On Image Representing in Image Analysis

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Abstract: The presentation is devoted to the research of mathematical fundamentals for image analysis and recognition procedures being conducted currently in the Dorodnicyn Computing Centre of the Russian Academy of Sciences, Moscow, Russian Federation. The paper presents and discusses the main results obtained using the Descriptive Approach to Analysing and Understanding of Images when solving fundamental problems of the formalization and systematization of the methods and forms of representing information in the problems of the analysis, recognition, and understanding of images. In particular, the problems arise in connection with the automation of information extraction from images in order to make intelligent decisions (diagnostics, prediction, detection, evaluation, and identification of patterns). The final goal of this research is automated image mining: a) automated design, test and adaptation of techniques and algorithms for image recognition, estimation and understanding; b) automated selection of techniques and algorithms for image recognition, estimation and understanding; c) automated testing of the raw data quality and suitability for solving the image recognition problem.

1 INTRODUCTION

The automation of processing, analysing, evaluating, and understanding of information provided in the form of images is one of the critical breakthrough problems of theoretical computer science. The image is one of the main means of representing and transmitting information needed to automate intelligent decision making in a variety of application domains.

To date, in the analysis and evaluation of images, there is extensive experience in the application of mathematical methods from different branches of mathematics, computer science, and physics, in particular algebra, geometry, discrete mathematics, mathematical logic, probability theory, mathematical statistics, mathematical analysis, mathematical theory of pattern recognition, digital signal processing, and optics.

On the other hand, the variety of methods used does not replace the need for some regular basis for ordering and selecting appropriate methods of image analysis, a uniform representation of the processed data (images) that meet standard requirements of pattern recognition algorithms to the source data, the construction of mathematical models of images focused on the identification problem, and the general availability of a universal language for the uniform description of images and their transformations

This paper presents the main results on the formalization and systematization of methods and forms of information representation in problems of analysis, recognition, and understanding of images. We have summarized the development of a descriptive approach (DA) to analysing and understanding images formulated by I.Gurevich (Gurevich, 1989, 1991, 2005). This is a direction of research concerning the formalization and representation of images. Recall that DA is a specialization of the algebraic approach of Yu.Zhuravlev (Zhuravlev, 1998) to the case of the representation of information in the form of images.

Axiomatics and formal structures of the DA provide methods and tools for presenting and describing images for their subsequent analysis and evaluation. The theoretical basis of the research is the DA; general algebraic methods; and methods of the mathematical theories of image processing, image analysis, and pattern recognition.

It is established that the overall success and effectiveness of the analysis and evaluation of

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information provided in the form of images are determined by the possibilities of reducing images to a form suitable for recognition (RIFR).

RIFR processes are crucial for solving applied problems of image analysis and, in particular, to make intelligent decisions based on information extraction from images. The DA provides the ability to solve both problems associated with the construction of formal descriptions of images as objects of recognition and problems of synthesis of procedures of pattern recognition and image understanding. The operational approach to characterizing images requires that processes of analysing and evaluating information provided in the form of images (the trajectory of problem solving) as a whole could be viewed as a sequence/combination of transformations and computing of a set of interim and final (defining the solution) evaluations. These transformations are defined on the equivalence classes of images and their representations. The latter are defined descriptively, i.e., using a base set of prototypes and corresponding generative transformations that are functionally complete with respect to the equivalence class of admissible transformations.

Now we outline the goals of theoretical development in the DA framework (and image analysis algebraization) ("What for") and necessary steps to finalize the DA ("What to Do or What to be Done") and the global problem of an image reduction to a recognizable form.

2 DESCRIPTIVE APPROACH TO IMAGE ANALYSIS AND UNDERSTANDING

This section contains a brief description of the principal features of the DA needed to understand the meaning of the introduction of the conceptual apparatus and schemes of RIFR proposed to formalize and systematize the methods and forms of representation of images.

The automated extraction of information from images includes (1) automating the development, testing, and adaptation of methods and algorithms for the analysis and evaluation of images; (2) the automation of the selection of methods and algorithms for analysing and evaluating images; (3) the automation of the evaluation of quality and adequacy of the initial data for solving the problem of image recognition; and (4) the development of standard technological schemes for detecting, assessing, understanding, and retrieving images.

The automation of information extraction from images requires complex use of all the features of the mathematical apparatus used or potentially suitable for use in determining transformations of information provided in the form of images, namely in problems of processing, analysis, recognition, and understanding of images.

Experience in the development of the mathematical theory of image analysis and its use to solve applied problems shows that, when working with images, it is necessary to solve problems that arise in connection with the three basic issues of image analysis, i.e., (1) the description (modelling) of images; (2) the development, exploration, and optimization of the selection of mathematical methods and tools for information processing in the analysis of images; and (3) the hardware and software implementation of the mathematical methods of image analysis.

The main purpose of the DA is to structure and standardize a variety of methods, processes, and concepts used in the analysis and recognition of images.

The DA is proposed and developed as a conceptual and logical basis of the extraction of information from images. This includes the following basic tools of analysis and recognition of images: a set of methods of analysis and recognition of images, RIFR techniques, conceptual system of analysis and recognition image, descriptive image models (DIM) classes, the descriptive image algebra (DIA) language, statement of problems of analysis and recognition of images, and the basic model of image recognition.

The main areas of research within the DA are (1) the creation of axiomatics of analysis and recognition of images, (2) the development and implementation of a common language to describe the processes of analysis and recognition of images (the study of DIA), and (3) the introduction of formal systems based on some regular structures to determine the processes of analysis and recognition of images (see (Gurevich, 1989, 1991)).

Mathematical foundations of the DA are as follows: (1) the algebraization of the extraction of information from images, (2) the specialization of the Zhuravlev algebra (Zhuravlev, 1998) to the case of representation of recognition source data in the form of images, (3) a standard language for describing the procedures of the analysis and recognition of images (DIA) (Gurevich, 2006), (4) the mathematical formulation of the problem of image recognition, (5) mathematical theories of image analysis and pattern recognition, and (6) a model of the process for solving a standard problem of image recognition.

The main objects and means of the DA are as follows: (1) images; (2) a universal language (DIA); (3) two types of descriptive models, i.e., (a) an image model and (b) a model for solving procedures of problems of image recognition and their implementation; (4) descriptive algebraic schemes of image representation (DASIR); and (5) multimodel and multiaspect representations of images, which are based on generating descriptive trees (GDT) (Gurevich, 2005).

The basic methodological principles of the DA are as follows: (1) the algebraization of the image analysis, (2) the standardization of the representation of problems of analysis and recognition of images, (3) the conceptualization and formalization of phases through which the image passes during transformation while the recognition problem is solved, (4) the classification and specification of admissible models of images (DIM), (5) RIFR, (6) the use of the standard algebraic language of DIA for describing models of images and procedures for their construction and transformation, (7) the combination of algorithms in the multialgorithmic schemes, (8) the use of multimodel and multiaspect representations of images, (9) the construction and use of a basic model of the solution process for the standard problem of image recognition, and (10) the definition and use of nonclassical mathematical theory for the recognition of new formulations of problems of analyzing and recognizing images.

Note that the construction and use of mathematical and simulation models of studied objects and procedures used for their transformation is the accepted method of standardization in the applied mathematics and computer science.

The creation of the DA was significantly influenced by the following basic theories of pattern recognition: (1) the algebraic approach to pattern recognition of Zhuravlev (Zhuravlev, 1998) and their algorithmic algebra and (2) the theory of images of Grenander (Grenander, 1993, 1996), in particular algebraic methods for the representation of source data in image recognition problems developed in it.

As was already noted, in the DA, it is proposed to carry out the algebraization of the analysis and recognition of images using DIA. DIA was developed from studies in the field of the algebraization of pattern recognition and image analysis carried out since the 1970s. The creation of a new algebra was directly influenced by algorithms of Zhuravlev (Zhuravlev, 1998) and the research of Sternberg (Sternberg, 1985) and Ritter (Ritter, 2001), (Ritter, Wilson, 2001), which identified classic versions of image algebras.

A more detailed description of methods and tools of the DA obtained in the development of its results can be found in (Gurevich, 1989, 1991, 2005), (Gurevich, Zhernova, 2003), (Gurevich, Koryabkina, 2006), (Gurevich, Yashina, 2006, Computer-Aided), (Gurevich, Yashina, 2006, Operations), (Gurevich, Yashina, 2008), (Gurevich et al., 2008), (Gurevich, Yashina, 2012).

3 WHAT TO DO OR WHAT TO BE DONE. BASIC STEPS

The critical points of an image analysis problem solution are: 1) precise setting of a problem; 2) correct and "computable" representation of raw and processed data for each algorithm at each stage of processing; 3) automated selection of an algorithm: a) decomposition of the solution process for main stages; b) indication points of potential improvement of the solution ("branching points"); c) collection and application of problem solving experience; d) selection for each problem solution stage of basic algorithms, basic operations and basic models (operands); e) classification of the basic elements; 4) performance evaluation at each step of processing and of the solution: a) analysis, estimation and utilization of the raw data specificity; b).diversification of mathematical tools used for performance evaluation; c) reduction of raw data to the real requirements of the selected algorithms.

Basic steps of the development of image analysis theory are: a) mathematical settings of an image recognition problem (step 1); b) image formalization space and descriptive image models (step 2); c) generating descriptive trees and multimodel representation of images (step 3); d) image equivalence (step 4); e) image metrics (step 5); f) descriptive image algebras (step 6).

3.1 Mathematical Settings of an Image Recognition Problem

DONE:

1) Descriptive Model of Image Recognition Procedures
2) Mathematical Setting of an Image Recognition Problem. Image Equivalence Case.

Image analysis and recognition deal with properties of the object (scene) shown and deformations associated with the way and procedure of obtaining the image. In this case, to formalize image processing, we need to specify three sets (models) of images, on which we postulate the existence of classes of equivalence and sets of admissible transformations given on the classes of equivalence (Gurevich, Yashina, 2006). Introducing classes of equivalence on the sets of image models, we accept that any image possesses some regularity or a mix of regularities of different types. Under this assumption, analysis and recognition problem is reduced to making a difference between images that preserve their own regularity and images, the regularity of which can be broken.

Figure 1 shows the descriptive model of the image recognition problem.

$$J, J^*, J^R; T^F, T^R, T^F: J \Rightarrow J^*, T^R: J^* \Rightarrow J^R; \{J\} = \bigcup_j K_j.$$

Figure 1: Descriptive model of the image recognition problem.

Here, {J} is the set of ideal images, {J*} is the set of observable images, $\{J^R\}$ is the set of images obtained as a result of solving the recognition problem, $\{T^F\}$ is the set of admissible transformations to form the image, $\{T^R\}$ is the set of admissible transformations to recognize the image, and $\{K^i\}$ are classes of equivalence.

Let J be some true image of the object involved. We can consider processes of obtaining, forming, discretization, etc. (all procedures that make it possible to work with the image) as if the true image were transferred via the noisy channel. As a result, we analyze some real (observable) image J* rather than the true image. This real image is to be classified in the course of analysis, i.e., we should determine the prototype in the true class of equivalence or find the regularity (regularities) of the given type JR on the observable image J*. Thus, we can specify the sets {J}, {J*}, and $\{J^R\}$ and transformations to form (T^F) and recognize (T^A) the image

$$T^F: J\text{-}{>}\ J^*, \quad (1)$$

$$T^A: J^*\text{-}{>}J^R. \quad (2)$$

To perform image recognition, we need to give algebraic systems of transformations $\{T^F\}$ and $\{T^A\}$ on classes of equivalence of the set {J} and apply them to observable images J* to perform the backward analysis, i.e., classify images according to the nature of their regularity (restore true images, i.e., indicate classes of equivalence they belong to), and the forward analysis, i.e., search the image J* for regularities of the certain type J^R and localize them.

Stating the analysis problem in such a way, we can give the class of image processing procedures, analysis process of which is of fixed structure, with interpretation (particular implementation) depending on the purposes and type of analysis. There are following main stages of analysis.

Now three mathematical statements of image recognition problems are considered. The first one Z is introduced by Yu. Zhuravlev (Zhuravlev, 1998).

When solving real image recognition problems, we deal with the images of objects rather than with the objects themselves. Therefore, we will assume that the whole set of images is somehow divided into equivalence classes. We also assume that there is correspondence between the equivalence classes of images and the objects; however, in the future, we will not mention objects in the statement of the recognition problem. Taking into account the concept of equivalence of images introduced above, we can formulate the image recognition problem as follows.

The difference between problems Z^2 and Z^1 is that each equivalence class in problem Z^2 is replaced by a unique image—a representative of the class—with the number n_i , $1 \le n_i \le p_i$, where i is the number of the equivalence class. This replacement is performed by introducing the concept of an admissible transformation.

Problem Z^1 differs from Z by the fact that it explicitly uses equivalence classes of images. To reduce the image recognition problem Z^1 to the standard recognition problem Z, one should pass from the classification of a group of objects to the classification of a single object. Under certain constraints on admissible transformations, problem Z^2, which differs from Z^1 in that it contains admissible transformations that do not take the image beyond the equivalence class, allows one to handle a single image for each equivalence class—a representative of this equivalence class.

TO BE DONE:

1) establishing of interrelations and mutual correspondence between image recognition problem classes and image equivalence classes;

2) new mathematical settings of an image recognition problem connected with image equivalency;
3) new mathematical settings of an image recognition problem connected with an image multimodel representation and image image data fusion.

3.2 Image Formalization Space and Descriptive Image Models

DONE:

1) the conceptualization of a system of concepts that describe the initial information (images) in recognition problems has been carried out;
2) descriptive models of images focused on the recognition problem have been defined;
3) the image formalization space has been introduced, the elements of which include different forms (states, phases) of representing the image transformed from the original form into the recognizable one, i.e., into the image model;
4) the basic axioms of the descriptive approach were introduced.

Image formalization space (IFS) is the space including sets of an image "states" and sets of image transforming schema for formalization and systematization of techniques and forms of information representations in image analysis, recognition and understanding problems. More detailed description of IFS (Gurevich, Yashina, 2012) includes: a) construction of algorithmic schema generating phase trajectories for solving image analysis and recognition problems; b) DIM - mathematical objects providing representation in a form acceptable for a recognition algorithm of information carried by an image and by an image legend (context); c) multiple DIM and multi-aspect image representations; d) topological properties of the Image Formalization Space.

Recall that, in the DA, the processes of analyzing and evaluating the information presented in the form of images are considered as sequences of the transformations and calculations of a set of intermediate and final (defining the solution) evaluations. These estimates are essential characteristics of representations of the source image obtained at each stage of RISR. Final estimates are used in the final stage of solving the problem of recognition/classification of the source image in the application of algorithms for the recognition/classification of the image model created by RIFR.

The descriptive algebraic scheme of the image representation (DASIR), which is a formal scheme designed to produce a standardized formal description of surfaces, point configurations, shapes that form the image, and the relations between them, is recorded using the DIA.

DASIR reflect sequential and/or parallel use of transformations from the set of transformations to the initial information from the space of initial data. In (Gurevich, et al., 2008), there is an example of constructing DASIR to solve the problem of the morphological analysis of blood cells. All steps of the algorithmic scheme for training recognition algorithms for problems of analyzing cytological preparations and classifying the new image using three diagnoses based on a recognition algorithm with adjusted parameters were defined and described using the DIA with one ring.

In the DA, three classes of admissible transformations of images are considered (Gurevich, Yashina, 2006), i.e., procedural transformations, parametric transformations, and generative transformations. The basic classes of transformations of images (procedural, parametric, and generating) are defined, as well as related concepts of the structuring element, which generates rules and correct generative transformation.

All transformations for processing and analyzing images are conducted using the DIA record on the transformations of images. It makes it possible to vary the methods for solving the subproblem using different operations of the image analysis of fixed DIA and keeping the whole scheme of the technology of RISF and the extraction of information from images unchanged.

In order to apply of pattern-recognition algorithms to the created formal descriptions of images, it is necessary to implement the created schemes (to set specific transformations from the fixed DIA and parameters of transformations selected in the schemes) and apply them the initial information, i.e., to create models of images.

An example of the DASIR implementation can be found in (Gurevich, Yashina, 2006), i.e., at each step of the algorithmic scheme the created DASIR was concretized by the selection of a transformation that belongs to a specific DIA that describes the step.

In the general case, we can say that the use of the convolution of structuring elements and admissible transformations of the image to the initial information about the image leads to the transformation of the initial information in the image model. Specific allowable image transformations and specific methods of applying them to the initial information are selected based on the set problem of the analysis and recognition of images.

Axiomatization of algebraic image analysis constitutes a base for unification of image analysis algorithms representations and image models representations. The axioms define properties and structure of the Image Formalization Space (IFS).

It was shown that all image representations and procedures of RFSR form a topological space (IFS). The main properties of this space, as well as the conceptual basis of the synthesis of image models, are defined by the following axioms that constitute basic provisions of the DA.

Image models are the results of RIFR (taking into account all the information about the image). On the set of image models, basic DIA are introduced on image models of three classes in accordance with operations used for their construction. Note that these models are descriptive image models (DIM).

TO BE DONE:

1) Creation of image models catalogue
2) Selection and study of basic operations on image models for different types of image models (including construction of bases of operations)
3) Use of information properties of images in image models
4) Study of multimodel representations of images.

3.3 Generating Descriptive Trees and Multimodel Representation of Images

DONE:

Generating Descriptive Tree (GDT) - a new data structure for generation plural models of an image is introduced.

The introduction of axiomatics of DA and definition of three classes of DIM has led to the introduction of a new mathematical object for structuring representations of images and generation of image models.

Three types of appropriate conversions generating rules and a source image are necessary for constructing the three classes of representations of images (procedural, parametrical, and generating representations of images). The source image is described by means of a set of its implementations and by means of context-sensitive and semantic information.

According to the introduced axiomatics and definitions of various classes of representations of images, in this way, for merging and combination of various properties of image models, it is necessary to introduce the following hierarchies: the hierarchy of possible implementations of images; the hierarchies of semantic and context-sensitive information in images; the hierarchies of parametrical, procedural, and generating conversions; and hierarchies of generating rules. It is suggested to implement such structures in the form of special trees.

Specialization of the concept of a tree on the whole is related to specialization of nodes of a tree. As nodes we will select objects, operations, or rules of image analysis tasks used to construct different image models. Such nodes are called GDT descriptors. The definitions of parent, calculated, fixed, objective, and abstract GDT descriptors have been introduced, but will not be dwelt on in this work.

Definition 1 (Gurevich, 2005). The generating descriptive tree (GDT) is the structure intended for classification and automated generation of image models and it possesses the following properties:(1) GDT descriptors are GDT nodes; (2) Every GDT combines the descriptors of one type; that is, GDTs represent the same type of properties of an image; (3) Each GDT element can be united with another element to generate new partial multiaspect image models; (4) Descriptors are linked among themselves by parent–daughter relationships; (5) Each descriptor has a relationship with a unique parent descriptor and can have some links with derived descriptors. If the descriptor has no parent, it is called a radical GDT. If the descriptor has no derived descriptors, it is called a leaf.

Note that parametrical GDTs are GDTs intended for classification and automation of the generation of parametrical image models. A parametrical GDT, thus, contains GDT descriptors describing the properties of parametrical conversions, leading to an evaluation of features of images. A procedural GDT is a GDT intended for classification and automation of the generation of procedural image models. A procedural GDT, thus, contains the GDT descriptors describing the properties of procedural conversions.

TO BE DONE:

1) to define and to specify GDT;
2) to set up image recognition problem using GDT;
3) to define descriptive image algebra using GDT;
4) to construct a descriptive model of image recognition procedures based on GDT using;
5) to select image feature sets for construction of P-

GDT;
6) to select image transform sets for construction of T-GDT;
7) to define and study of criteria for selection of GDT-primitives.

3.4 Image Equivalence

DONE:
There were introduced several types of image equivalence: image equivalence based on the groups of transformations; image equivalence directed at the image recognition task; image equivalence with respect to a metric.

We consider the problem of searching for a correct algorithm for the image recognition problem. We consider various methods for defining the equivalence of images, namely, equivalence on the basis of transformation groups, equivalence oriented to a special statement of the image recognition problem, and equivalence with respect to metric. In the case of definition of equivalence based on transformation groups, we construct examples of equivalence classes. It is shown that the concept of equivalence is one of the key concepts in image recognition theory. We study the relationship between equivalence and invariance of images.

Using the introduced concept of equivalence of images, we modify the standard mathematical statement of the image recognition problem and formulate an image recognition problem in terms of equivalence classes. We prove that, under certain constraints on the image transformations, the problem of image recognition in the standard statement can be reduced to an abridged problem for which there exists a correct algorithm within the algebraic closure of the class of recognition algorithms for calculating estimates (ACEs).

TO BE DONE:
1) to study image equivalence based on information properties of the image;
2) to define and construct image equivalence classes using template (generative) images and transform groups;
3) to establish and to study links between image equivalence and image invariance;
4) to establish and to study links between image equivalence and appropriate types of image models;
5) to establish and to study links between image equivalence classes and sets of basic image transforms.

3.5 Image Metrics

It is an open problem.

TO BE DONE:
1) to study, to classify, to define competence domains of pattern recognition and image analysis metrics;
2) to select workable pattern recognition and image analysis metrics;
3) to construct and to study new image analysis-oriented metrics;
4) to define an optimal image recognition-oriented metric;
5) to construct new image recognition algorithms on the base of metrics generating specific image equivalence classes.

3.6 Descriptive Image Algebras

DONE:
1) Descriptive Image Algebras (DIA) with a single ring were defined and studied (basic DIA);
2) it was shown which types of image models are generated by main versions of DIA with a single ring;
3) the technique for defining and testing of necessary and sufficient conditions for generating DIA with a single ring by a set of image processing operations were suggested;
4) the necessary and sufficient conditions for generating basic DIAs with a single ring were formulated;
5) the hierarchical classification of image algebras was suggested;
6) it was proved that the Ritter's algebra could be used for construction DIA's without a "template object".

This object is studied in developing a mathematical apparatus for analysis and estimation of information represented in the form of images. For a structural description of possible algorithms for solving these problems, we need a formal instrument that allows us to describe and justify the chosen way of solution. As formalization tools, we chose the algebraic approach, which should provide a unique form of procedures for describing the objects–images and transformations of these objects–images.

The need to develop a mathematical language that ensures that solutions of problems of image processing, analysis, and understanding may be uniformly described by structural algorithmic schemes is justified by the following factors:

(1) there are many algorithms (designed and introduced into practice) for analysis, estimation, and understanding of information represented in the form of images;
(2) the set of algorithms is neither structured nor ordered;
(3) as a rule, methods for image analysis and understanding are designed on the basis of intuitive principles, because the information represented in the form of images is hardly formalized;
(4) the efficiency of these methods is estimated (as is usual in experimental sciences) by the success in solving actual problems—as a rule, the problem of rigorous mathematical justification of an algorithm is not considered.

"Algebraization" is one of the most topical and promising directions of fundamental research in image analysis and understanding. The main goal of the algebraic approach is the development of a theoretical basis for representations and transformations of images in the form of algebraic structures that enable one to use methods from different areas of mathematics in image analysis and understanding.

An object that lies most closely to the developed DIA is the image algebra proposed and developed by Ritter (Ritter, 2001). Ritter's main goal in developing the image algebra is the design of a standardized language for description of algorithms for image processing intended for parallel execution of operations. A key difference in the new image algebra from the standard Ritter image algebra is that DIA is developed as a descriptive tool, i.e., as a language for description of algorithms and images rather than a language for algorithm parallelizing.

The conceptual difference of the algebra under development from the standard image algebra is that objects of this algebra are (along with algorithms) descriptions of input information. DIA generalizes the standard image algebra and allows one to use (as ring elements) basic models of images and operations on images or the models and operations simultaneously. In the general case, a DIA is the direct sum of rings whose elements may be images, image models, operations on images, and morphisms. As operations, we may use both standard algebraic operations and specialized operations of image processing and transformations represented in an algebraic form. In more detail, the definition of the standard image algebra and that of DIA are considered in (Gurevich, Yashina, 2006).

To use DIA actively, it is necessary to investigate its possibilities and to attempt to unite all possible algebraic approaches, for instance, to use the standard image algebra as a convenient tool for recording certain algorithms for image processing and understanding or to use Grenander's concepts for representation of input information.

The main attention was given to DIAs with one ring, which form the main subclass of basic DIAs. In future, we are going to consider DIAs based on superalgebras and investigate other possibilities of application of other algebraic concepts in the theory being developed.

TO BE DONE:

1) to study DIA with a single ring, whose elements are image models;
2) to study DIAs with several rings (super algebras);
3) to define and study of DIA operation bases;
4) to construct standardized algebraic schemes for solving image analysis and estimation problems on the DIA base;
5) to generate DIA using equivalence and invariance properties in an explicit form;
6) to demonstrate efficiency of using DIA in applied problems;
7) to study alternative algebraic languages for image analysis, recognition and understanding.

4 CONCLUSIONS

In principle, the success of image analysis and recognition problem solution depends mainly on the success of image reduction to a recognizable form, which could be accepted by an appropriate image analysis/recognition algorithm. All above mentioned steps contribute to the development techniques for this kind of image reduction/image modeling. It appeared that an image reduction to a recognizable form is a critical issue for image analysis applications, in particular for qualified decision making on the base of image mining. The main tasks and problems of an image reduction to a recognizable form are listed below:

1. Formal Description of Images:

1) Study and construction of image models (Step 2);
2) Study and construction of multimodel image representations (Step 3);
3) Study and construction of metrics (Step 5).

2. Description of Image Classes Reducible to a Recognizable Form:

1) Introduction of new mathematical settings of an image recognition problem (Step 1);
2) Establishing and study of links between multimodel representation of images and image metrics (Steps 3, 5);
3) Study and use of image equivalencies (Step 4).

3. Development, Study and Application of an Algebraic Language for Description of the Procedures of an Image Reduction to a Recognizable Form (Step 6).

We hope that after passing through the above mentioned steps we'll be able to formulate the axiomatics of the descriptive (mathematical) theory of image analysis.

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