

Modeling of Nonlinear Dynamics of Active Components in Intelligent Electric Power Systems

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Keywords: Smart Grid, Volterra Polynomials, Power Quality, Control Systems, Electric Power Systems.

Abstract: The research is aimed at developing algorithms for the construction of automated systems to control active components of the electrical network. The construction of automated systems intended for the control of electric power systems requires high-speed mathematical tools. The method applied in the research to describe the object of control is based on the universal approach to the mathematical modelling of nonlinear dynamic system of a black-box type represented by the Volterra polynomials of the N-th degree. This makes it possible for the input and output characteristics of the object to obtain an adequate and fast mathematical description. Results of the computational experiment demonstrate the applicability of the mathematical tool to the control of active components of the intelligent power system.

1 INTRODUCTION

One of the main directions in power engineering is the adoption of components applicable to the implementation of a smart grid concept. This requires:

- Transmission lines with variable characteristics (active and reactive impedance components);
- Devices for electromagnetic conversion of energy with wide capabilities to adjust parameters;
- Systems of energy storage and accumulation;
- Switching devices with a high breaking capacity and large commutation life;
- Executive mechanisms that make it possible to act on the active network components on-line by changing the network parameters and topology.

An integral part of modern power system is positioned sensors and current state variables in the amount sufficient for the on-line estimation of the network state in normal, emergency and post-emergency conditions.

Therefore, the objective is to create control systems which operate in real time and allow fast generation of control signals to all active network components in order to generate optimal control actions.

This method of control is only possible if new algorithms and techniques of power system control are implemented, in particular when the methodology

on selection of input vectors that characterize operating conditions of power systems in terms of system topology are developed.

2 STATEMENT OF THE PROBLEM

In order to estimate the objective current state it is necessary to take into account the parameters characterizing power quality.

The application of appropriate mathematical tools will make it possible to solve the stated problem.

These mathematical tools should meet the following requirements:

- appropriately reflect the object of control in the entire range of change in its characteristics;
- afford the possibility of obtaining an adequate mathematical description based on real characteristics of the object;
- have high performance in its technical implementation.

Generally speaking, the analysis of dynamic characteristics of wind power unit is based on the methods using differential equations. Most of the researches are devoted to the specification of characteristics of individual components of wind turbine (Li, 2011, He, 2009), specification of various

coefficients (Manyonge, 2012) or consideration of a mechanical part of the turbine as a n-mass system (Bhandari, 2014). In practice, the initial data are known with some error. In this case, as a rule, solutions to the inverse problem turn out to be unstable with respect to an error in the initial data. Therefore, to construct stable methods we use the theory of ill-posed problems. (Kabanikhin, 2011).

It is also obvious that these mathematical tools are difficult to use in the microprocessor software which in turn makes it difficult to perform control.

The goal of this research is to test the algorithms for the construction of computer-aided systems for power system control, in which the mathematical models are used in the form of integral Volterra polynomials.

We will name only some of the research areas, in which the Volterra integral power series find their use. These are: modelling of technical systems (Venikov and Sukhanov, 1982, Pupkov, 1976) and electronic devices (Stegmayer, 2004), nonlinear identification of communications channels (Tong Zhou and Giannakis, 1997, Cheng and Powers, 1998) and visualization systems (Lin and Unbehauen, 1992), analysis of non-stationary time series (Minu and Jessy, 2012), and description of automatic feedback control systems (Belbas and Bulka, 2011).

Nowadays there are many methods for the determination of dynamic characteristics, and the universality of this mathematical apparatus makes it possible to create the software for doing experiments on computer. In particular, the Voltaire XL package (American company Applied Wave Research) has shown its performance in describing the electronic scheme by the finite sums of the Volterra series.

An isolated electrical energy source represented by a horizontal-axis wind turbine was used as a real physical object (Solodusha, 2014).

3 REFERENCE DYNAMIC SYSTEM

It should be noted that renewable energy sources are an active component of modern electric power systems. As a reference dynamic system, we will consider a mathematical model of horizontal-axis wind turbine represented using the techniques (Pronin and Martyanov, 2012, Perdana, 2004, Sedaghat and Mirhosseini, 2012) in the following form:

$$z(t) = \left(\frac{1}{Z(t) + 0.08b(t)} - \frac{0.035}{b^3(t) + 1} \right)^{-1}, \quad (1)$$

$$C_p(t) = 0.22 \left(\frac{116}{z(t)} - 0.4b(t) + 5 \right) \exp \left(-\frac{12.5}{z(t)} \right), \quad (2)$$

$$Z(t) = \frac{\omega_T(t)R}{V(t)}, \quad M_T(t) = \frac{\rho S C_p(t) V^3(t)}{2\omega_T(t)}, \quad (3)$$

$$\frac{d\omega_T}{dt} = \frac{M_T(t) - M_C(t)}{J}, \quad (4)$$

where ω_T (rad/s) is rotational speed of wind turbine elements, M_T (N·m) is torque created by aerodynamic force, M_C (N·m) is load resistance torque, J (kg·m²) is moment of inertia of the wind turbine rotating parts, ρ (kg·m⁻³) is air density, S (m²) is blade – swept area, R (m) is wind wheel radius, b (deg) is blade lean angle, V (m/s) is wind speed; dimensionless magnitudes: C_p is wind energy efficiency, Z is speed, z is current value of speed.

One of the key tasks is to reduce the dynamic loads on the structure during strong winds. Control of blade turning makes it possible to considerably decrease the load on the structure. The research is aimed at studying the impact of the blade lean angle b and wind speed V on the angular velocity of rotation ω_T .

4 INTEGRAL MODELS

The mathematical model of the input-output type system can be represented by the Volterra polynomial of the N -th degree:

$$y(t) = \sum_{n=1}^N \sum_{1 \leq i_1 \leq \dots \leq i_n \leq p} f_{i_1, \dots, i_n}(t), \quad (5)$$

$$f_{i_1, \dots, i_n}(t) =$$

$$\int_0^t \dots \int_0^t K_{i_1, \dots, i_n}(s_1, \dots, s_n) \prod_{m=1}^n x_{i_m}(t - s_m) ds_m, \quad (6)$$

where $t \in [0, T]$, $y(0) = 0$, $y(t) \in C_{[0, T]}^{(1)}$.

To construct an integral model in the form (5), (6) means to restore multidimensional transient characteristics of the nonlinear dynamic system K_{i_1, \dots, i_n} . Currently, there are quite many methods developed to determine the dynamic characteristics

(Doyle and Pearson, 2002, Rugh, 1981). The most widely used approach is presented in (Danilov, 1990). It suggests setting a multiparametric family of test signals consisting of a combination of Dirac delta functions to recover the Volterra kernels. However, such an approach has limited application (Ljung, 1987).

The technique for the identification of (5), (6) (Apartsyn, 2003, 2000, 2013) which is used in the paper is based on setting a group of test signals represented by special linear combinations of Heaviside functions with deviating argument. Here the problem of identification is reduced to solving the Volterra linear integral equations of the first kind, which allow explicit inversion formulas.

Further in (5), we will consider only the case where $N=2$, which is the most important for applications. The Volterra kernels will be identified by the technique (Apartsyn, 2003, 2000, 2013), using the midpoint rule to numerically solve (5), (6).

The numerical procedure for solving the system (1) - (4) will be considered as a reference for the assessment of the integral model accuracy. To approximately solve (1) - (4) we apply the 4-th order Runge-Kutta method.

The integral models are constructed to describe the nonlinear dynamics of the output signal $\Delta\omega_T(t) = \omega_T(t) - \omega_{T_0}$ in the case of scalar input signal $\Delta b(t)$ (or $\Delta V(t)$).

Below consideration is given to the case for the input signal $\Delta b(t)$. Practical identification of transient characteristics in the model

$$y_1(t) = \int_0^t K_1(s_1)\Delta b(t-s_1)ds_1 + \int_0^t \int_0^t K_{11}(s_1, s_2)\Delta b(t-s_1)\Delta b(t-s_2)ds_1 ds_2 \quad (7)$$

was carried out on the basis of the experimental data for the test disturbance signals

$\Delta b_\omega^\alpha(t) = \alpha(e(t) - e(t-\omega))$, $\Delta V(t) = 0$, where $\alpha = \pm 10$, $0 \leq \omega \leq t \leq 20$ (s), $e(t)$ - Heaviside function:

$$e(t) = \begin{cases} 0, & t < 0; \\ 1, & t \geq 0. \end{cases}$$

Figure 1 presents the outputs of the reference model (1)-(4) to the input disturbances

$$\Delta b_1(t, \omega) = -10(e(t) - e(t-\omega)).$$

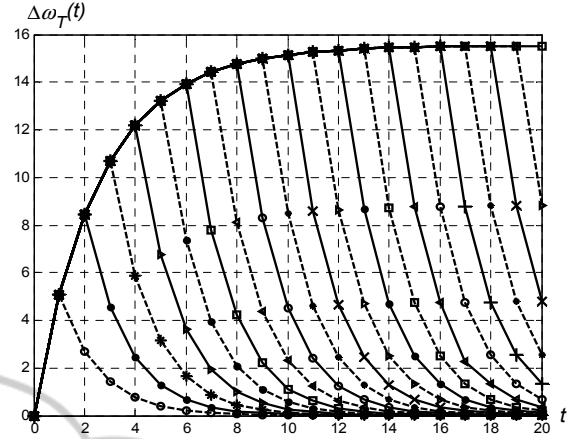


Figure 1: Experimental outputs. $y^{h_1}(t, \omega)$.

The outputs $y^{h_1}(t, \omega)$ of the reference model (1)-(4) took part in the recovery procedure of the sought transient characteristics of the system in the scalar model (7).

It should be specified that the recovery of kernels $K_1(t)$, $K_{11}(t, t-\omega)$ in (7) as a result of the application of the approach (Apartsyn, 2003, 2000, 2013) can be reduced to solving special Volterra linear integral equations of the first kind. Search for the difference analog to the kernels $K_1(t)$, $K_{11}(t, t-\omega)$ was carried out on a uniform grid $t_i = ih$, $i = \overline{1, n}$, $nh = T$. The total number of the unknowns taking part in the experiment of constructing one model of form (7) was equal to $n + \frac{n(n+1)}{2}$.

Along with the scalar model of form (7) we have developed and implemented an algorithm for the construction of the quadratic Volterra polynomial

$$y_2(t) = \sum_{i=1}^2 \int_0^t K_i(s_1)x_i(t-s_1)ds_1 + \sum_{i=1}^2 \int_0^t \int_0^t K_{ii}(s_1, s_2)x_i(t-s_1)x_i(t-s_2)ds_1 ds_2 + \int_0^t \int_0^t K_{12}(s_1, s_2)x_1(t-s_1)x_2(t-s_2)ds_1 ds_2 \quad (8)$$

For the case of vector input $x(t) = (x_1(t), x_2(t))$, where $x_1(t) = \Delta b(t)$, $x_2(t) = \Delta V(t)$.

Figure 2 presents the output $y^{hV_1}(t, \omega)$ of the reference model (1)-(4) to the input disturbance of form

$$\begin{cases} x_1^{h_1}(t, \omega) = -10(e(t) - e(t - \omega)), \\ x_2^{V_1}(t) = 5e(t), \end{cases}$$

which was used in the recovery of the kernel K_{12} from the integral model (8).

The total number of the unknowns participating in the experiment of constructing one model of form (8), was equal to $2n + n(n + 1) + n^2$.

5 CASE STUDY

The computational experiment consists of two stages. In the first stage we build the integral models of form (7), (8) by solving the problem of the identification of transient characteristics of a dynamic system. In the second stage we consider the problem of determining the control action $x_1(t) \equiv \Delta b(t)$, that maintains the output signal $\Delta \omega_T(t)$ at a set level ω^* . Considering the transient characteristics K_{i_1, \dots, i_n} and output $y(t)$ in (7), (8) to be known, we determine the input signal $x(t)$ which corresponds to the specified output $y(t)$.

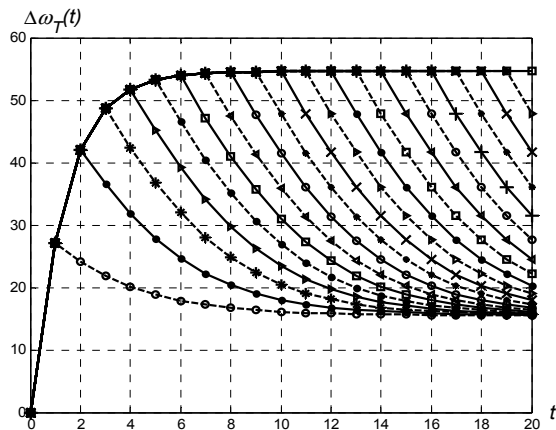


Figure 2: Experimental outputs. $y^{h_1}(t, \omega)$.

In this section we present the results demonstrating the first of the indicated stages of the mathematical modeling. To ensure better accuracy, the amplitude α of test signals used to determine the Volterra kernels in (7), (8) was aligned with the magnitude of the acting disturbances. It should be noted that the model built using only one group of signals cannot be considered equally suitable for the

calculation in the entire range of admissible changes in the input signals. In order to improve the accuracy of modeling we introduced reference initial conditions for which the models of form (7) were constructed. The calculations were performed on the uniform grid with a step $h = 1$ (s).

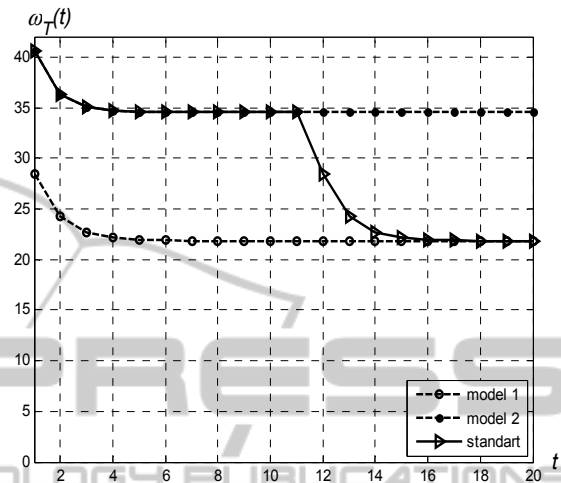


Figure 3: Application of two integral models calculated using (7). Notations: “model 1”, “model 2” are responses of integral models for the reference integral model conditions $V_0 = 8$ (m/s), $V_0 = 10$ (m/s), respectively, “standart” – a response of the standard model (1) - (4).

Figure 3 illustrates the application of the quadratic Volterra polynomial (7) to forecast the output to the input signals:

$$\Delta V(t) = 10(e(t) - e(t - 11)) + 8(e(t - 11) - e(t - 20)), \Delta b(t) = 10e(t), \quad t \in [0, 20],$$

for $b_0 = 10$ (deg), $V_0 = 8$ (m/s), $V_0 = 10$ (m/s).

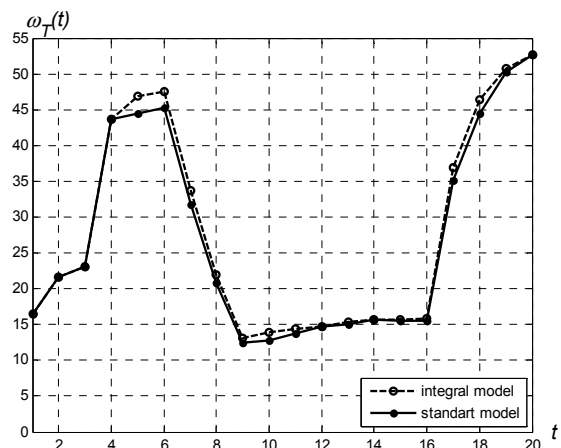


Figure 4: Comparison of the application of the integral model of form (8) and the standard model (1)-(4).

Figure 4 illustrates the result of modeling the output of the system $\Delta\omega_r(t)$ to the input signals:

$$\Delta V(t) = 5(e(t-3) - e(t-6)) - 5(e(t-6) - e(t-16)), \Delta b(t) = 20(e(t) - e(t-8)) + 10(e(t-8) - e(t-11)) + 10(e(t-16) - e(t-20)), t \in [0, 20],$$

for $b_0 = 20$ (deg), $V_0 = 5$ (m/s) using the integral model (8). The maximum relative error in computations made up 4.4%.

Table 1 presents relative and absolute errors.

Table 1: Relative and absolute errors.

Examples of the input signals	ϵ_1	ϵ_2	ϵ_3	ϵ_4
$\Delta b(t) = -10e(t), \Delta V(t) = 5e(t)$	0.00	0.000	0.00	0.00
$\Delta b(t) = -20e(t), \Delta V(t) = 10e(t)$	0.00	0.000	0.00	0.00
$\Delta b(t) = -10e(t), \Delta V(t) = 5(e(t) - e(t-3))$	1.47	0.004	6.26	0.02
$\Delta b(t) = -10(e(t) - e(t-1)), \Delta V(t) = 5e(t)$	1.17	0.003	4.98	0.01
$\Delta b(t) = -10e(t), \Delta V(t) = 5(e(t) - e(t-12))$	1.89	0.068	8.04	0.29
$\Delta b(t) = -20(e(t) - e(t-11)), \Delta V(t) = 10e(t)$	1.39	0.102	5.91	0.43
$\Delta b(t) = -20(e(t) - e(t-4)), \Delta V(t) = 10e(t)$	1.28	0.006	5.45	0.03
$\Delta b(t) = -20e(t), \Delta V(t) = 10(e(t) - e(t-1))$	1.29	0.007	5.49	0.03

The notations used in Table 1:

$$\epsilon_1 = \max_{1 \leq i \leq T} |\Delta\omega_r(t_i) - y_2(t_i)| \text{ (rad/s),}$$

$$\epsilon_2 = |\Delta\omega_r(T) - y_2(T)| \text{ (rad/s),}$$

$$\epsilon_3 = \frac{\epsilon_1}{\omega_{r_0}} \cdot 100\% \text{ (in \%), } \epsilon_4 = \frac{\epsilon_2}{\omega_{r_0}} \cdot 100\% \text{ (in \%),}$$

$$b_0 = 20 \text{ (deg), } V_0 = 5 \text{ (m/s), } t_i = i \cdot h, i = \overline{1, 20}, \omega_{r_0} = 23.5 \text{ (rad/s), } h = 1 \text{ (s), } T = 20 \text{ (s).}$$

The calculations show that the constructed integral models describe the physical process with admissible accuracy.

For solving (8) with respect to the control action $x_1 \equiv \Delta b(t)$ we use the algorithms developed in (Solodusha, 2009). The study employs stable difference methods in which a grid step is used as a regularization parameter (Apartsyn, 2003). As applied to the problem of automatic control it is planned to compare the techniques for the identification of Volterra polynomials of form (8) which are based on the introduction of special classes of piecewise constant test input signals. The analysis of the studied approaches will allow us to identify the preferable ranges for one or another algorithm, for the reference model (1) - (4).

6 CONCLUSIONS

The presented results of the mathematical modeling using the finite interval of the integro-power Volterra series were for the first time applied to describe the dynamics of the horizontal-axis wind turbine.

The technique was developed to construct the integral model and technically implement the high-speed system of control. A computational experiment aimed at constructing the integral models of the wind power unit was done.

The results of the computational experiment demonstrates the applicability of this mathematical tool to the control of active components of the electric power system.

To improve the accuracy of modeling, it is planned to introduce a structure with switchable kernels, which will envisage the adaptive behavior of the model in the case the input signal amplitudes go beyond some limited interval.

The computer modeling was carried out using the author's software created in Matlab.

Further it is planned to apply this approach to the research into complex dynamic systems which contain an arbitrarily large amount of components of the active-adaptive isolated system.

ACKNOWLEDGEMENTS

The research was partly funded by the grant of the Russian Foundation of Basic Research, project No.15-01-01425a.

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