

Discrete Sliding Mode Control for a VCM Positioning System

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Abstract: In this paper, a discrete control system is implemented for a positioning device using a voice-coil motor (VCM). The VCM positioning system is configured with a proportional-integrator observer (PIO) and discrete sliding mode controller (DSMC). Since the PIO could estimate system unmeasurable parameters for compensation, the implemented control system subject to uncertainty might feature high robustness. Through experimental examinations of step response for a sliding stage under dry friction and with a mass of 728 g, the position error of 7.3 μm was obtained for a step command of 3 mm. The percentage of position error is 0.25%. Compared with that obtained by using the PID controller is 0.57%, the superiority of the implemented control system is demonstrated.

1 INTRODUCTION

Positioning system is one of the fundamental technologies that supports the development of precision machinery such as machine tool. Recently, due to significant progress in precision industry, the needs for miniaturized devices are increasingly presented. In positioning system, the direct drive that can transmit power without gear reducer might feature compact size. This is very attractive in miniaturization. Therefore, the direct drives using linear actuators such as piezoelectric actuator, piezo-magneto actuator, ultrasonic motor, linear motor, voice-coil motor (VCM), and static electric actuator, etc., are well found in industry.

In this paper, the control performance of the positioning system using the VCM is studied. The actuation of VCM is based on the principle of electromagnetic effect. Due to its simple structure, compact size, and high precision, popular applications can be well found in hard disc drive and automatic image focusing device. However, these applications are with constraint conditions of small load and almost friction-free motion. It is well known that the friction force behaves remarkable nonlinearity in microscopic motion. Therefore, the positioning device under dry friction is very difficult

to obtain high precision positioning. To cope with the nonlinear system, controller design with excellent robustness is very essential. In this study, the control scheme of the discrete sliding mode controller (DSMC) coupled with proportional-integrator observer (PIO) is proposed for the VCM positioning system under dry friction. The proposed DSMC coupled with PIO is aimed at improving robustness of the positioning system and providing compensating function for external disturbances.

Relating to the controller design, a PIO was verified as effective in estimating system state and disturbance (Hsu, 2007). Regarding the DSMC, two approximation laws of exponential and variable rate were examined. Due to the drawbacks of current approximation laws, two new approximation laws being capable of reducing control chattering phenomenon of switching surface were proposed. Based on numerical simulations, a more stable system locating at the origin was verified (Yan, 2006). Moreover, an adaptive DSMC coupled with new exponential approximation law was proposed. This system was shown as stable at the origin and successfully applied to a DC motor driving system capable of tracing a reference signal (Lizhong, 2007). Different to the control algorithm, the switching control of delay time was focused for examining the

DSMC. Based on the Lyapunov function, a sufficient condition was given for designing a controller, which could drive the system moving to the sliding surface and guarantee the existence of the sliding surface. Through numerical examination, the effectiveness of the system having controllability was demonstrated (Yu, 2013). In addition, the DSMC being applied to a higher-order system plus delay time was proposed. Based on the Lyapunov function, a stable existing condition of sliding mode was obtained. This method without the need of reducing system order could obtain better control and tracing performance compared to that using the PID controller (Khandekar, 2013). Recently, the DSMC was proposed for precision position control of the piezoelectric actuating system. A model was derived for compensating the hysteresis effect. Through experimental examination, the proposed DSMC was verified with more excellent performance by comparing with that using the PID controller (Xu, 2013).

Relating to the positioning device using the VCM, a method of simultaneous perturbation stochastic approximation (SPSA) was used to suitably tune the PID gains for the VCM positioning system. Through experimental examination, a more stable and faster positioning performance could be obtained (Ming, 2005). To determine the optimal actuating condition, the nonlinear double dynamic Taguchi Method was applied to the combined piezo-VCM actuating system (Liu, 2007). A new type of VCM having shorted turn was developed. This type of VCM featured faster rising time in establishing magnetic field, thus with higher acceleration (Liu, 2010). A neural network based on radial basis function and coupled with the PID was proposed for the VCM positioning system. Through numerical simulations, excellent robustness of the system was shown (Gao, 2011). Through the above-mentioned literature survey, the sliding mode control (SMC) having excellent robustness can be widely found in various applications. The proposed control scheme in this paper will be verified as effective for the system subject to high nonlinearity and external disturbance through the following approaches.

2 PHYSICAL MODEL

In this section, main components of the VCM positioning system are described, and then the physical model is established, which will be used for controller design.

2.1 Positioning Device

Figure 1 shows the experimental setup for examining the positioning device using the VCM. The main components of the positioning system include a VCM, a sliding stage, and a frictional adjusting mechanism. The sliding stage having a mass of 728 g and with a dimension of $35 \times 25 \times 130 \text{ mm}^3$ is set on a V-grooved base for the motion with one degree-of-freedom. A linear encoder with a resolution of 5 nm is mounted beside the sliding stage.

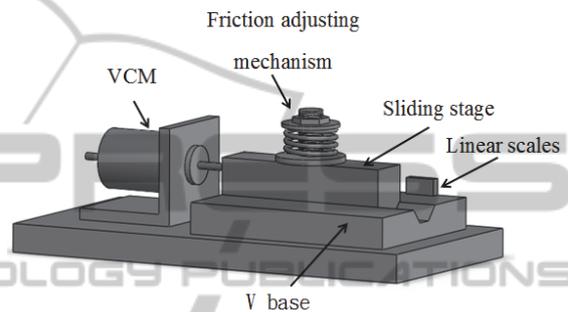


Figure 1: Main components of the VCM positioning system.

2.2 VCM Model

The VCM used is a linear actuator featuring compact size, high precision, and high response actuating ability. Figure 2 shows the schematic drawing of the VCM structure. The equivalent circuit based on the motor structure can be depicted as shown in Figure 3, where V_{vcm} is the applied voltage for the VCM, i is coil current, R is coil resistance, L is coil inductance, e_m is back electromotive force(emf), and x is the displacement of the moving shaft.

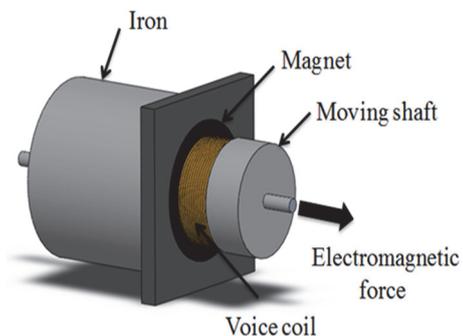


Figure 2: Schematic drawing of VCM.

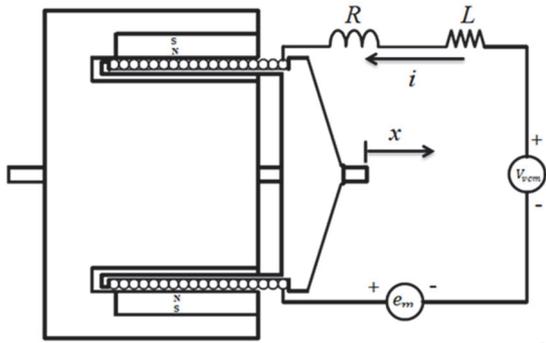


Figure 3: Equivalent electric circuit of VCM.

According to the Kirchhoff's voltage law, the equivalent circuit can be expressed as follows,

$$\begin{aligned} V_{vcm} &= Ri + L \frac{di}{dt} + e_m \\ e_m &= K_m \dot{x} \end{aligned} \quad (1)$$

where K_m is a back emf constant, and \dot{x} is a time derivative of displacement, i.e., the speed of the moving shaft.

2.3 Positioning System Model

Referring to the schematic drawing shown in Figure 1, a brief drawing expressing the main components is shown in Figure 4.

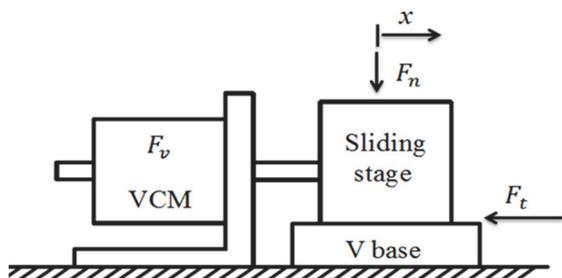


Figure 4: Main components of the VCM positioning system.

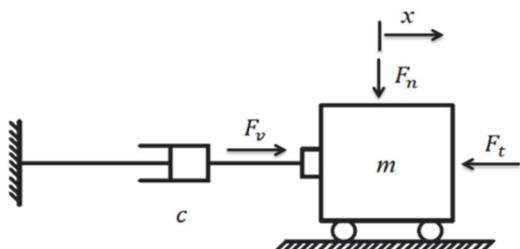


Figure 5: Free diagram of the VCM positioning system.

Focusing on the physical parameters, the model of the positioning device can be depicted as shown in Figure 5, where m is the mass of the sliding stage, c is the damping coefficient representing the nature of the VCM, F_n is the normal force subjected to the frictional adjusting device, and F_t is the dry frictional force existing between the sliding surfaces. According to the Newton's second law, a dynamic equation can be derived as follows,

$$\begin{aligned} m\ddot{x} + c\dot{x} &= F_v - F_t \\ F_v &= K_v i \end{aligned} \quad (2)$$

where F_v is the electromotive force generated by the VCM, and K_v is a force constant; F_t can be represented by a nonlinear continuous equation as follows,

$$\begin{aligned} F_t &= \mu(\dot{x}) F_n' \operatorname{sgn}(\dot{x}) + \sigma \dot{x} \\ \mu(\dot{x}) &= \mu_k + (\mu_s - \mu_k) \exp\left[-\left(\frac{|\dot{x}|}{\dot{x}_s}\right)^2\right] \end{aligned} \quad (3)$$

where F_n' is the total normal force, σ is the viscous damping coefficient, \dot{x}_s represents a reference velocity and shows the sensitivity level of the sliding velocity affecting on the frictional coefficient, and μ_k and μ_s are kinematic and static friction coefficients, respectively.

2.4 State Variables

In this study, the objective is to precisely control the displacement of sliding stage via the controlled voltage, V_{vcm} for the VCM. The state variables relating the positioning system are given as,

$$\begin{aligned} x_1 &= x, \text{ the displacement of sliding stage} \\ x_2 &= \dot{x}, \text{ the speed of sliding stage} \\ x_3 &= i, \text{ the coil current of VCM} \\ u &= V_{vcm}, \text{ the applied voltage for VCM} \end{aligned}$$

Using the state variables, Eqs. (1) and (2) can be represented by the following forms,

$$\begin{aligned} m\dot{x}_2 + cx_2 &= K_v x_3 - F_t \\ u &= Rx_3 + L\dot{x}_3 + K_m x_2 \end{aligned} \quad (4)$$

Eq. (4) is rearranged into the state equation as follows,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{c}{m} & \frac{K_v}{m} \\ 0 & -\frac{K_m}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \frac{F_t}{m} \quad (5)$$

Table 1: Parameters of the VCM positioning system.

Symbol	Unit	Value
m	kg	0.63
c	Ns/m	1.778
L	H	94×10^{-3}
R	Ω	3.657
K_v	N/A	4.029
K_m	V/(m/s)	4.029
μ_k		0.25
μ_s		0.3
F_n	N	6.18
\dot{x}_s	m/s	0.001
σ		0.4

With the parameters listed in Table 1 (Liu, 2005), the continuous state equation is transformed into the discrete state equation by using MATLAB package with the zero-order hold (ZOH) input and sampling period of 0.01 s.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.009819 & 0.002787 \\ 0 & 0.9603 & 0.05195 \\ 0 & -0.3482 & 0.6672 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1.023 \times 10^{-5} \\ 0.002965 \\ 0.08773 \end{bmatrix} u + \begin{bmatrix} -7.84610 \times 10^{-5} \\ 0.002965 \\ 0.08773 \end{bmatrix} d \quad (6)$$

where u and d are the controlled input and external disturbance, respectively.

3 CONTROLLER DESIGN

3.1 Proportional-Integral Observer

Considering Eq. (6), the state equation can be expressed in the form as,

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ed(k) \\ y(k) &= Hx(k) \end{aligned} \quad (7)$$

where $x(k) = [x_1(k) \ x_2(k) \ x_3(k)]^T$ represents state vector, $A \in \mathfrak{R}^{n \times n}$ is the system matrix, $B \in \mathfrak{R}^{n \times 1}$ is an input vector, $E \in \mathfrak{R}^{n \times 1}$ is an error vector, $u(k)$ is the controlled input, $d(k)$ is an external disturbance, $H \in \mathfrak{R}^{1 \times n}$ is an output vector, and $y(k)$ is the system output. In this study, a proportional-integral observer (PIO) is proposed to estimate unknown system states and external disturbance of the VCM positioning system. The structure of the PIO is given as,

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + L_1[y(k) - \hat{y}(k)] + E\hat{d}(k) \\ y(k) &= H\hat{x}(k) \\ \hat{d}(k+1) &= \hat{d}(k) + L_2[y(k) - \hat{y}(k)] \end{aligned} \quad (8)$$

where $L_1 \in \mathfrak{R}^{n \times 1}$ and $L_2 \in \mathfrak{R}$ are the designed gains of the PIO, $\hat{x}(k)$ and $\hat{d}(k)$ are the observed values of the system states and external disturbance. The observed error vector of system state and the observed error of system output are defined as $e(k) = x(k) - \hat{x}(k)$ and $\tilde{y}(k) = y(k) - \hat{y}(k)$, respectively. From Eqs. (7) and (8), the error term can be derived as,

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= Ax(k) + Bu(k) + Ed(k) - A\hat{x}(k) \\ &\quad - Bu(k) - L_1(y(k) - \hat{y}(k)) - E\hat{d}(k) \\ &= Ax(k) + Ed(k) - A\hat{x}(k) - L_1(y(k) \\ &\quad - \hat{y}(k)) - E\hat{d}(k) \\ &= (A - L_1H)e(k) + Ed(k) - E\hat{d}(k) \end{aligned} \quad (9)$$

Similarly, let $\eta(k) = d(k) - \hat{d}(k)$, then $\eta(k)$ can be expressed as,

$$\begin{aligned} \eta(k+1) &= d(k+1) - \hat{d}(k+1) \\ &= \eta(k) + (d(k+1) - d(k)) \\ &\quad - (\hat{d}(k+1) - \hat{d}(k)) \\ &= \eta(k) - L_2He(k) + (d(k+1) - d(k)) \end{aligned} \quad (10)$$

Rearrange Eqs. (9) and (10) into state equations as,

$$\begin{bmatrix} e(k+1) \\ \eta(k+1) \end{bmatrix} = \begin{bmatrix} A - L_1H & E \\ -L_2H & I_1 \end{bmatrix} \begin{bmatrix} e(k) \\ \eta(k) \end{bmatrix} + \begin{bmatrix} 0 \\ d(k+1) - d(k) \end{bmatrix} = (M - LG) \begin{bmatrix} e(k) \\ \eta(k) \end{bmatrix} \quad (11)$$

$$\tilde{y} = [H \ 0] \begin{bmatrix} e(k) \\ \eta(k) \end{bmatrix} = G \begin{bmatrix} e(k) \\ \eta(k) \end{bmatrix}$$

where $M = \begin{bmatrix} A & E \\ 0 & I_1 \end{bmatrix}$, $L = [L_1^T \ L_2^T]^T$, and $G = [H \ 0]$.

From Eq. (11), if (M, G) is observable, the gain of observer L could be designed by the pole-placement method such that the eigenvalues of the matrix $M - LG$ might lie in the unit circle, resulting in asymptotically stable control of the positioning system.

3.2 Design of Sliding Mode Controller

The controller is implemented by the DSMC coupled with PIO. In designing the controller, at first, the sliding surface is constructed based on the error term between the reference values and the estimated state variables from the PIO; then, the control input u is derived based on the estimated state variables coupled with the approaching law provided in the work (Li, 2011). The approaching law is expressed as

$$s(k+1) = (1 - \gamma T)s(k) - \ln(|s(k)| + 1) \cdot \varepsilon T \operatorname{sgn} s(k) \quad (12)$$

where $0 < \gamma T < 1$ and $0 < \varepsilon T < 1$.

If the reference target is given as x_d , and the error between the reference target and the system state is expressed as $\hat{e}(k) = \hat{x}(k) - x_d$, then the sliding surface can be designed as:

$$\begin{aligned} s(k) &= c\hat{e}(k) = c(\hat{x}(k) - x_d(k)) \\ c &= [c_1 \quad c_2 \quad c_3](k) \end{aligned} \quad (13)$$

Using Eqs. (8), (12), and (13), the control input $u(k)$ is to be determined. Eq. (12) can be rewritten as:

$$\begin{aligned} s(k+1) &= c\hat{e}(k+1) \\ &= c(\hat{x}(k+1) - x_d(k+1)) \\ &= cA\hat{x}(k) + cBu(k) + cL_1[y(k) - \hat{y}(k)] \\ &\quad + cE\hat{d}(k) - cx_d(k) \end{aligned} \quad (14)$$

Based on Eq. (14), the control law is derived as

$$\begin{aligned} u(k) &= (cB)^{-1}[(1 - \gamma T)\hat{s}(k) - \ln(|s(k)| + 1)\varepsilon T \operatorname{sgn} s(k) \\ &\quad - cA\hat{x}(k) - cL_1(y(k) - \hat{y}(k)) - cE\hat{d}(k) \\ &\quad + cx_d(k+1)] \end{aligned} \quad (15)$$

4 EXPERIMENTAL RESULTS

To experimentally examine the control performance of the VCM positioning system, control experiments based on a step command of 3 mm were performed for both the traditional PID controller and the

proposed control scheme. Performance comparison is carried out in the following sections.

4.1 Configuration of Positioning System

The control system for performing positioning practice is shown in Figure 6. A 16-bit DAQ card is used to send the control signal via amplifier to the VCM and decode the position information from the linear encoder (scale). The control program is implemented with the LabVIEW package which is commercially available software.

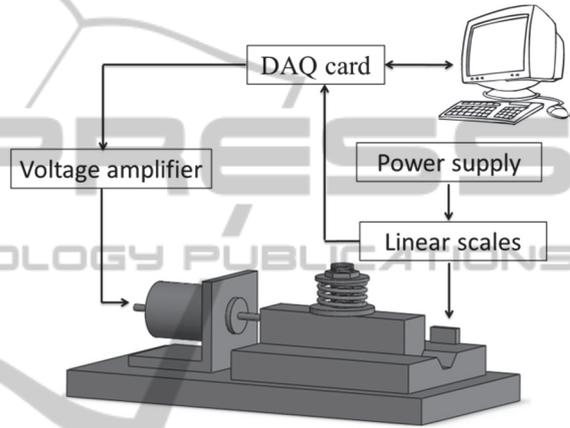


Figure 6: Experimental configuration of the VCM positioning system.

4.2 PID Controls

The control input of the PID controller can be expressed as:

$$u = K_p e_1 + K_i \int_0^t e_1 dt + K_d \frac{de_1}{dt} \quad (16)$$

where $e_1 = Q(x - x_d)$ with a constant vector $Q = [1 \ 0 \ 0]$, is the system output tracking error; K_p , K_i , and K_d are the proportional, integral, and derivative gains, respectively. For control practice, Eq. (16) is transferred to the discrete form as

$$u(k) = K_p e_1(k) + K_i T_s \sum_{i=0}^k e_1(i) + \frac{K_d}{T_s} [e_1(k) - e_1(k-1)] \quad (16)$$

The initial conditions of the PID control system were given as $x_1(0) = 0$, $x_2(0) = 0$, and $x_3(0) = 0$. The PID gains were suitably tuned as $K_p = 1380$, $K_i = 720$, $K_d = 3.8$. To prevent the VCM from damage, the control input was limited in the range of ± 3 V.

Figure 7 shows the experimental results of the PID control under a step command of 3 mm. From the displacement shown in Figure 7(a), the VCM could reach the target position at time 5.2 s. However, the sliding stage behaved significantly unsmooth motion. For example, the sliding stage stuck to the sliding surface even though the control command shown in Figure 7(c) was increasingly given during the time interval of 1.2 s to 2.8 s. This also could be seen from Figure 7(b) showing the time history of position error. From the enlarged position error in the time interval of 7 s to 10 s, the maximum error was recorded as 17.1 μm with an error percentage of 0.57%.

4.3 DSMC Coupled with PIO

Figure 8 shows the experimental configuration of the VCM positioning system based on the DSMC coupled with PIO. The program was implemented with the LabVIEW package.

The initial conditions were the same as that of the PID control system, i.e., $x_1(0) = 0$, $x_2(0) = 0$, and $x_3(0) = 0$. The designed parameters were

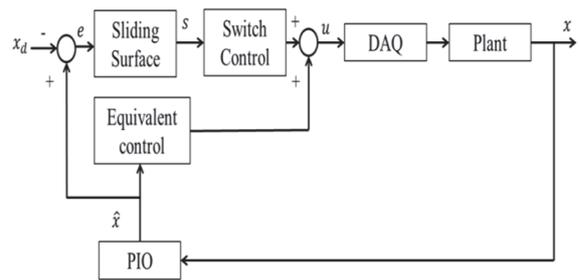


Figure 7: Configuration of the DSMC coupled with PIO.

determined as $c = [920 \ 2.3 \ 4.3]$, $\gamma T = 0.001$, and $\varepsilon T = 0.85$.

Figure 9 shows the experimental results of the DSMC coupled with PIO. From the displacement shown in Figure 9(a), the VCM could reach the target position at time 3 s, which was faster than that using the PID controller. Although the control input was varying a large range of positive and negative voltages as shown in Figure 9(b), a smooth motion could be found from the displacement and the position error shown in Figure 9(c). Also, from the enlarged position error between the time interval of

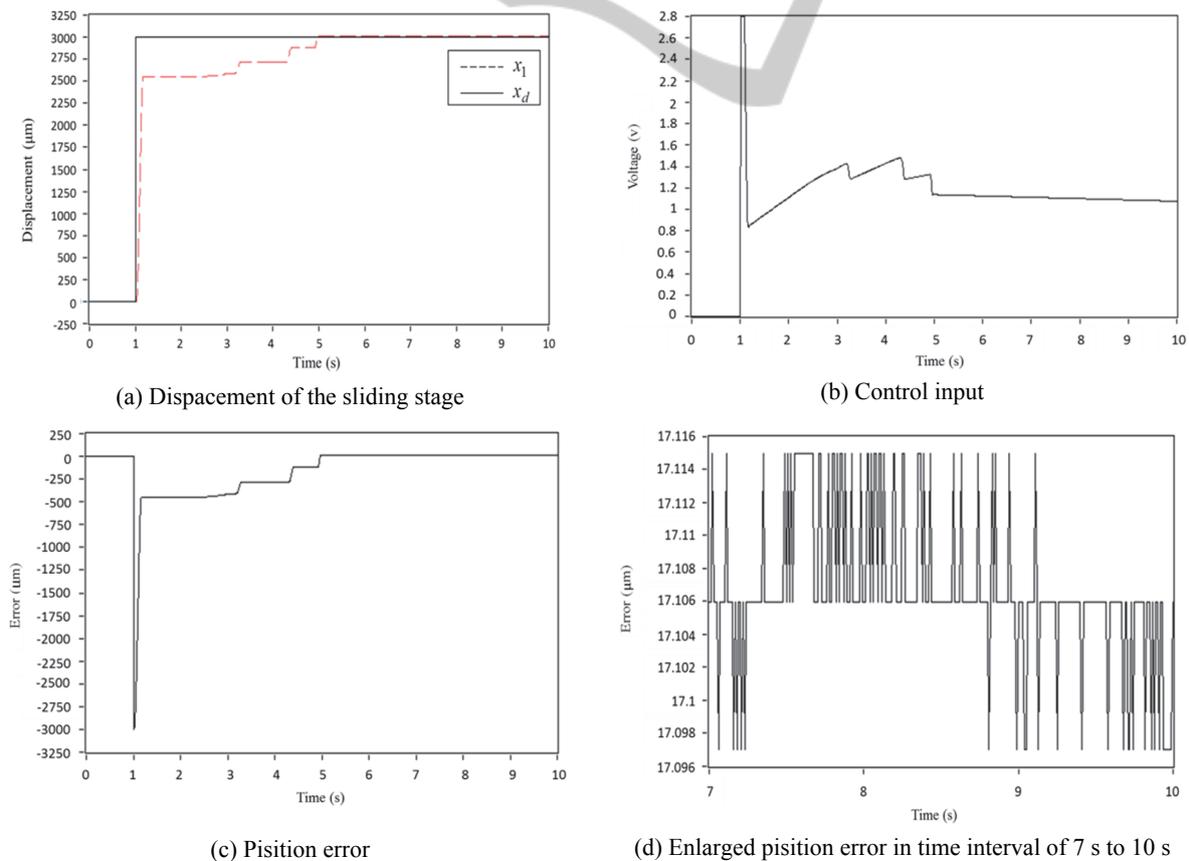


Figure 8: Experimental results of the PID controller.

7 s to 10 s, the maximum error was recoded as 7.3 μm with an error percentage of 0.25%. These results revealed that the control performace using the proposed control scheme was superior to that using the PID controller.

5 CONCLUSIONS

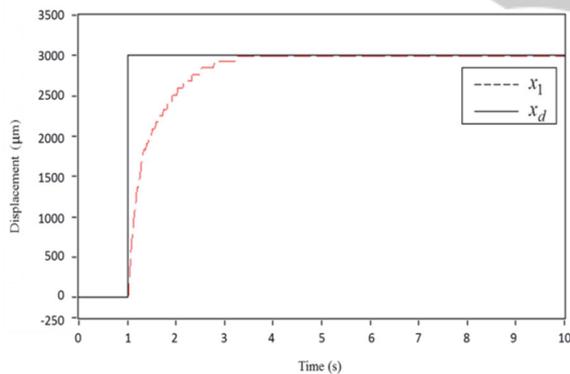
In this paper, the DSMC coupled with PIO was implemented to the applicaiton of the VCM positioning system. Through experimental examinations, the VCM positioning system using the proposed control scheme could reach a position error of 7.3 μm with an error percentage of 0.25%, and a smooth motion control was obtained. Compared with that using the traditional PID controller, the proposed control scheme having significant performance improvement in positioning error and smooth motion was verifeid.

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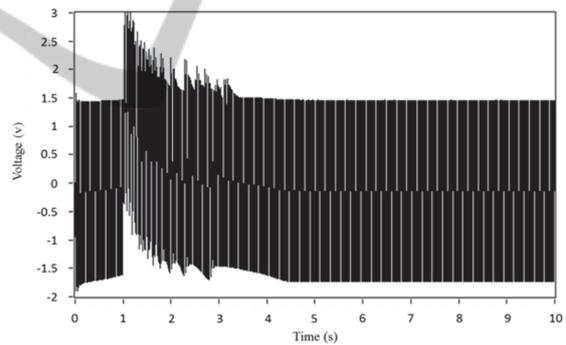
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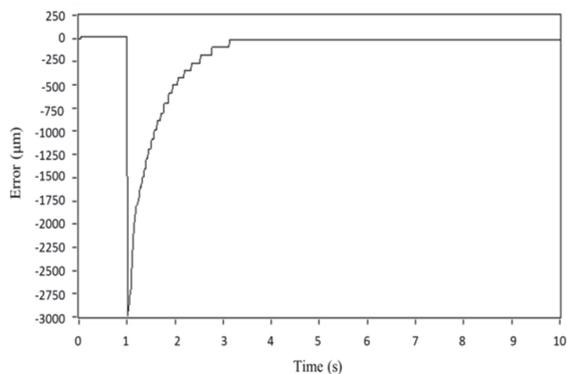
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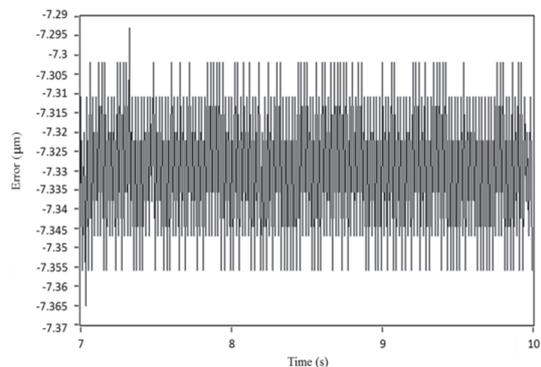
(a) Displacement of the sliding stage



(b) Control input



(c) Position error



(d) Enlarged position error in time interval of 7 s to 10 s

Figure 9: Experimental results of the DSMC coupled with PIO controller.

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