Construction of a Bayesian Network as an Extension of Propositional Logic

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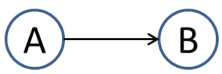
Keywords: Bayesian Network, Association Rule Mining, Propositional Logic.

Abstract: A Bayesian network is a probabilistic graphical model. Many conventional methods have been proposed for its construction. However, these methods often result in an incorrect Bayesian network structure. In this study, to correctly construct a Bayesian network, we extend the concept of propositional logic. We propose a methodology for constructing a Bayesian network with causal relationships that are extracted only if the antecedent states are true. In order to determine the logic to be used in constructing the Bayesian network, we propose the use of association rule mining such as the Apriori algorithm. We evaluate the proposed method by comparing its result with that of traditional method, such as Bayesian Dirichlet equivalent uniform (BDeu) score evaluation with a hill climbing algorithm, that shows that our method generates a network with more necessary arcs than that generated by the traditional method.

1 INTRODUCTION

A Bayesian network is a probabilistic graphical model that represents the causal relationships between random variables as a directed acyclic graph (Pearl, 1985). Figure 1 shows an example of a Bayesian network model.



A=F	A=T
0.1	0.9

	B=F	B=T
A=F	0.9	0.1
A=T	0.1	0.9

F:False
T:True

Figure 1: Example of a Bayesian network model.

In Fig. 1, circles having a character denote random variables. Causal relationships between these random variables are represented by arcs; the tip of the arc points represents a resulting event (consequent), and the end of the arc points represents a causal event (antecedent). The tables in Fig. 1 are called conditional probability tables (CPTs) and show the conditional probabilities of a consequent (B) given an antecedent (A).

Many methods have been proposed for constructing Bayesian networks. They optimize an evaluation function based on a marginal likelihood function, such as Bayesian information criterion (Schwarz, 1978), Akaike's information criterion (Akaike, 1973), K2 score (Cooper and Herskovits, 1992) and Bayesian Dirichlet equivalent uniform (BDeu) (Hecerman and Chickering, 1995). In addition, they search for an optimal value in their function using a heuristic algorithm, such as a greedy algorithm (Cormen et al., 2009), a genetic algorithm (Holland, 1992),(S. Fukuda and T.Yoshihiro, 2014) and a hill climbing algorithm (I. Tsmardinos and Aliferis, 2006),(J. A. Gamez and Puerta, 2015). However, we presume that they often incorrectly identified the Bayesian network structures.

In this study, to correctly construct a Bayesian network, we employ the concept of propositional logic. A propositional logic can handle a statement that is represented as true or false. In propositional logic, a logical expression is represented as a statement, e.g., gif A, then B,h because A and B both deal with causal structures.

The Venn diagrams shown in Fig. 2 illustrate a causal relationship between an antecedent (A) and a consequent (B) expressed in a CPT of a Bayesian net-

Enomoto, T. and Kimura, M..
Construction of a Bayesian Network as an Extension of Propositional Logic.
In *Proceedings of the 7th International Joint Conference on Knowledge Discovery, Knowledge Engineering and Knowledge Management (IC3K 2015) - Volume 1: KDIR*, pages 211-217
ISBN: 978-989-758-158-8

work (on the left) and an implicational relation rule of propositional logic ($A \rightarrow B$) (on the right).

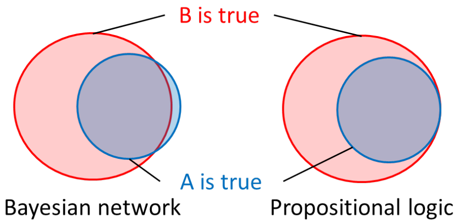


Figure 2: Venn diagram on the left illustrates a causal relationship according to a Bayesian network, and the one on the right illustrates an implicational relation rule of propositional logic.

In a propositional logic, whether or not a causal relationship holds is expressed in binary. On the other hand,in a Bayesian network, the existence of a causal relationship is not necessarily represented in binary but instead by a probability. Figure 2 illustrates these situations with Venn diagrams to clarify their differences. Realistically, whether or not a causal relationship is established cannot be treated in binary as done in propositional logic because there is no guarantee that the causal relation infallibly holds true for any case in an actual setting. In fact, in a case where the probability of the occurrence of an event is extremely high, we consider that the causal relationship holds true. Therefore, we regard a Bayesian network as an extension of propositional logic.

If we limit ourselves to a one-to-one relation between an antecedent and a consequent, as in Fig. 1, the combinations of values that A and B can be are as follows:

$$\begin{aligned} \{A\,is\,T \quad &\Rightarrow \quad B\,is\,T\} \\ \{A\,is\,T \quad &\Rightarrow \quad B\,is\,F\} \\ \{A\,is\,F \quad &\Rightarrow \quad B\,is\,T\} \\ \{A\,is\,F \quad &\Rightarrow \quad B\,is\,F\} \end{aligned}$$

The traditional methods for constructing a Bayesian network optimized all the combinations of causal relationships. However, in the implication relationship of propositional logic, if an antecedent is true, a causal relationship can either be true or false, and if an antecedent is false, the causal relationship is true. In other words, when an antecedent is false, a consequent has nothing to do with the establishment of a causal relationship. Therefore, we regard a Bayesian network as an extension of propositional logic, and causal relationships with a false antecedent state need not be taken into account when constructing a Bayesian network.

Therefore, we have proposed a methodology for constructing a Bayesian network whose causal relationships are removed if the antecedent state is false using association rule mining.

2 METHODS

In order to construct a Bayesian network as an extension of propositional logic, we need to determine the existence or non-existence of causal relationships in all random variables. In addition, we must extract only the causal relationships whose antecedent is T. Therefore, we can effectively extract causal relationship candidates using association rule mining.

Association rule mining is a typical method for identifying relationships between variables in large-scale transactional data (Piatetsky and Frawley, 1991), (R. Agrawal and Swami, 1993). This method uses three indices called support, confidence, and lift to evaluate a causal relationship, as illustrated in Equations (1), (2) and (3) below. Let P(A) denote a marginal probability of a random variable A, then

$$support(A \Rightarrow B) = P(A,B), \tag{1}$$

$$confidence(A \Rightarrow B) = P(B|A), \tag{2}$$

$$lift(A \Rightarrow B) = \frac{P(A,B)}{P(A)P(B)}. \tag{3}$$

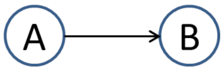
Equation 1 expresses the co-occurrence frequency between random variables and is defined as a percentage of the transactions that contain both an antecedent and a consequent in all data transactions. Equation 2 is a conditional probability and is defined as a percentage of the transactions that contain both an antecedent and a consequent in all data transactions that include the antecedent. Equation 3 is a measurement of the interdependence between the random variables. If its value is higher than 1.0, the antecedent and consequent are regarded as having a dependency.

Apriori is a classic algorithm for use in association rule mining. In order to extract candidates of random variable combinations that have a correlation, this algorithm focuses only on combinations of random variables that frequently appear in a data store. The Apriori algorithm is applied when either a random variable or combination of random variables occur infrequently.

2.1 Bayesian Network and CPT Relationships

Since the values in CPTs are probabilities, they range from 0 to 1. However, the value assignments in CPTs cannot be independent from the structure of the

Bayesian network. Figure 3 demonstrates a situation that is an example of a contradictory Bayesian network model where the diagram assumes a causal relationship between A and B, although its CPT indicates no causal relationship between them.



A=F	A=T
0.1	0.9

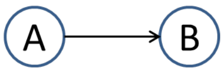
	B=F	B=T
A=F	0.1	0.9
A=T	0.5	0.5

F:False
T:True

Figure 3: Bayesian network model that cannot predict a consequent if an antecedent is T.

This Bayesian network model has an arc from A to B. However, the probabilities of any combination of values of random variable A and random variable B are identical; namely, when A is true, the value of random variable B cannot be predicted to be true or false.

Moreover, Fig. 4 indicates no causal relationship between A and B. For all values of random variable A, the value of random variable B is true with a probability of 0.8. As such, A and B are independent, $P(B|A) = P(B)$.



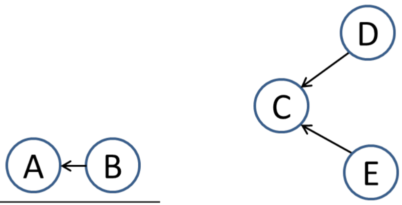
A=F	A=T
0.1	0.9

	B=F	B=T
A=F	0.1	0.9
A=T	0.1	0.9

F:False
T:True

Figure 4: Bayesian network model that shows no causal relationship.

In order to exclude these types of contradictory networks, our method targets networks whose probabilities in the CPTs are less than θ_l or more than θ_m when the antecedent is T, as in Fig. 5, where the values of θ_l and θ_m are 0.2 and 0.8, respectively.



(a)

	A=F	A=T
B=F	0.1	0.9
B=T	0.9	0.1

(b)

		C=F	C=T
D=F	E=F	0.5	0.5
D=F	E=T	0.5	0.5
D=T	E=F	0.5	0.5
D=T	E=T	0.2	0.8

Figure 5: Bayesian network examples containing causal relationships. When an antecedent is T, the consequent is F(a), and when all random variables in an antecedent are T, the consequent is T(b).

2.2 Causal Relationship Extraction using Confidence Values

Association rule mining can efficiently extract causal relationship candidates whose support values are frequently high using the Apriori algorithm. In other words, even if random variables have strong associations, a candidate is not extracted when its support value is low. To extract these causal relationship candidates, we focus on the use of confidence values in the Apriori algorithm. Confidence measures the association strength for a random variable combination. Even though the Apriori algorithm only extracts causal relationship candidates possessing high support values, in our study, we must extract causal relationships possessing strong associations. Therefore, we extract the random variable combination that is a causal relationship candidate if its confidence and support exceeds certain threshold values. In this study, we use a high threshold for confidence and a low threshold for support to efficiently extract causal relationship candidates.

2.3 Extraction of Causal Relationship with Low Consequent Probability

In this study, causal relationships whose antecedent is T, namely $\{\text{An antecedent is } T \Rightarrow \text{A consequent is } T\}$ and $\{\text{An antecedent is } T \Rightarrow \text{A consequent is } F\}$, are taken into account. However, the Apriori algorithm only extracts causal relationship candidates whose antecedents and consequents are frequently T, i.e., causal relationships whose consequents are frequently F are not extracted. In order to extract these candidates, if the causal relationships are not extracted by the Apriori algorithm, the random variable of the consequent in the CPT is swapped for its complement. Figure 6 illustrates this method using the Bayesian network.

In this Bayesian network, imagine that $\{B \Rightarrow A\}$ is not extracted by the Apriori algorithm because the consequent of the causal relationship is frequently F. Therefore, in order to extract it, we convert the consequent A to its complement A^c. Figure 7 shows this

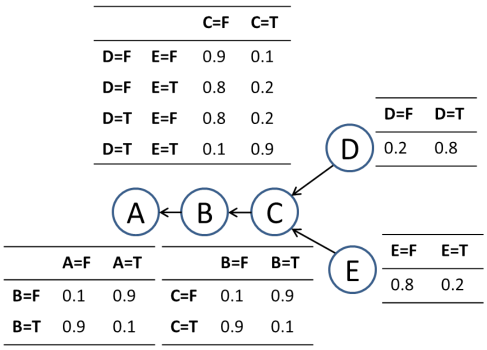


Figure 6: A Bayesian network for explaining our proposed method.

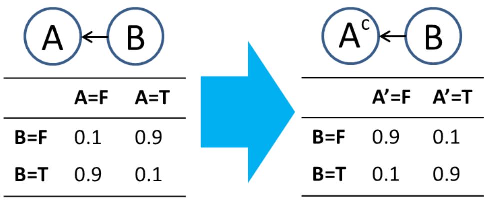


Figure 7: Swapping of probabilities in the CPT values.

conversion.

The Bayesian network and table on the right in Fig. 7 are obtained after the conversion that enables the Apriori algorithm to extract this causal relationship if the support is greater than the given threshold value.

2.4 Extraction of Causal Relationships with Plural Random Variables in Antecedents

When causal relationships have plural random variables in their antecedents, it is possible that they may not be extracted even if there is a strong association. For example, $\{D,E \Rightarrow C\}$ in Fig. 6 is not extracted because random variable C tends to be T only when both random variable D and E are T. In other words, when one of the random variables is not T, C is hardly ever T. Therefore, the confidence values of $\{D \Rightarrow C\}$ and $\{E \Rightarrow C\}$ are low. In such a case, when the Apriori algorithm is executed, these causal relationships are eliminated during the process. Because of this, $\{D,E \Rightarrow C\}$ is not extracted even if its confidence value exceeds the threshold.

In order to extract such candidates, if the confidence value of a causal relationship is lower than the given threshold, we compare it with the confidence value of the causal relationship that then adds an optional random variable to the antecedent. If the latter value is higher than the original, it is not eliminated until the next iteration of the Apriori algorithm. In the case of Fig. 7, this allows us to keep $\{D \Rightarrow C\}$ and $\{E \Rightarrow C\}$ until $\{D,E \Rightarrow C\}$ is extracted.

2.5 Selecting and Linking for Causal Relationship Candidates

With our method, a Bayesian network is obtained by combining the causal relationship candidates.

Keep in mind that each random variable in a Bayesian network appears only once as a consequent variable. Based on this fact, we combine causal relationships with the highest lift from candidates that share the same consequent. For example, assume that the following causal relationship candidates are found for the Bayesian network in Fig. 6, which are sorted in descending order of lift.

1. $\{D,E \Rightarrow C\}$
2. $\{B \Rightarrow C\}$
3. $\{B \Rightarrow A\}$
4. $\{A,C \Rightarrow B\}$

In this case, first, $\{D,E \Rightarrow C\}$ is integrated in a Bayesian network. Second, whether or not $\{B \Rightarrow C\}$ is taken account. Since the causal relationship possessing random variable C in the consequent has already been integrated, it is not employed. Then $\{B \Rightarrow A\}$ and $\{A,C \Rightarrow B\}$ are integrated. Figure 8 shows a Bayesian network constructed from these causal relationship candidates.

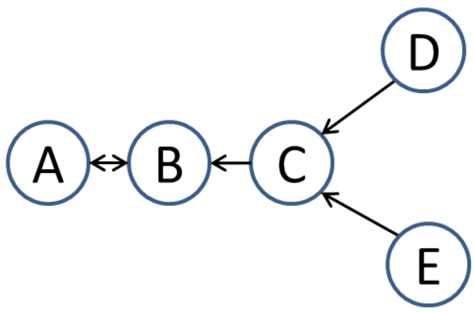


Figure 8: Bayesian network constructed from causal relationships candidates.

2.6 Direction Determination for Dual-directional Causal Relationship

When causal relationships are integrated, dual-directional causalities can be created, as shown in Fig. 8. If we find such causalities, we must establish the correct direction by comparing the products of the causal relationship confidence values. With respect to the dual-directional causalities in the Bayesian network in Fig. 8, the equations to be compared are $P(B|C)P(B|A)$ and $P(B|A,C)$.

3 EXPERIMENT

In order to evaluate the proposed method, we compared the Bayesian network created by the proposed method with one created by the BDeu score evaluation with a hill climbing algorithm(J. A. Gamez and Puerta, 2015). We created sampling data from an original Bayesian network as shown in Fig. 9, where the values of θ_l and θ_m are 0.2 and 0.8, respectively.

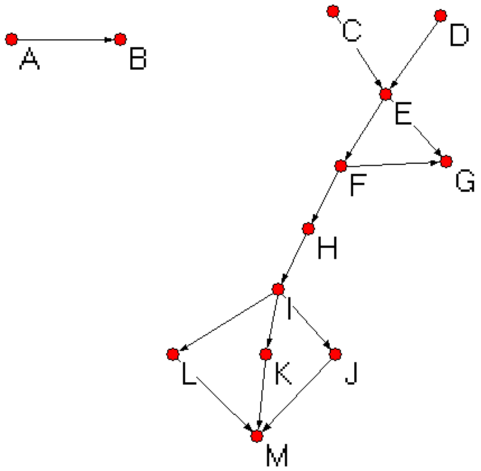


Figure 9: Bayesian network for our experiment.

The parameters of association rule mining in the proposed method were as follows:

- The support threshold = 0.01 (s_1), 0.015 (s_2), 0.02 (s_3),
- The confidence threshold = 0.7.

In this experiment, we used three support values. Confidence value was fixed to be 0.7, because experimental results did not change, while it was changed to 0.6, 0.65, 0.7 and 0.75.

After constructing the Bayesian networks, we used Hamming distance in the original Bayesian network to compare the constructed Bayesian networks. The Hamming distance counts the difference between the two off-diagonal matrices representing directed graphs. The Hamming distance of exactly the same graph is zero. The more different the graphs are, the greater the value of the Hamming distance is. Therefore, if the Hamming distance for the proposed method is shorter than that for BDeu with a hill climbing algorithm (hereafter, we simply call BDeu), we can say that our method can give more accurate Bayesian network than BDeu.

3.1 Results and Discussions

Figure 10 shows the results obtained using BDeu, and Fig. 11, 12 and 13 show the results obtained by the proposed method. Table 1 shows the numbers of the arcs and Hamming distances.

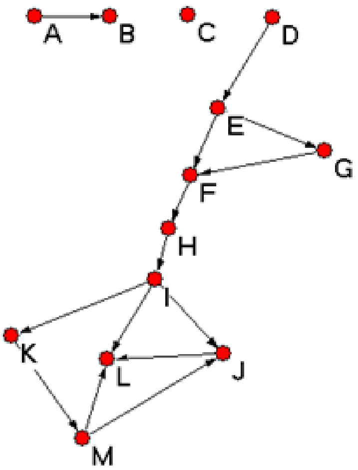


Figure 10: A Bayesian network constructed by BDeu with a hill climbing algorithm.

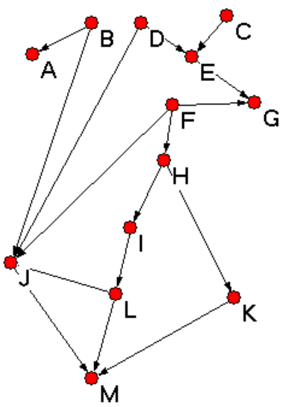


Figure 11: A Bayesian network constructed by our proposed method with the support threshold s_1.

Table 1: The numbers of arcs and Hamming distance.

	BDeu	s_1	s_2	s_3
Total number of arcs	14	16	13	9
Total number of correct arcs	10	10	10	3
Hamming distance	8	10	7	17

In the Bayesian network constructed by BDeu, mainly inverted forks, such as $\{C,D \Rightarrow E\}$ and $\{J,K,L \Rightarrow M\}$ were not reproduced. This is because

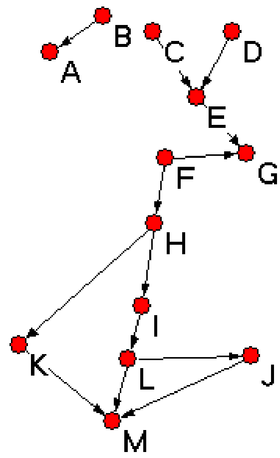


Figure 12: A Bayesian network constructed by our proposed method with the support threshold s_2.

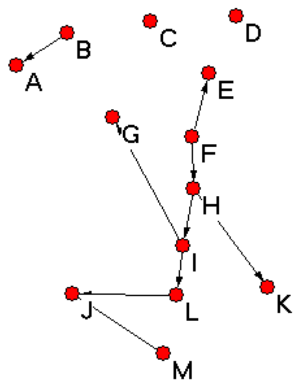


Figure 13: A Bayesian network constructed by our proposed method with the support threshold s_3.

the number of random variables in an antecedent was too many to be optimized. In the proposed method, s_2 had most shortest Hamming distance of all. In addition, this method reproduced all inverted forks. In the case that the support value was 0.01, the Bayesian network had causal relationship that did not exist in the original one, such as $\{A, D, L \Rightarrow J\}$. This is because a rule whose antecedent had many random variables tended to have low suppprt value and high confidence value. On the other hand, in the case of support value was 0.02, inverted forks such as $\{C, D \Rightarrow E\}$ and $\{J, K, L \Rightarrow M\}$ were not reproduced because of the same reason for the case with the low support value, 0.01.

Even in the case with the support value s_2, there were some causal relationships that could not be reproduced. We explain its reason with the relationships as follows:

- $\{B \Rightarrow A\}$,
- $\{I \Rightarrow K\}$.

The causal relationship $\{B \Rightarrow A\}$ had antecedent and consequent opposite to $\{A \Rightarrow B\}$ in the original Bayesian network. Both $\{A \Rightarrow B\}$ and $\{B \Rightarrow A\}$ were extracted by our Apriori algorithm. However, since $P(A|B)$ was higher than $P(B|A)$, the causal relationship $\{B \Rightarrow A\}$ was adapted in our method. The reason why $P(A|B)$ was higher was that $P(A)$ was higher than $P(B)$. Remembering that we generated data of B from the value of A following the probability $P(B|A)$ and $P(B|A^c)$ in our simulation, this can occur if both $P(B|A)$ and $P(B|A^c)$ are reasonably low. In practice, we might draw an arc from B to A if we manually build the network, if $P(A|B)$ is higher than $P(B|A)$. It can be said that our method simulated such cases.

The causal relationship $\{I \Rightarrow K\}$ was extracted in Apriori algorithm. However, the lift value of $\{H \Rightarrow K\}$ is higher than that of $\{I \Rightarrow J\}$. Therefore, $\{I \Rightarrow K\}$ was not integrated.

We can see that the Hamming distance is longer in the network generated by BDeu than that in the s_2. However, Hamming distance of s_1 and s_3 are longer than it. Therefore, our proposed method is effective for creating a Bayesian network when appropriate support and confidence value are determined.

4 CONCLUSION AND FUTURE WORKS

Many conventional methods have been proposed for constructing Bayesian networks that optimize combinations of the occurrences of causal relationships. However, they often yield an incorrect Bayesian network structure.

If we correspond the establishment of a causal relationship in a Bayesian network to a probability with the value 0.0 or 1.0, we can regard the Bayesian network as a combination of propositional logic statements. In this study, based on this concept, we proposed a method to construct a Bayesian network as an extension of propositional logic to determine a correct Bayesian network structure.

The proposed construction method uses association rule mining. We extracted causal relationship candidates using the confidence values in the Apriori algorithm. We have also proposed a method for extracting causal relationships whose antecedents have plural random variables and consequents are less frequently T. Finally, we linked the obtained causal relationship to obtain a Bayesian network.

We compared the resultant Bayesian network constructed using our method with those constructed using BDeu score evaluations. We found that, when appropriate support and confidence value are used, our proposed method is effective to create a Bayesian network.

In future studies, we will discuss improvements to calculate appropriate values of thresholds for support and confidence. We need to confirm the limit of θ_l and θ_m that we can apply this method. In addition, we will discuss the extension of our approach to multiple-valued logic.

REFERENCES

Akaike, H. (1973). Information theory and an extension of the maximum likelihood princi- ple. *Proceedings of the 2nd International Symposium on Information Theory*, pages 267–281.

Cooper, G. and Herskovits, E. (1992). A bayesian method for the induction of probabilistic networks from data, machine learning. *Machine Learning*, 9:309–347.

Cormen, T., Leiserson, C., and Rivest, R. (2009). *Introduction to Algorithms, 3rd Edition*. MIT.

Hecerman, D. and Chickering, D. (1995). Learning bayesian networks: The combination of knowledge and statistical data. *The combination of knowledge and statistical data*, 20:197–243.

Holland, J. H. (1992). *Adaptation in natural and artificial systems: An introductory analysis with applications to biology, control, and artificial intelligence*. MIT.

I. Tsmardinos, L. E. B. and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network bayesian network structure learning algorithm. *Machine Learning*, 65.

J. A. Gamez, J. L. M. and Puerta, J. M. (2015). Structural learning of bayesian networks via constrained hill climbing algorithms: Adjusting trade-off between efficiency and accuracy. *International Journal of Intelligent Systems*, 30.

Pearl, J. (1985). Bayesian networks: A model of self-activated memory for evidential reasoning. *Proceedings of the 7th Conference of the Cognitive Science Society*, pages 329–334.

Piatetsky, G. and Frawley, W. J. (1991). *Know ledge Discovery in Databases*. MIT.

R. Agrawal, T. I. and Swami, A. (1993). Mining association rules between sets of items in large databases. *, In Proceedings of the 1993 ACM SIGMOD International Conference on Management of Data*, pages 207–216.

S. Fukuda, Y. Y. and T.Yoshihiro (2014). A probability-based evolutionary algorithm with mutations to learn bayesian networks. *International Journal of Artificial Intelligence and Interactive Multimedia*, 3:7–13.

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, 6:461–464.