# A Diagnosis Scheme for Dynamical Systems: Approach by Guaranteed Parameter Estimation

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Abstract: Through parameter estimation schemes, one could be able to detect, localize and identify the occurring fault via simple computation. Yet, certain faults may not be discovered even be mistaken in a normal condition with unknown noises by trend checking or state monitoring. A more informative way when a correct model is present to analyses the data via parameter estimation. In this paper, we propose by using interval analysis a diagnosis scheme, from which we can extract the guaranteed diagnostic results to inform the supervisor so that appropriate actions could be taken. Sending them the results in a guaranteed way to tell the diagnostician which kind of fault exist is firstly taken care in diagnosis context. Our original fault detection and localization procedure has been firstly proposed in an interval analysis context for the constant fault in parameters. Moreover, another new technique in parameter estimation is the distance check, which speed up the estimation procedure. Some drawbacks have been discussed in the end.

## 1 INTRODUCTION

The fault detection, isolation and identification is a key aspect in the reinforcement of the manipulability of operation systems. Generally, the fault is considered as a nonpermitted deviation from an expectoration process, a value from sensors, outputs. These monitoring measurable variables in some supervisor mechanism can be very difficult to implement (Bailey, 1984). Moreover these surveillance technique does not allow a profound investigation to fault diagnosis (Isermann, 1993). Even if one or more deviations are found, no further information on which actuator or which sensor is needed to be taken care of. Such situation requires more sophisticated methods to extract information based on measurements. To do this, methods such as state estimation diagnosis, parameter estimation diagnosis and fault model diagnosis have been developed (Isermann, 2005).

In stochastic case, the diagnosis could be arrived by using the confidence interval to prove the correctness and the failure mode in case of some extreme situation occurs. This intervals are some times very large that one could no further rely on. In the mean time, the so called estimated value has intrinsic value error due to the model inaccuracy and data uncertainty, which shadow the estimated results for further usage.

When dealing with fault detection and identification (FDI) problem, particularly for process models, parameter estimation method is the first to consider. The computational simplicity and direct understandable parameter information make it very popular among the diagnostician. Whereas, the main drawbacks of parameter estimation techniques are their weak robustness to external disturbances as one have to set a nominal value to start searching (Jauberthie et al., 2013) and also, the identifiability of a studied system must be done at first. With interval analysis approach, no more identifiability information is required for parameter estimation (Jaulin and Walter, 2001) in general. The simplicity of implementation and applicable for a large range of system make this approach popular in automatic control field (Jaulin, 2001). Whereas, a great consumption of computation time of such method is inevitable (Jaulin and Walter, 1993) due to the inner brand and bound solution finding scheme which limits the diagnostic capabilities (fault size, appearance time, etc.) (Isermann, 1984) (Cimpoesu et al., 2012). Once the parameter is obtained, the diagnostic procedure is completed by two steps: the comparison between the quantities obtained from actual model and the normal system. The fault will be detected by using some range decision or in-


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tersection computation. Next step is the fault identification. The faults may be classed according to its form, the corresponding reactions will be taken according to the fault tree. If the fault is tolerable, the system may continue functioning with a fault model or if a full stop is needed and a sound diagnosis is required to eliminate the fault.

In this paper, parameter estimation based fault diagnosis procedure is described. Only single fault during each experiment is considered. To address the fault condition as soon as possible, a distance based method has been proposed. This new elimination method originally proposed in interval analysis context allowing us to find the unfeasible parameter as soon as the output abnormality observed.

This paper is organized as follows: Section 2 will give a brief explication on the treating problem in set membership context, the general idea of parameter estimation approach in interval analysis. In section 3, the guaranteed diagnosis procedure via parameter estimation is firstly proposed. In section 4, a case study is conducted, the so called method has been compared with results obtained with normal condition. At last, the section 5 will give a brief summary on active diagnosis and further research direction. Some notices of use has delivered.

## 2 MODEL-BASED DIAGNOSIS

This article follows the standard notation of interval analysis (Kearfott et al., 2005) where $x$ in bold referring to an interval box. Let us consider a nonlinear dynamic system of the following form:

$$\begin{cases} \dot{x}(t,p) &= f(x(t),p,u(t)), \\ y(t,p) &= h(x(t,p)). \end{cases} \tag{1}$$

where $x(t) \in \mathbb{IR}^n$ and $y(t) \in \mathbb{IR}^m$ denote respectively the state variables and the measured outputs. $u(t)$ is the system input. The initial conditions $x(0)$ is supposed to belong to an initial bounded "box" $x_0 = [\underline{x_0}, \overline{x_0}]$. The parameter vector $p$ is constant and is assumed to belong to a bounded "box" $p_0 = [\underline{p_0}, \overline{p_0}]$. Time $t$ is assumed to belong to $[0, t_{max}]$. The functions $f$ and $h$ are nonlinear functions. $f$ is real and analytic on $M$ for every $p \in P_0$, where $M$ is an open set of $\mathbb{R}^n$ such that $x(t) \in M$ for every $p \in P_0$ and $t \in [0, t_{max}]$). Moreover the function $f$ is assumed to be sufficiently differentiable in the domain $M$.

In diagnosis context via parameter estimation approach, the system parameters have always some connections with state deviation or physical meanings. A fully rigorous and easy implementable procedure to estimate the abnormality of parameters or states should be taken into account.

The parameter estimation procedure is based on set membership computation which is concerning to find $p$ such that $y_m(p)$ fits best in an inclusion test to be specified. The parameters are considered consistent if the error $v(t_i)$ is assumed to satisfy:

$$y(t_i,\hat{p}) - y_m(t_i,p) \in v(t_i) = [\underline{v}(t_i), \overline{v}(t_i)], \quad i = 1, ..., N. \tag{2}$$

where $y(t_i)$ represents the output of system with exact parameter $\hat{p}$ and $y_m(t_i)$ represents the measured output, this inclusion is to be taken component-wisely. We assume that $\underline{v}(t_i)$ and $\overline{v}(t_i)$ are known as lower and upper bounds for the acceptable output errors. Such bounds may, for instance, correspond to a bounded measurement noise. The integer $N$ is the total number of sample times.

Interval analysis provides tools for computing with sets which are described using outer-approximations formed by union of non-overlapping boxes. Some basic tools on interval analysis are proposed in (Li et al., 2014).

## 3 GUARANTEED FAULT DIAGNOSIS

The word "guaranteed" is referring to a parameter estimation (detection) phase in diagnosis. Generally, the model based diagnosis could be achieved by comparing the behaviors from the real output of a process and the model output, where is later is obtained from state estimator (Gertler, 1998) (Isermann, 1993). To ensure a fault is actually arrived, one has to check with an occurrence counter or a redundant acceptance scheme to make sure that indeed a fault is arrived. Thus, a rigorous and robust method could be used, which detects and confirms the fault in one shot. Interval analysis computes with guaranteed bounds together with a rigorous solver is a good tool to realize the job.

A general process for FDI in interval analysis may be conducted by three essential steps:

1. Detect the faults from the measurements via parameter estimation
2. Locate the faults
3. Identification of faults, its frequency or magnitude.

The first step should make the data in bounded form. This could be done by adding the bound error to the measured data. Then, guaranteed parameter estimation procedure is conducted on observed quantities. At last, the obtained sets intersect with admissible parameter range to check if an error appears.

In practice, suppose that the acceptable range of parameters is represented by $p$ and the estimated acceptable range of parameters in each diagnosis is represented by $\hat{p}$. So, the residual part of two sets can be represented by:

$$r = \hat{p} \cap p. \tag{3}$$

If the intersection part $r = \emptyset$, a fault is detected.

In the diagnosis context, the deviation of the model output and the real output of the studied system can be described by the distance (Equ 4). This indicator could be served as a tool in the second place (step 2) to evaluate if one solution sets if enough close to one another. One possible tool is the middle points distance, which could be described by the following formula:

$$D(y(t_i) - y_m(t_i,p)) = |m(t_i,y) - m(y_m(t_i,p))|, \tag{4}$$

A direct use of this criterion to eliminate certain points is inappropriate. The width information will lost due to the midpoint measurement. Another possible tool is Hausdorff distance $H(\cdot)$, which is given by:

$$\begin{aligned} H(y(t_i), y_m(t_i,p)) &= |m(y(t_i)) - m(y_m(t_i,p))| + \\ & |rad(y(t_i)) - rad(y_m(t_i,p))| \end{aligned} \tag{5}$$

Suppose we have the two sets of value, this distance could be useful to drag out the unfeasible points.

**Proposition.** There are two sets of parameters $p_1$ and $p_2$, if the distance difference between them are satisfying:

$$Dist(y - y_m(p_1)) - Dist(y - y_m(p_2)) > C, \tag{6}$$

where $C$ is a constant, $Dist(\cdot) = \sum_{i=1}^{N} D(\cdot)$ or $Dist(\cdot) = \sum_{i=1}^{N} H(\cdot)$, $N$ represents the number of available points. We can say that the $p_1$ is an unfeasible set, because it is sufficiently away from the measurements.

As we can see, sometimes the solution from middle points happens to be in the solution sets, so its distance with measurements may be very small. Any other sets of parameters have larger value than it. So the value of $C$ is always be settled large to avoid such situation.

The following procedure may be integrated in parameter estimation algorithm (Li et al., 2014), as there is no initial computation to calculate this midpoint distance:

**Algorithm 1:** Distance criterion elimination ($y$, $h([x_e(1:j)])$), $Dist_{pre}$).

**Input:** $x_e$;
**Output:** $\mathcal{P}_{rejected}$;
**if** $Dist(y, h([x_e(1:j)])) - Dist_{pre} > C$ **then**
  $\mathcal{P}_{rejected} := \mathcal{P}_{rejected} \cup p$;
**else**
  $Dist_{pre} := Dist(y, h([x_e(1:j)]))$
**end if**

where $x_e = [x, p]$, $j$ represents the value of $x(t_j)$ . When first use of this algorithm, $Dist_{pre}$ is initialized by 0. This algorithm requires no extra writing of system sensitivity equation. All information needed is available and must calculated for the set inversion procedure after.

In interval analysis context, the parameter estimation requires an inclusion test to separate the feasible and unfeasible sets. All the feasible $p \in$ P must satisfy:

$$y_m(t,p) \in y = [\underline{y}(t), \overline{y}(t)] \tag{7}$$

where $y_m(t,p)$ presents the model output, $y$ represents the actual measurement. This test is time consuming. A contractor could be applicable to reduce the computation time. When there is an analytical output model exists, one could use a contractor like: Newton, Krawczyk to eliminate the unfeasible boxes. Or, if no such model is available, we can use a mean value form to represent the inclusion test (Kieffer and Walter, 2011) with more computation effort on sensitivities parts, so that a new parameter set could be achieved by displacing the $p_j$ to one side.

In the following section, we will use the distance check to replace the contractor in parameter estimation algorithm, and a comparison of execution time between the method set inversion with contractor, the method set inversion with distance elimination, the method uses only set inversion will be compared and commented.

## 4 APPLICATION

The results are obtained using a Core i7 at 3.6GHz with 8G RAM on a Linux system. The state estimation is using VNODE-LP. Single permanent fault in constant parameters is considered, but multiple faults are also admissible for such solution scheme.

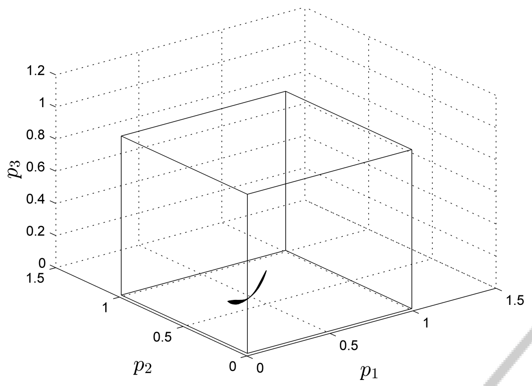


Figure 1: Outer approximation of admissible sets for $p_1$, $p_2$ and $p_3$

## 4.1 Two Compartments Model

The case study that we consider is a linear compartment system, which has been studied in (Kieffer and Walter, 2011).

$$\begin{aligned} \dot{x}_1 &= -(p_1+p_3)x_1+p_2x_2+u \\ \dot{x}_2 &= p_1x_1-p_2x_2 \end{aligned} \tag{8}$$

the fault parameter is appeared on $p_1$ at the beginning of time, which is considered to be equal with 0.2 as a fault in system, the other parameters are $p_2 = 0.15$ and $p_3 = 0.35$. Besides, the initial state is $x_1 = 1$ and $x_2 = 0$, assume there is no input. We generate the faulty measurements with these configuration over every second in $t = 15s$ and add a constant interval noise $v = [-0.005, 0.005]$. In normal situation, we know $\hat{p}_1 \in [0.5891, 0.6115]$ considering the measurement error. The parameter estimation step should give us a necessarily good estimation for $p_1$ so that this fault may be detected and isolated with other potential faults in diagnosis procedure. In this paper, we will also be interested in time efficiency.

The parameter space is supposed to be included in

$$p \in \begin{bmatrix} 0.01 & 1.0 \\ 0.01 & 1.0 \\ 0.01 & 1.0 \end{bmatrix}. \tag{9}$$

The stop criterion is supposed to $\varepsilon = 0.0005$. We set $C = 0.85$ for the midpoint distance method. The three parameters are projected to the 3-D dimension plan. Only the admissible and rejected sets are pictured of interest. The blue parts represent the unfeasible sets (Fig. 1). As we are seeing a 3D figure from a 2D view point, the more the parts superposed, darker color they will get.

To be clear, the obtained admissible parameters are equal with:

$$p_{admis} \in \begin{bmatrix} 0.1889, & 0.2121 \\ 0.1241, & 0.4267 \\ 0.1241, & 0.4267 \end{bmatrix} \tag{10}$$

As we can see, the parameter has no intersection between the admissible parameter $p_1$ and $\hat{p}_1$. The constant fault appeared on parameter $p_1$ has been detected. To identify the fault, the Hausdorff distance can be useful to make a decision map so that certain actions could be taken. In this case, we have $H(p_1, \hat{p}_1) = 0.4002$. Based on the distance information, we need to check the conveyance tunnel as a lower transfer rate is observed.

As the identification step is not time consuming, it is clear that if the parameter estimation process could be faster, better diagnosis performance could be achieved. Here, we use the proposed approach with other methods to show their different time consumptions :

Table 1: Time consumption for each configuration of parameter estimation.

| Time (s) / ε | SIVIA | Distance check | SIVIA with Contractor |
|---|---|---|---|
| 0.005 | 29 | 27 | 147 |
| 0.001 | 144 | 124 | 3781 |
| 0.0005 | 548 | 496 | 16089 |

The column SIVIA represents the parameter estimation is achieved by algorithm proposed in (Li et al., 2014). We can conclude that with the distance check, the time for parameter estimation can be relatively reduced. The contractor may delay the whole process when precise bounding is required. A moderate usage of contractor is needed, one could use the distance criterion as the start condition for contractor or some other volume criterion could be also helpful.

## 4.2 Pharmaceutical Model

Let us consider a nonlinear model about glucose-oxydase pharmacokinetics studied by (Verdiere et al., 2005). The system is given by:

$$\begin{cases} \dot{x}_1 = & k_{12}(x_2-x_1)-k_v\dfrac{x_1}{1+x_1}, \quad x_1(0)=x_{10}, \\ \dot{x}_2 = & k_{21}(x_1-x_2), \quad x_2(0)=0, \\ y = & x_1. \end{cases} \tag{11}$$

Where $x_1$ is the enzyme concentration in plasma (compartment 1), $x_2$ is concentration in compartment 2 and $k_{12}$ is the rate constant of the transfer from compartment 1, practically plasma, to compartment 2, is the sum of all the transcapillary transfers in all the organs. Furthermore, $k_{21}$ is the rate constant of the transfer from compartment 2 to compartment 1, $k_v$ is the maximum rate of an uptake by macrophages through the mannose receptor. The receptor-mediated uptake is a cellular process taking place at the level of the macrophage membrane. The parameters to be estimated are $p = [k_{12}, k_{21}, k_v]$ which are assumed to be uncertain.

The initial condition is supposed to $x_1 = 1$ and the measurement is only available on $x_1$. The fault is supposed single and no time variant, already existed before the diagnosis, which indicating $k_v = 0.092$ at $t = 0$. The interchange parameters are supposed to $k_{12} = 0.011$ and $k_{21} = 0.02$ which are in right value. Knowing that the right parameters are equal to $p = [0.011, 0.02, 0.1]$. Under the measurements error assumption $v = [-0.005, 0.005]$ over all time, we know that $\hat{k}_v \in [0.0974, 0.1026]$. For a global consideration of any other fault, we set the a priori parameter interval as following :

$$p = \begin{bmatrix} 0.0099, & 0.0121 \\ 0.018, & 0.022 \\ 0.08, & 0.11 \end{bmatrix}. \quad (12)$$

The sampling time is $t = 117s$ and we measure the $x_1$ at every second. The following figures have been obtained by the same parameter estimation algorithm with $\varepsilon = 0.0001$. Smaller values have been tested by no more obvious improvement in quantity of acceptable sets are obtained. The results for admissible sets are presented in figure 2. The admissible sets for each parameter are supposed to be included in:

$$p_{admis} \in \begin{bmatrix} 0.0099, & 0.0121 \\ 0.0180, & 0.0220 \\ 0.0891, & 0.0948 \end{bmatrix} \quad (13)$$

The fault parameter has been well localized as the intersection of admissible sets for $k_v$ between the estimated parameter and the a priori known parameter is empty which refers to a fault. Besides, as the $k_v = [0.0891, 0.0948]$ is resided completely in unacceptable parts, this resolution of fault distinction could be described by the Hausdorff distance $H(p_3, \hat{p}_3) = 0.0083$. Next, a fault tree which indicates the level of fault and gives corresponding actions.

In the figure 2, the searching domain of $k_{12}$ and $k_{21}$ are full filled with acceptable boxes, which implies that the measurement has too big error or the sampling time is not sufficient, that set inversion algorithm is not able to generate the solution with such data. Such situation will be problematic during multiple occurrence of fault. Besides, it may happen that the intersection of estimated parameter and the admissible right parameter is not empty. To ameliorate this

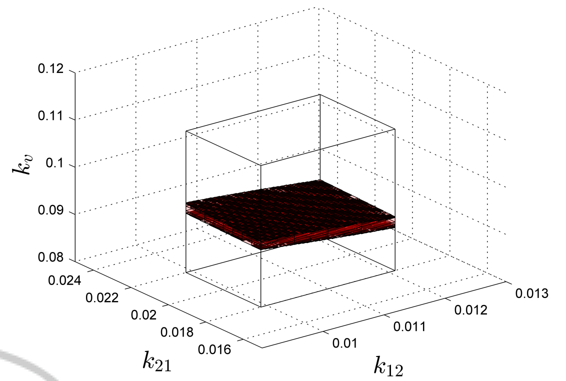


Figure 2: Outer approximation of admissible sets of $k_{12}$, $k_{21}$ and $k_v$.

phenomena, one could use an optimal input design or an optimal initial state design to get better estimated results from presumed bounding errors on system output or use more accurate sensors. As we are seeing a 3D figure from a 2D view point, the parts are superposed, the ones in the middle are darker than the outsides.

We have tried different configurations for parameter estimation, the distance check technique combines the parameter estimation procedure is the optimal one in term of time efficiency, see Table 2.

Table 2: Time consuming for each configuration of parameter estimation.

| Time (s) / $\varepsilon$ | SIVIA | Distance check | SIVIA with Contractor |
|---|---|---|---|
| 0.0005 | 10 | 9 | 13 |
| 0.0001 | 321 | 310 | 4897 |
| 0.00005 | 1267 | 1242 | 18972 |

## 5 CONCLUSION

The original proposed diagnosis procedure using set inversion of interval analysis may be an effective tool to detect, localize and identification of fault. The proposed distance check is the first talked about in such context. The only question is the choice of $C$ which is experimental. In practice, as the diagnosis is off line, one could start with a value compared to the sum of width on error bounds of measurement, decreasing it when no more pessimistic solutions are mistaken.

In diagnosis, a guaranteed solution is always useful to localize surely the faulty parameters. With enough precision and multi fault occurrences on estimated results, this information could be evaluated through a fault tree analysis. According to the presumption made on the error bound, the results are fully rigorous.

In this paper, we proposed an original diagnosis procedure, the distance check technique is at the first time talked about, which helps to eliminate the unfeasible sets more efficiently in parameter estimation context using interval analysis. Comparing other available methods, one could use our approach to get a more rapid parameter estimation. Similar to the contractor with set inversion algorithm, this technique is useful when the unacceptable parts are large of the initial search domain. If it is not the case, one have to use other optimization methods to obtain the results more quickly.

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