Order-up-to Networked Policy for Periodic-Review Goods Distribution Systems with Delay

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Abstract: In this paper, inventory control problem in goods distribution networks with non-negligible transshipment delay is addressed. In contrast to the majority of earlier approaches, system modeling and policy design do not assume simplified system structure, such as a serial, or a tree-like one. The network nodes, in addition to satisfying market demand, answer internal requests with delay spanning multiple periods. The stock in the network is refilled from uncapacited outside sources. A dynamic model of the considered class of goods distribution systems is constructed and a new inventory policy is formulated. The proposed policy shares similarity with the classical order-up-to one, yet provide improved performance owing to the networked perspective assumed in the design process.

1 INTRODUCTION

The formal and computational difficulties have directed the research effort in logistic system optimization and control mainly to single stage (Hoberg et al., 2007; Ignaciuk & Bartoszewicz, 2011), serial (Song, 2009, Movahed & Zhang, 2013), or tree-like configurations (Kim et al., 2005; Ignaciuk, 2014). The new information and communication technology advancements permit now large-scale deployment of management solutions in more complex – networked – settings. However, as is the case of simplified structures considered so far in the literature, the crucial aspect behind the efficient operation and cost reduction in a networked system is implementation of an appropriate inventory control policy.

In this work, logistic networks with arbitrary, mesh topology are considered in periodic-review mode of operation. The stock replenishment orders are realized with non-negligible lead-time delay that may span multiple review periods. The external demand is represented by uncertain, time-varying functions, accepting any stochastic process typically considered in inventory control problems. A dynamic model of network node interactions is constructed. Since the localized view of the goods flow process reflected in the classical ordering policies, e.g. order-up-to (OUT) one, may generate significant cost increase in a real-world installation (Cattani et al., 2011), an alternative – networked – policy is proposed. The designed policy, while sharing functional similarities with the classical one, shows improved dynamical characteristics and generates smaller costs.

2 PROBLEM SETTING

2.1 System Dynamics

The goods distribution system to be controlled encompasses N nodes with the indices from the set $\Omega_N = \{1, 2, ..., N\}$. The overall amount of goods in the system is refilled from external sources. The set of all node indices, including the controlled nodes and external sources, $\Omega_M = \{1, 2, ..., M\}$, $M \geq N$.

Let $k = 0, 1, 2, ...$ be the independent variable denoting subsequent review periods. The stock balance equation at controlled node c, $c \in \Omega_N$:

$$x_c(k+1) = x_c(k) - d_c(k) + \underbrace{\sum_{s\in\Omega_M} a_{sc} u_c(k - L_{sc})}_{\text{goods delivered to node c}} - \underbrace{\sum_{r\in\Omega_N} a_{cr} u_r(k - T_c)}_{\text{goods served by node } c} \quad (1)$$

where:

- $x_c(k)$ is the amount of goods (on-hand stock) readily available at node c in period k;

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- $u_c(k)$ is the amount of goods requested by node c in period k to replenish its stock;
- a_{sc} is the part of the overall order $u_c(k)$ to be acquired from node $s \in \Omega_M$ by node c;
- $L_{sc} = T_s + T_{sc}$ is the lead-time delay in providing the goods from node s to node c, $L_{sc} \in \{2, 3, ..., L\}$, L denotes the maximum lead-time delay in goods transfer between any two neighboring nodes;
- T_s is the order processing time at node s, including all the activities related to preparing the order for the requesting node, $T_s \in \{1, 2, ..., L-1\}$,
- T_{sc} is the time of transporting the goods from node s to node c, $T_{sc} \in \{1, 2, ..., L-1\}$,

The (external) demand is modeled as an uncertain, bounded function of time $0 \le d_c(k) \le d_c^{\max}$, where $d_c^{\max} \ge 0$ denotes the upper estimate of $d_c(k)$. No assumption is taken regarding the nature of stochastic process describing the evolution of d_c.

Without loss of generality the network is assumed connected (there is no isolated node) and full order partitioning takes place, i.e.

$$\forall_c \sum_{s \in \Omega_M} a_{sc} = 1. \qquad (2)$$

When treated as a graph, although arbitrary flow orientation is permitted, the network is also assumed directed, i.e. if $a_{sc} \neq 0$, then $a_{cs} = 0$ for c, $s \in \Omega_N$. Moreover, for any $c \in \Omega_N$, $a_{cc} = 0$, so that no controlled node is a source of goods for itself.

According to (1), the goods to other nodes within the controlled network are sent with at least one period delay that covers the time to prepare all the necessary documentation and shipment. Meanwhile, the demand is served immediately in period k, which implies that answering the external requests (demand) takes precedence over the internal goods traffic. Each node may serve the requests coming from other nodes inside the managed network as well as answer the external demand, which closely reflects the actual real-life settings (Cattani et al., 2011).

2.2 Classical Out Policy

In order to replenish the stock depleted according to market demand (and internal requests) at a network node, the OUT policy may be applied. When demand forecasting is not used, the OUT policy calculates the order quantities according to (Silver et al, 1998):

$$u_c(k) = x_c^{out} - x_c(k) - \sum_{s \in \Omega_M} \sum_{j=k-L_{sc}}^{k-1} a_{sc} u_c(j), \qquad (3)$$

where the sum represents the work-in-progress (the order placed but not yet realized as a result of delay).

In the serial and tree-like topologies, it can be shown that with sufficiently large x_c^{out}, for $k > 0$, the orders generated by the OUT policy according to (3) satisfy (Ignaciuk & Bartoszewicz, 2012)

$$u_c(k) = d_c(k-1), \qquad (4)$$

i.e. the ordering signal issued in a current period matches the demand from the previous one. In consequence, the bullwhip effect is prevented. Unfortunately, in the case of networked system with full (mesh) connectivity this favorable property does not hold. In order to mitigate the negative influence of demand variability on the costs in the network, one may apply alternative (local) strategies with smoothing properties, e.g. (Ignaciuk, 2012), or, as is proposed in this work, construct a new policy taking into account the network dynamics.

3 NETWORKED MODEL

In order to formulate a networked inventory policy, it is convenient to describe model (1) in an appropriately chosen state space. The following state-space representation is proposed:

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \sum_{j=1}^{L} \mathbf{B}_j \mathbf{u}(k-j) - \mathbf{d}(k), \qquad (5)$$

where:

- $\mathbf{x}(t) = [x_1(t) \ \ldots \ x_N(t)]^T$ is the vector of on-hand stock levels inside the controlled network;
- $\mathbf{u}(t) = [u_1(t) \ \ldots \ u_N(t)]^T$ is the vector of stock replenishment signals;
- $\mathbf{d}(t) = [d_1(t) \ \ldots \ d_N(t)]^T$ is the vector of demands imposed at the controlled nodes with $\mathbf{d_{max}} = [\, d_1^{\max} \ldots \ d_N^{\max} \,]^T$ grouping the information about the demand upper bounds;
- matrices

$$\mathbf{B}_j = \begin{bmatrix} \sum_{i:L_{i1}=j} a_{i1} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & \sum_{i:L_{i1}=j} a_{i2} & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & \sum_{i:L_{i1}=j} a_{i3} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & b_{n-1,n} \\ b_{n1} & b_{n2} & b_{n3} & \cdots & \sum_{i:L_{in}=j} a_{in} \end{bmatrix} \qquad (6)$$

for $j = 1, ..., L$ hold the information about the node interconnections; the elements on the main diagonal reflect the goods acquisition with lead time j (incoming shipments), whereas the off-diagonal ones

$$b_{iw} = \begin{cases} -a_{iw}, \text{ if } T_i = j, \\ 0, \text{ otherwise,} \end{cases} \tag{7}$$

with $w \in \Omega_N$, correspond to the goods provision with processing time j (the outgoing shipments inside the network).

For further derivations, it is also convenient to define

$$\mathbf{B} = \sum_{j=1}^{L} \mathbf{B}_j. \tag{8}$$

It follows from (2) that $\mathbf{B} = \mathbf{I} + \mathbf{E}$, where $\mathbf{I}$ denotes an $N \times N$ identity matrix and $\mathbf{E}$ is a hollow matrix with entries $a_{ij} \in [-1, 0]$ column-wise summing at most to –1, is invertible.

4 NETWORKED OUT POLICY

4.1 Proposed Inventory Policy

Let $\mathbf{x_d} = [\, x_1^{out} \;...\; x_N^{out} \,]^T$ denote the vector of target inventory levels. The proposed policy for the considered goods distribution system establishes the orders using the equation

$$\mathbf{u}(k) = \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(k) - \sum_{i=1}^{L}\sum_{j=i}^{L} \mathbf{B}_j \mathbf{u}(k-i)]. \tag{9}$$

4.2 Policy Properties

Assuming zero initial input, i.e. $\mathbf{u}(k) = 0$ for $k < 0$, from (5), the stock level in arbitrary period $k > 0$ can be expressed as

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{x}(0) + \sum_{i=1}^{L}\sum_{j=0}^{k-1} \mathbf{B}_i \mathbf{u}(j-i) - \sum_{j=0}^{k-1} \mathbf{d}(j) \\ &= \mathbf{x}(0) + \sum_{i=1}^{L} \mathbf{B}_i \sum_{j=0}^{k-i-1} \mathbf{u}(j) - \sum_{j=0}^{k-1} \mathbf{d}(j). \end{aligned} \tag{10}$$

At the initial time, $\mathbf{u}(0) = \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(0)]$. Afterwards, for any $k > 0$, the ordering signal satisfies the condition specified in the following theorem.

Theorem 1. For $k > 0$, the stock replenishment signal established according to (9) for system (5) satisfies

$$\mathbf{u}(k) = \mathbf{B}^{-1}\mathbf{d}(k-1). \tag{11}$$

Proof. First note that (9) can be equivalently written as

$$\mathbf{u}(k) = \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(k) - \sum_{i=1}^{L} \mathbf{B}_i \sum_{j=k-i}^{k-1} \mathbf{u}(j)]. \tag{12}$$

Substituting (10) into (12), yields

$$\mathbf{u}(k) = \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(0) - \sum_{i=1}^{L} \mathbf{B}_i \sum_{j=0}^{k-1} \mathbf{u}(j) + \sum_{j=0}^{k-1} \mathbf{d}(j)], \tag{13}$$

and using (8),

$$\mathbf{u}(k) = \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(0) - \mathbf{B}\sum_{j=0}^{k-1} \mathbf{u}(j) + \sum_{j=0}^{k-1} \mathbf{d}(j)]. \tag{14}$$

Then, applying (14), $\mathbf{u}(k+1)$ can be expressed as

$$\begin{aligned} \mathbf{u}(k+1) &= \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(0) - \mathbf{B}\sum_{j=0}^{k} \mathbf{u}(j) + \sum_{j=0}^{k} \mathbf{d}(j)] \\ &= \mathbf{B}^{-1}[\mathbf{x_d} - \mathbf{x}(0) - \mathbf{B}\sum_{j=0}^{k-1} \mathbf{u}(j) + \sum_{j=0}^{k-1} \mathbf{d}(j)] \\ &\quad - \mathbf{B}^{-1}[\mathbf{B}\mathbf{u}(k) - \mathbf{d}(k)] \\ &= \mathbf{u}(k) - \mathbf{B}^{-1}\mathbf{B}\mathbf{u}(k) + \mathbf{B}^{-1}\mathbf{d}(k) = \mathbf{B}^{-1}\mathbf{d}(k), \end{aligned} \tag{15}$$

which ends the proof.

Let

$$\mathbf{z}(k) = \mathbf{x}(k) + \sum_{i=1}^{L}\sum_{j=i}^{L} \mathbf{B}_j \mathbf{u}(k-i), \; \mathbf{z} \in \mathbb{R}^N, \tag{16}$$

which represents a network analogue of inventory position (sum of on-hand stock and open orders).

Proposition 2. The dynamics of $\mathbf{z}(t)$ can be described by

$$\mathbf{z}(k+1) = \mathbf{z}(k) + \mathbf{B}\mathbf{u}(k) - \mathbf{d}(k). \tag{17}$$

Proof. Taking into account the zero initial input, directly from the definition of $\mathbf{z}$ one obtains $\mathbf{z}(0) = \mathbf{x}(0)$. In turn, applying (5) to (17), yields

$$\begin{aligned} \mathbf{z}(1) &= \mathbf{x}(1) + \sum_{i=1}^{L}\sum_{j=i}^{L} \mathbf{B}_j \mathbf{u}(1-i) \\ &= \mathbf{x}(0) + 0 - \mathbf{d}(0) + \sum_{j=1}^{L} \mathbf{B}_j \mathbf{u}(0) \\ &= \mathbf{z}(0) + \mathbf{B}\mathbf{u}(0) - \mathbf{d}(0), \end{aligned} \tag{18}$$

thus showing that (17) is satisfied in period $k = 0$. Then, using (5), for arbitrary $k > 0$ the following relation can be established

$$\begin{aligned} \mathbf{z}(k+1) &= \mathbf{x}(k+1) + \sum_{i=1}^{L}\sum_{j=i}^{L} \mathbf{B}_j \mathbf{u}(k+1-i) \\ &= \mathbf{x}(k) + \sum_{i=1}^{L}\sum_{j=i}^{L} \mathbf{B}_j \mathbf{u}(k-i) + \mathbf{B}\mathbf{u}(k) - \mathbf{d}(k) \\ &= \mathbf{z}(k) + \mathbf{B}\mathbf{u}(k) - \mathbf{d}(k), \end{aligned} \tag{19}$$

which ends the proof.

Theorem 3. System (5) under the control of policy (9) is bounded-input-bounded-output stable.

Proof. Since finite $\mathbf{u}$ and $\mathbf{x}$ yield finite $\mathbf{z}$, and $\mathbf{d}$ influences (5) and (17) in the same way, the stability assessment of both systems subject to policy (9) is equivalent. Substituting $\mathbf{u}(k) = \mathbf{B}^{-1}\mathbf{d}(k-1)$ into (17) yields

$$\mathbf{z}(k+1) = \mathbf{z}(k) + \mathbf{d}(k-1) - \mathbf{d}(k), \tag{20}$$

which implies that $\mathbf{z}(t)$ (and thus $\mathbf{x}(t)$) is bounded for any bounded demand. This conclusion ends the proof.

4.3 Selection of Target Stock Level

A successful control policy in modern supply networks is expected to achieve a high service level. In this work, the service level is quantified by the demand fill rate, i.e. the part of imposed demand realized from readily available resources at the nodes. The fill rate is influenced by the choice of target stock level. Owing to the overall complexity of the networked system interconnections, the optimal target stock $\mathbf{x_d}$ needs to be determined through numerical computations for given network parameters – demands imposed at the nodes and inter-node lead time. Below, an intuitive procedure to calculate $\mathbf{x_d}$ for minimizing backlog and thus obtaining high fill rate is shown. The procedure assumes only the knowledge about the demand upper estimate $\mathbf{d_{max}}$ (not its statistical parameters).

It follows from Theorem 1 that steady-state replenishment signal $\mathbf{u_{ss}}$ in response to steady-state demand $\mathbf{d_{ss}}$ satisfies

$$\mathbf{u}_{ss} = \mathbf{B}^{-1}\mathbf{d}_{ss}. \tag{21}$$

Then, using (9) and (21), the steady-state stock level is determined as

$$\begin{aligned}\mathbf{x}_{ss} &= \mathbf{x_d} - \sum_{i=1}^{L}\sum_{j=k-i}^{k-1}\mathbf{B}_i\mathbf{u}_{ss} - \mathbf{B}\mathbf{u}_{ss} \\ &= \mathbf{x_d} - \sum_{i=1}^{L} i\mathbf{B}_i\mathbf{u}_{ss} - \mathbf{B}\mathbf{u}_{ss} \\ &= \mathbf{x_d} - \sum_{i=1}^{L} i\mathbf{B}_i\mathbf{B}^{-1}\mathbf{d}_{ss} - \mathbf{d}_{ss}.\end{aligned} \tag{22}$$

In the worst case, $\mathbf{d_{ss}} = \mathbf{d_{max}}$. Thus, setting

$$\mathbf{x_d} > (\sum_{i=1}^{L} i\mathbf{B}_i\mathbf{B}^{-1} + \mathbf{I})\mathbf{d}_{\max} \tag{23}$$

will result in reduced backlog.

On the other hand, in the absence of external demand $\mathbf{x_{ss}} = \mathbf{x_d}$. It follows from the numerical analysis presented in the next section that the proposed policy provides oscillation and overshoot free stock level evolution. Therefore, setting the warehouse capacity at the network nodes equal to $\mathbf{x_d}$ gives enough space to store the goods locally. The stock level $\mathbf{x}(k) \le \mathbf{x_d}$ and costly emergency storage outside the controlled network is not required.

5 SIMULATION EXAMPLE

Let us consider the goods distribution network illustrated in Fig. 1. Nodes 1–5 are managed by a single organization – they constitute the controlled elements – while nodes 6–8 are the exogenous sources used to replenish the stock inside the network. Nodes 1 and 2 constitute the contact points with the external market, responding to demands d_1 and d_2. Nodes 3 and 5 serve as intermediate suppliers and node 4 represents a distribution centre. The arrows in the graph indicate the flow of goods. With each connection there is associated a pair of values (a_{ij}, L_{ij}): a_{ij} denotes the fraction of stock replenishment signal issued by node j for node i and L_{ij} is the delay in procuring orders from node i to j. The processing time at each node equals one period.

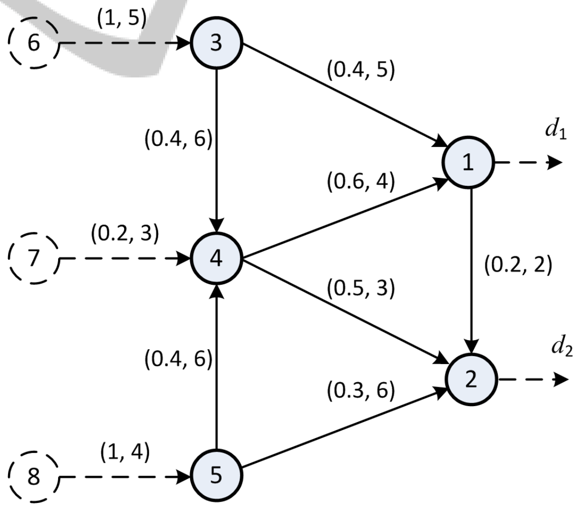

Figure 1: Goods distribution system.

The initial condition $\mathbf{x}(0) = [80\ \ 80\ \ 60\ \ 60\ \ 60]^T$ units and the target stock level, selected according to (23) with $\mathbf{d_{max}}$ estimate $[20\ \ 20\ \ 0\ \ 0\,0]^T$ units as $\mathbf{x_d} = [125\ \ 100\ \ 80\ \ 110\ \ 50]^T$ units. It is also assumed that there is no goods in transit before the control process commences, i.e. $\mathbf{u}(k) = 0$ for $k < 0$.

The performance of proposed networked policy (9) is compared with benchmark local policy (3). The test proceeds in two phases: for $k < 15$ – the goods accumulation phase (from $\mathbf{x}(0)$ to $\mathbf{x}_d$) in the absence of demand; for $k \geq 15$ – reaction to the highly variable, uncertain demand following the Poisson process with mean equal to 15 units.

Table 1: Bullwhip indicator.

Policy	**Network**		**Local**	
Node	1	2	1	2
BI	0.974	1	1.024	1

The graphs depicted in Figs. 2 and 3 indicate that both policies make the stock level converge to the target value and, afterwards, follow the trend set by mean demand. Local OUT policy (graph b) requires larger safety stock to prevent backlog (occurring for negative stock level) which translates to higher holding costs with respect to the networked policy (graph a). The local OUT policy also generates a larger ripple in response to highly variable demand. According to the bullwhip indicator (BI) data – order-to-demand variance ratio (Chen et al., 2000) – grouped in Table 1, the networked policy successfully counteracts demand variations from affecting the ordering signal.

6 CONCLUSIONS

The paper presents a new inventory control policy for networked goods distribution systems. The policy ensures stable system performance in the presence of arbitrary delay in goods provision. The proposed policy outperforms the classical OUT one by avoiding oscillations and backlog, thus showing the benefits of adopting *networked* perspective in

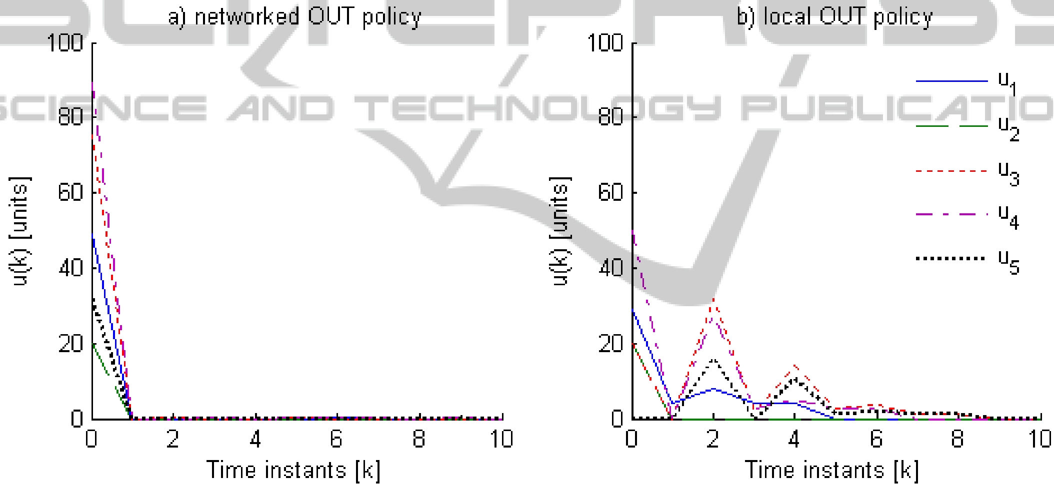

Figure 2: Control input: a) networked policy, b) local policy.

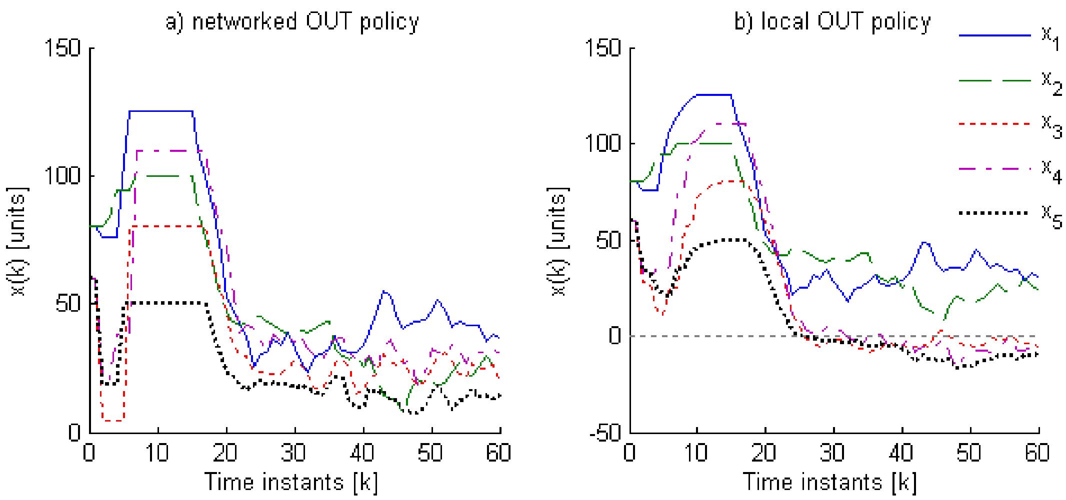

Figure 3: Stock level at the nodes: a) networked policy, b) local policy.

The paper presents a new inventory control policy for networked goods distribution systems. The policy ensures stable system performance in the presence of arbitrary delay in goods provision. The proposed policy outperforms the classical OUT one by avoiding oscillations and backlog, thus showing the benefits of adopting *networked* perspective in control scheme design. However, the internal traffic may still lead to the bullwhip effect. If order smoothening is of priority, one should seek alternative networked strategies. Also new, more realistic measures of the bullwhip effect in the networked environment should be developed.

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