{LE}=[a_1+\alpha_L(b_1-a_1)]+[(a_2-a_1)+\alpha_L(a_1+b_2-a_2-b_1)]\mu \tag{5}$$

where $\alpha_L\epsilon[0,1]$ and $\mu\epsilon[0,1]$

And the right uncertain border of the aggregated MF is analogously determined:

$$x_{RE}=[a_3+\alpha_R(b_3-a_3)]-[(a_3-a_2)+\alpha_R(a_2+b_3-a_3-b_2)]\mu \tag{6}$$

where $\alpha_R\epsilon[0,1]$ and $\mu\epsilon[0,1]$

The full horizontal model of the aggregated MFs takes the following form:

$$x_{AgB}=x_{LE}+\alpha_E(x_{RE}-x_{LE}) \tag{7}$$

where $\alpha_E\epsilon[0,1]$ and $\mu\epsilon[0,1]$

The numerical example described in next section shows how to use in practice above equations to aggregate two inconsistent expert opinions.

3 NUMERICAL EXAMPLE

Two experts made assessments, but their opinions in form of two triangular membership functions are disjoint. Fig.3. presents these opinions.

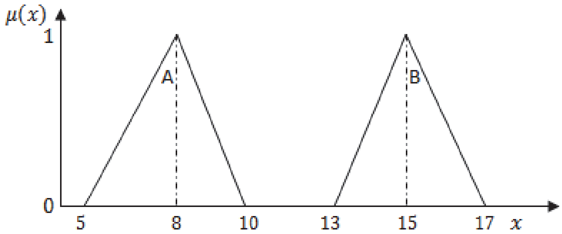

Figure 3: Two inconsistent expert opinions.

The first step to get the full horizontal model of the aggregated MFs is to determine the possible tops of aggregated membership function using equations (1)-(4): $c_1 = 5 + 8\alpha_L$, $c_2 = 8 + 7\alpha_L$, $c_3 = 8 + 7\alpha_R$, $c_4 = 10 + 7\alpha_R$, where $\alpha_R \geq \alpha_L$.

Hence, the left and right embedded borders are achieved:

$$x_{LE} = 5 + 8\alpha_L + (3 - \alpha_L)\mu$$

$$x_{RE} = 10 + 7\alpha_R - 2\mu$$

where $\alpha_R \geq \alpha_L$and $\alpha_L, \alpha_R \epsilon [0,1]$ and $\mu \epsilon [0,1]$

The full horizontal model of these two inconsistent opinions takes form:

$$x_{AgB} = 5 + 8\alpha_L + (3 - \alpha_L)\mu + \alpha_E(5 + 7\alpha_R - 8\alpha_L - 5\mu + \alpha_L\mu)$$

The formula x_{AgB} is multidimensional function and it depends on four parameters $x_{AgB} = f(\mu, \alpha_L, \alpha_R, \alpha_E)$. In addition, if the values of variables α_L and α_R are the same the result x_E is triangular membership functions, if they are different then it takes trapezoidal MF. E.g. if $\alpha_L = 0$ and $\alpha_R = 0$ then $x_E = 5 + 3\mu + \alpha_E(5 - 5\mu)$ and if $\alpha_L = 1$ and $\alpha_R = 1$ then $x_E = 13 + 2\mu + \alpha_E(4 - 4\mu)$. The result for different values of α_L and α_R is presented in Fig.4.

The exact result of aggregation of two inconsistent expert opinions is a multidimensional granule as presented in Fig. 4. For practical use we can seek for low-dimensional representation in the form of optimal distributions representing full multidimensional solution (as shown in Fig. 5.).

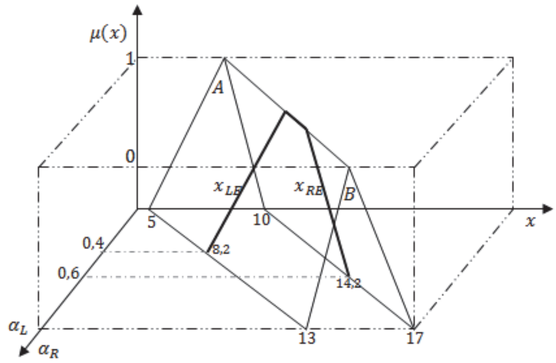

Figure 4: Visualization of 4D horizontal membership function $x_{AgB} = f(\mu, \alpha_L, \alpha_R)$Type-2 and one of embedded MF for $\alpha_L = 0{,}4$and $\alpha_R = 0{,}6$.

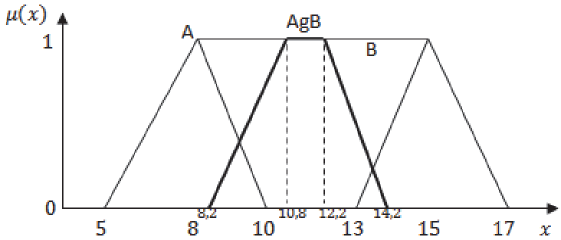

Figure 5: Visualization of 4D horizontal MF $x_{AgB} = f(\mu, \alpha_L, \alpha_R)$Type-2 and one embedded MFT1 for $\alpha_L = 0{,}4$and $\alpha_R = 0{,}6$in 2D-space.

It can be assumed that in some problems the parameters α_L and α_R can mean the credibility of expert opinions and setting the appropriate values of them makes x_{AgB} function only 2-dimensional.

4 CONCLUSIONS

Inconsistent FSs A and B generate one fuzzy set Type-2 which is a family of embedded fuzzy sets Type-1. It means that the true but precisely unknown x-value that was evaluated by the experts A and B can be contained in one of FSsT-1 imbedded in the achieved FST-2, which membership function is visualized in Fig.4. Each possible embedded MFT1 can be achieved by choice of values of RDM variables. Interval-valued fuzzy sets theory and horizontal RDM membership functions allow to aggregate uncertain fuzzy sets which express inconsistent expert opinions. Each opinion delivers right and left border of fuzzy set. Two opinions deliver two right and two left borders. It means that the aggregated borders are uncertain and they generate fuzzy set with uncertain borders. Hence the aggregated MF has uncertainty of higher order than each of the single component opinions.

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