Accelerating the Performance of Parallel Depth-First-Search Branch-and-Bound Algorithm in Transportation Network Design Problem

Amirali Zarrinmehr and Yousef Shafahi
Department of Civil and Environmental Engineering, Sharif University of Technology, Azadi Avenue, Tehran, Iran

Keywords: Transportation Network Design Problem, Parallel Branch-and-Bound Algorithm, Depth-First-Search, Greedy Algorithm, Super-linear Speedup.

Abstract: Transportation Network Design Problem (TNDP) aims at selection of a subset of proposed urban projects in budget constraint to minimize the network users' total travel time. This is a well-known resource-intensive problem in transportation planning literature. Application of parallel computing, as a result, can be useful to address the exact solution of TNDP. This paper is going to investigate how the performance of a parallel Branch-and-Bound (B&B) algorithm with Depth-First-Search (DFS) strategy can be accelerated. The paper suggests assigning greedy solutions to idle processors at the start of the algorithm. A greedy solution, considered in this paper, is a budget-wise feasible selection of projects to which no further project can be added while holding the budget constraint. The paper evaluates the performance of parallel algorithms through the theoretical speedup and efficiency which are based on the number of parallel B&B iterations. It is observed, in four cases of TNDP in Sioux-Falls transportation network, that achieving high-quality solutions by idle processors can notably improve the performance of parallel DFS B&B algorithm. In all four cases, super-linear speedups are reported.

1 INTRODUCTION

Transportation Network Design Problem (TNDP) is a well-know infrastructural problem in transportation planning. As a combinatorial problem, TNDP targets the selection of the optimal subset of a set of proposed projects, i.e. construction of urban highways, in budget constraint, so as to minimize the users' total travel time. This is an NP-Hard problem which has been addressed by various heuristic or meta-heuristic approaches (see for example Vitins and Axhausen (2010), or Farahani et al., (2013) as a more recent survey). The exact solution of TNDP, however, is a resource-intensive problem which becomes intractable soon as the problem enlarges (Poorzahedy, 1980).

By advent of parallel computing facilities in recent decades, much work has been directed to address the exact solution of NP-Hard problems. Parallel algorithms have been devised to tackle many of moderate or rather large size combinatorial problems (Roucairol, 1996). Parallel Branch-and-Bound (B&B) algorithms with Depth-First-Search (DFS) strategy have been among these algorithms for which promissing results have been reported (Anstreicher et al., 2002).

This paper is going to investigate accelerating the performance of parallel B&B algorithm with DFS strategy in TNDP. The B&B algorithm proposed by LeBlanc (1975) and parallelized by Zarrinmehr (2011) is taken into consideration. The paper demonstrates how assigning greedy high-quality solutions to the initial idle processors can improve the parallel performance of the algorithm.

The rest of this paper is organized as follows: Section 2 provides a formal discreption of TNDP. Section 3 overviews in brief the related literature and prerequisite concepts which will be refered in subsequent sections. Section 4 presents the methodology of this paper which will be about feeding idle processors. In section 5, detailed results of running the program on four cases of Sioux-Falls transportation network are reported and discussed, which will be followed by concluding remarks in section 6.

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2 FORMAL DESCRIPTION OF TNDP

TNDP is often formulated as a bi-level optimization problem. In this formulation, the optimal subset of projects is selected at the Upper Level Problem (ULP) and the users' behaviour in route selection is simulated at the Lower Level Problem (LLP), by solving a Traffic Assignment Problem (TAP). To formally describe the problem at the upper level, assume that (Poorzahedy and Abulghasemi, 2005):

A_y: set of proposed projects (links) to be added to the current network,

y_a: Binary decision variable corresponding to project a, $a \in A_y$, taking values 1 or 0 indicating whether to construct a project or not,

y: binary vector of decision variables,

$A(y)$: set of network links after decision y has been made, $A(y) = A \cup \{a \in A_y : y_a = 1\}$,

x_a: Amount of flow in link "a", $a \in A(y)$,

$x(y)$: vector of traffic flow on network links after decision y has been made,

$t_a(x_a)$: Volume-delay function of link a, $a \in A(y)$, assumed to be convex and differentiable for $x_a \geq 0$,

c_a: Cost related to the construction of project a, $a \in A_y$,

B: available budget for the construction of given projects,

Now the TNDP at the upper level can be written as (1)-(4).

$$\text{ULP: Min}_y\, T(y) = \sum_{a \in A(y)} x_a t_a(x_a) \quad (1)$$

$$\text{s.t: } y_a = 0 \text{ or } 1 \;\forall a \in A_y \quad (2)$$

$$\sum_{a \in A_y} c_a y_a \leq B \quad (3)$$

$x(y)$: the user equilibrium flow, from solution of LLP(y), corresponding to decision vector y (4)

In the above formulation, the objective function (1) is the sum of users' total travel time over all links of the network. Decision variables are defined in equation (2) as binary values. Equation (3) means that the selection of projects must meet the budget constraint. Constraint (4) expresses that the amounts of traffic flows on the network links must follow the pattern of User Equilibrium (UE) which is obtained through a TAP. This problem, as shown in (5)-(8), is formulated at the LLP and the following definitions are further required for its description:

P: set of origin-destinations (ODs), $P \subseteq V \times V$,

d_{rs}: The travel demand from r to s, $(r,s) \in P$,

$K_{rs}(y)$: Non-empty set of different paths form r to s after decision y has been made, $(r,s) \in P$,

f_k: Amount of flow in path k from r to s, $k \in K_{rs}(y)$,

δ_{ak}: Binary variable taking values 1 or 0 indicating whether link "a" belongs to path k or not, $a \in A(y)$, $k \in K_{rs}(y)$,

$$\text{LLP}(y)\text{: Min } \sum_{a \in A(y)} \int_0^{x_a} t_a(u)\, du \quad (5)$$

s.t.:

$$\sum_{k \in K_{rs}(y)} f_k = d_{rs} \quad \forall (r,s) \in P \quad (6)$$

$$f_k \geq 0 \quad \forall k \in K_{rs}(y), \forall (r,s) \in P \quad (7)$$

$$x_a = \sum_{(r,s) \in P} \sum_{k \in K_{rs}(y)} \delta_{ak} f_k \;\forall a \in A(y) \quad (8)$$

In the LLP formulation, the objective function (5) is the sum of the integrals of the link volume-delay functions, which is a mathematical construct without any clear interpretation (Sheffi, 1985). Equation (6) shows the flow conservation constraint, while the path flows are ensured to be non-negative in equation (7). The relationship between link flows and path flows is also considered as equation (8). It expresses that the amount of flow at each link is the summation of those path flows going through the link (Sheffi, 1985).

Solving LLPs makes the model consider how users select their route in the network, which is often known as the UE situation. The UE in a transportation network is a situation in which for any origin-destination, all paths used by travellers have an equal travel time which is less than or equal to the travel time of unused paths (Sheffi, 1985). The TAP formulation in (5)-(8) will be referred as a UE-TAP through this paper. There is still another kind of TAP which results in the System Optimal (SO) paths and flows in the network, and therefore, will be referred as SO-TAP. In the following is the formulation of a SO-TAP which is the same as UE-TAP unless in its objective function (Sheffi, 1985):

$$\text{Min } T(y) = \sum_{a \in A(y)} x_a t_a(x_a) \quad (9)$$

s.t. : (6), (7), (8)

The objective function (9) in SO-TAP has an intuitive meaning, which is minimization of the users' total travel time at the entire network. The resulting flow patterns, however, does not represent the actual behaviour of users, though it can be useful theoretically in transportation network problems.

It is easy to see that both formulations of UE-

TAP and SO-TAP are convex optimization problems and can be solved using the iterative convex combination algorithms (Sheffi, 1985).

3 RELATED LITERATURE

This section brief-reviews the related literature in three subsections. First, speedup and efficiency, as two basic measures in parallel computing are introduced as well as Amdahl's law. Parallel B&B algorithms with DFS strategy are introduced next. Finally the application of the B&B algorithm in TNDP and its parallelization are addressed.

3.1 Two Basic Measures in Parallel Computing

Two important parallelization measures used extensively in the literature are known as speedup and efficiency. Speedup is a measure indicating how many times the parallel algorithm performs faster due to the parallelization. Efficiency is also a normalized version of speedup, which is the speedup value divided by the number of processors (Barney, 2010).

A fundamental theorem in parallel computing is known as Amdahl's law. According to Amdahl's law, the speedup can never exceed the number of processors. This, in terms of efficiency measure, means that the efficiency will always be less than 1. A linear speedup is achieved when the speedup almost equals to the number of processors (Barney, 2010).

3.2 Parallel DFS B&B Algorithms

The B&B algorithms are general heuristic search procedures which can be applied in the exact solution of combinatorial problems (Li and Wah, 1990). In these algorithms, the search space of the problem is iteratively decomposed in the form of a rooted search tree, in which the root is the main problem and the descendents are partially solved problems. The B&B algorithm selects iteratively among the nodes of the search tree, expands the selected node, and updates the search tree information, until when there will be no node to be selected in the search tree (Gendron and Crainic, 1994).

Various search strategies of B&B algorithms arise at the stage of defining the selection priority over nodes of the search tree. DFS is a very famous strategy that selects the deepest node in the search tree in each iteration. Parallelization of this strategy has been an important research topic in 1980's and 1990's (Lai and Sahni, 1984; Li and Wah, 1986, Li and Wah, 1990). This was mainly because of some reports on super-linear speedup (i.e. a speedup which is greater than the number of processors) which seemed to be in contradiction with Amdahl's law, introduced previously in 3.1. The super-linear speedup in parallel DFS B&B algorithms was not, however, a contradiction because in reported cases the processors had been observed to access high quality solutions in early iterations, which in turn brought about a reduction in the search tree and problem size (Pruul et al., 1988).

3.3 B&B Algorithms in TNDP

Due to the NP-Hard complexity of TNDP, extensive research has investigated non-exact solutions for the problem (see Farahani et al., (2013) for example). Not much work has addressed the exact solution of TNDP. The only exact solution, to our knowledge, is the early study of LeBlanc (1975) which proposed a B&B algorithm to tackle the problem.

LeBlanc organized a binary rooted tree in correspondence with binary decision variables of the problem. In partial solutions, LeBlanc proposed solving a SO-TAP in which all undecided projects were assumed to be added to the underlying network. He proved that the users' total travel time, taken from such an assignment, would work as a lower-bound for complete solutions located at the last level of the search tree which are to be evaluated, in turn, with a UE-TAP. LeBlanc's algorithm was a working method to tackle small size TNDPs, but became inefficient soon as the problem enlarged (Poorzahedy, 1980).

In a recent study, Zarrinmehr (2011) discussed that the B&B algorithm proposed by LeBlanc (1975) well-adapts to a master-slave parallelization paradigm. In master-slave paradigm, one processor, namely the master, holds the main information of the problem. At the beginning of B&B iterations, the master processor assigns the nodes of the search tree to other processors, namely the slaves. Each slave processor receives one node from the master for which it solves a traffic assignment problem and sends back to the master processor the corresponding result. Receiving the results back from the slaves, the master updates the information and starts a new iteration if needed. This process will be addressed throughout this paper as a parallel iteration.

The results reported by Zarrinmehr (2011)

supported that, in low levels of parallelism, an almost linear speedup can be assured. It was also observed that, due to little idling overhead and negligible communication between master and slave processors, the performance of the parallel algorithm can be roughly estimated by the theoretical speedup, $S^{t}(p)$, and theoretical efficiency, $E^{t}(p)$, which account for the number of iterations in the parallel B&B algorithm rather than the running-times (Li and Wah, 1990).

Given that p is the number of parallel processors and $I(1)$ and $I(p)$ stand for the number of B&B iterations while using a single processor and p processors, respectively, the theoretical speedup and efficiency are defined as follows:

$$S^{t}(p) = I(1)/\ I(p) \quad (10)$$

$$E^{t}(p) = S^{t}(p)/p \quad (11)$$

4 METHODOLOGY

To address the solution of TNDP, this paper uses the parallel DFS B&B algorithm developed in Zarrinmehr (2011) as a base algorithm. The only difference is about feeding initial idle processors at the beginning of parallel B&B iterations.

The idling of processors in parallel B&B algorithms usually takes place when the algorithm is just started and there are not yet enough nodes available in the search tree to be distributed among processors (See Henrich (1993) for example). Exploiting these idle processors to evaluate high-quality solutions in parallel DFS B&B algorithms can be helpful in achieving better solutions while starting the search and it does not impose an extra computational burden on the performance of parallel program.

In many combinatorial problems (e.g. scheduling problems, integer-programming problems, etc) finding feasible solutions with good quality is an NP-Hard problem, itself, which is not easier than the main problem. In TNDP, however, simple greedy algorithms can be devised to generate such solutions. The next sub-section introduces one of these algorithms.

4.1 Greedy Solutions for TNDP

Although addition of more projects to a transportation network would not necessarily result in reducing the users' total travel time (known as Brass' paradox in transportation literature) it can provide a greedy solution for the problem. In other words, a transportation network with more projects, in a greedy approach, is likely to provide less congestion and travel time. As a result, a high-quality solution for TNDP may be a feasible selection of projects to which no further project can be added in budget constraint. Zarrinmehr and Shafahi (2014) named such a solution a dominant solution in one dimensional binary integer programming problems and proposed an algorithm for Enumeration of Dominant Solutions (EDS).

Given that there are n projects in TNDP to be selected, the EDS algorithm holds an n-length binary string as a candidate solution. The algorithm does not ignore any of dominant solutions and enumerates each of them in $O(3n)$ as a rough estimate. The interested reader can follow Zarrinmehr and Shafahi (2014) to find the details about the EDS algorithm and its functionality.

4.2 Feeding Initial Idle Processors

As mentioned in the beginning of this section, dominant solutions can serve as high-quality solutions to feed initial idle processors in the parallel DFS B&B solution of TNDP. Furthermore, the greedy solutions provided by this algorithm can be generated in no more that $O(3n)$ which is computationally negligible compared with other computations e.g. the iterative traffic assignment procedure. As a result, one way to feed initial idle processors in the parallel B&B solution of TNDP can be application of the EDS algorithm.

In the methodology used in this paper, whenever some idle slave processors receive nothing from master to work on, the master uses the EDS algorithm to assign dominant solutions to those processors. Assignment of dominant solutions to idle processors is done in the same order as they are computed by the EDS algorithm, as long as there are some idle processors to be assigned. These solutions are evaluated by idle slave processors and sent back to the master. The master, then, may update the best solution in starting iterations of the parallel DFS B&B algorithm which, in turn, can be helpful in directing the search and discarding useless nodes of the search tree.

5 NUMERICAL RESULTS

To explore the effect of feeding initial idle processors in the parallel DFS B&B algorithm in TNDP, this section considers a problem with 12 given projects in the Sioux-Falls transportation

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network (Bar-Gera, 2011) as shown in Figure. 1.

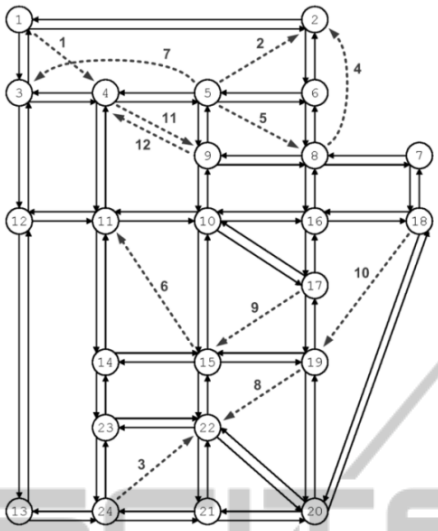

Figure 1: The Configuration of Sioux-Falls network and projects.

Given that the volume-delay functions of the network edges (i.e. highways) follow the pattern of equation (12), details about the projects definition are given in Table 1.

$$t_a(x_a) = fftt_a(1+0.15(\frac{x_a}{C_a})^4) \quad (12)$$

Table 1: Projects definition.

Project Number	Free Flow Travel Time (Minutes)	Practical capacity (Vehicles per Hour)	Project Cost (Units of Budget)
1 (1 to4)	8.0	5000	20
2 (5 to 2)	6.0	5000	22
3 (24 to 22)	5.0	5000	27
4 (8 to 2)	7.0	5000	27
5 (5 to 8)	4.0	5000	27
6 (15 to 11)	6.5	5000	28
7 (5 to 3)	10.0	5000	31
8 (19 to 22)	5.0	5000	32
9 (17 to 15)	4.5	5000	34
10 (18 to 19)	6.0	5000	40
11 (4 to 9)	3.0	5000	40
12 (9 to 4)	3.0	5000	42

In equation (12), t_a is the travel time in link 'a' (minutes), $fftt_a$ is the free flow travel time in link 'a' (minutes), x_a is the amount of traffic flow on link 'a' (vehicles per hour), and C_a is the practical capacity of link 'a' (vehicles per hour).

The parallel programs of DFS B&B algorithm were run before and after feeding the initial idle processors. Each program was only run once, because the performance of the parallel algorithm is not subjected to a stochastic behaviour (Zarrinmehr, 2011). Four budget levels of 60, 100, 140, and 200 (in units of budget) have been considered. The performance measures are theoretical speedup and efficiency which are rough estimates for real speedup and efficiency, according to Zarrinmehr (2011). Results are reported for processor numbers of 1, 2, 4, ..., 20. Table 2 presents the detailed results.

There are some symbols in Table 2 that need to be introduced beforehand:

$\#p$: Number of processors,

$\#iter$: Number of parallel iterations in parallel B&B algorithm (as introduced earlier in subsection 3.3),

$S^t(p)$: Theoretical speedup,

$E^t(p)$: Theoretical efficiency,

Err: The gap, in percentage, between the objective function of the best found solution by idle processors and the global optimum.

Tables 2(a-d)-1 use the measure *Err* to show the quality of greedy solutions found by the EDS algorithm in the beginning of the parallel B&B algorithm. When this measure is close to zero, a better solution is initially available to direct the search of the DFS strategy.

For example, in Table 2(a)-1, when 6 processors are considered to parallelize the DFS B&B algorithm, the solution [000000011000] is achieved with the objective function (i.e. users' total travel time) of 6710836 (vehicle-minutes) which is a rather high-quality solution due to *Err* = 100*(6710836-6667832)/6667832 = 0.6%. This initial solution contributes to discard useless nodes in the search tree of DFS B&B algorithm and leads to a notable reduction in the number of parallel iterations of the algorithm. This is shown in Table 2(a)-2 for $\#p$=6 where the number of parallel iterations is reduced from 40 to 32.

Addressing the speedup and efficiency through the number of parallel iteration (as in 3.3), it is interesting also to see that the reduction in the number of parallel iterations due to feeding idle processors may bring about a super-linear speedup. Again, in Table 2(a)-2 when 6 processors are assumed to be available, after feeding the idle processors it can be observed that

$S^t(6)$=216/32=6.75 > 6 and $E^t(6)$=6.75/6=1.13 > 1.00.

Table 2: Detailed Results of Parallel DFS B&B Algorithm Before and After Feeding Idle Processors.

(a)-1: Solutions Found by Idle Processors When B=60

# p	Best Solution Found by Idle Processors	Corresponding Objective Function (vehicle-minutes)	Gap (%)
1	-	-	-
2	[000000000011]	7443194	11.6
4	[000000001010]	7127291	6.9
6	[000000011000]	6710836	0.6
8-20	[000000110000]	6667832	0.0
The Global Optimum of the Problem			
[000011000000]	*6667832*		

(a)-2: Performance Measures When B=60

	Before Feeding Processors			After Feeding Processors		
# p	*# iter*	$S^t(p)$	$E^t(p)$	*# iter*	$S^t(p)$	$E^t(p)$
1	216	1.00	1.00	216	1.00	1.00
2	109	1.98	0.99	109	1.98	0.99
4	57	3.79	0.95	57	3.79	0.95
6	40	5.40	0.90	32	**6.75**	**1.13**
8	31	6.97	0.87	25	**8.64**	**1.08**
10	26	8.31	0.83	21	**10.29**	**1.03**
12	24	9.00	0.75	18	**12.00**	**1.00**
14	22	9.82	0.70	18	12.00	0.86
16	19	11.37	0.71	16	13.50	0.84
18	18	12.00	0.67	16	13.50	0.75
20	17	12.71	0.64	15	14.40	0.72

(b)-1: Solutions Found by Idle Processors When B=100

# p	Best Solution Found by Idle Processors	Corresponding Objective Function (vehicle-minutes)	Gap (%)
1	-	-	-
2	[000000001111]	6951620	8.7
4	[000000011100]	6586257	3.0
6-20	[000000111000]	6395632	0.0
The Global Optimum of the Problem			
[000000111000]	*6395632*		

Table 2: Detailed Results of Parallel DFS B&B Algorithm Before and After Feeding Idle Processors. (cont.)

(b)-2: Performance Measures When B=100

	Before Feeding Processors			After Feeding Processors		
# p	# iter	$S^t(p)$	$E^t(p)$	# iter	$S^t(p)$	$E^t(p)$
1	468	1.00	1.00	468	1.00	1.00
2	236	1.98	0.99	236	1.98	0.99
4	120	3.90	0.98	115	**4.07**	**1.02**
6	83	5.64	0.94	59	**7.93**	**1.32**
8	64	7.31	0.91	46	**10.17**	**1.27**
10	53	8.83	0.88	37	**12.65**	**1.26**
12	47	9.96	0.83	32	**14.63**	**1.22**
14	41	11.41	0.82	30	**15.60**	**1.11**
16	37	12.65	0.79	25	**18.72**	**1.17**
18	34	13.76	0.76	24	**19.50**	**1.08**
20	33	14.18	0.71	22	21.27	1.06

(c)-1: Solutions Found by Idle Processors When B=140

# p	Best Solution Found by Idle Processors	Corresponding Objective Function (vehicle-minutes)	Gap (%)
1	-	-	-
2	[000000011111]	6575316	5.5
4	[000000111101]	6271596	0.6
6-20	[000000111110]	6259644	0.4
The Global Optimum of the Problem			
[001000111000]	*6233324*		

(c)-2: Performance Measures When B=140

	Before Feeding Processors			After Feeding Processors		
# p	*# iter*	$S^t(p)$	$E^t(p)$	*# iter*	$S^t(p)$	$E^t(p)$
1	717	1.00	1.00	717	1.00	1.00
2	362	1.98	0.99	362	1.98	0.99
4	183	3.92	0.98	166	**4.32**	**1.08**
6	124	5.78	0.96	111	**6.46**	**1.08**
8	95	7.55	0.94	84	**8.54**	**1.07**
10	76	9.43	0.94	68	**10.54**	**1.05**
12	65	11.03	0.92	58	**12.36**	**1.03**
14	59	12.15	0.87	51	**14.06**	**1.00**
16	53	13.53	0.85	45	15.93	1.00
18	47	15.26	0.85	42	17.07	0.95
20	45	15.93	0.80	39	18.38	0.92

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Table 2: Detailed Results of Parallel DFS B&B Algorithm Before and After Feeding Idle Processors. (cont.)

(d)-1: Solutions Found by Idle Processors When B=200

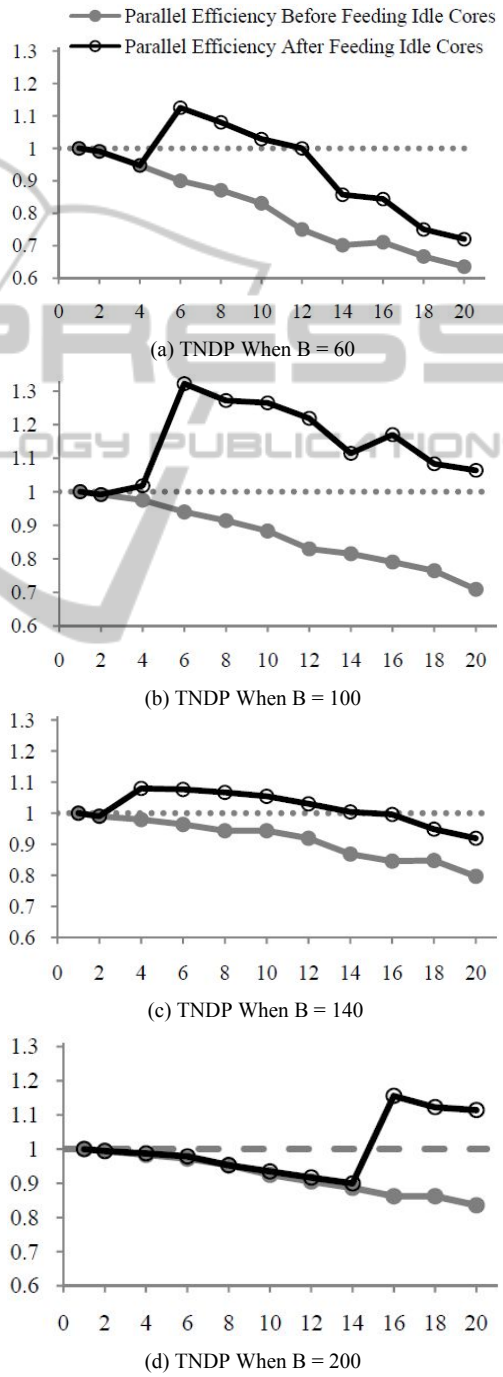
# p	Best Solution Found by Idle Processors	Corresponding Objective Function (vehicle-minutes)	Gap (%)
1	-	-	-
2	[000001111111]	6254867	3.6
4	[000010111111]	6183311	2.4
6	[000100111111]	6159487	2.1
8	[000100111111]	6159487	2.1
10	[000110111110]	6120960	1.4
12	[001000111111]	6108631	1.2
14	[001000111111]	6108631	1.2
16-20	[001010111100]	6035512	0.0
The Global Optimum of the Problem			
[001010111100]		*6035512*	

(d)-2: Performance Measures When B=200

	Before Feeding Processors			After Feeding Processors		
# *p*	# *iter*	$S^t(p)$	$E^t(p)$	# *iter*	$S^t(p)$	$E^t(p)$
1	869	1.00	1.00	869	1.00	1.00
2	437	1.99	0.99	437	1.99	0.99
4	221	3.93	0.98	220	3.95	0.99
6	149	5.83	0.97	148	5.87	0.98
8	114	7.62	0.95	114	7.62	0.95
10	94	9.24	0.92	93	9.34	0.93
12	80	10.86	0.91	79	11.00	0.92
14	70	12.41	0.89	69	12.59	0.90
16	63	13.79	0.86	47	**18.49**	**1.16**
18	56	15.52	0.86	43	**20.21**	**1.12**
20	52	16.71	0.84	39	**22.28**	**1.11**

Similar results can be observed in Table 2(a)-2 for #*p* = 8, 10, and 12, as well as in Tables 2(b), (c), and (d) where super-linear speedups, and efficiencies greater than one have been bolded. Figure 2 presents graphs for efficiencies before and after feeding idle processing cores.

The results in Table 2 suggest that, when assigning greedy TNDP solutions to idle processors, the improvement in the performance of the parallel DFS B&B algorithm is quite dependent on the quality of the best solution found by initial idle processors. This can be better found out in Figures 2(b) and 2(d). In Figure 2(b) the early access of processors to the optimal solution of the problem (i.e. *Err* =0.0) caused a notable increase in the efficiency (e.g. $E^t(6)$=1.32) of the parallel algorithm. On the other side, in Figure 2(d) the performance almost remains unchanged up to use of 14 processors, due to low-quality solutions accessed by idle processors.



(a) TNDP When B = 60

(b) TNDP When B = 100

(c) TNDP When B = 140

(d) TNDP When B = 200

Figure 2: Theoretical Efficiencies of Parallel Algorithms Before and After Feeding Idle Processors.

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6 CONCLUDING REMARKS

This paper addressed accelerating the exact solution of TNDP via a parallel DFS B&B algorithm with a master-slave parallelization paradigm. Theoretical measures of speedup and efficiency, based on the number of parallel iterations, were considered to evaluate performances. It was discussed that there were some initial idle processors at the start of the parallel iterations, as there were not sufficient nodes to be worked on. The paper suggested assigning greedy solutions in TNDP (namely dominant solutions), to these idle processors and applied a simple enumeration algorithm to provide such solutions. It was shown, in four case studies of TNDP with 12 projects, in Sioux-Falls transportation network, that feeding the idle processors can be helpful in reducing the number of parallel iterations and accelerating the performance of the algorithm. Super-linear speedups, and efficiency values greater than one, were observed in all examples.

Numerical results in this paper suggested that improving the performance of the algorithm was quite dependent on the quality of the best found solution by idle processors. Therefore, it is interesting to investigate how application of more promising solutions, rather than those computed first by the EDS algorithm, can affect improving the performance of the algorithm. Also, this paper addressed the performance measures through theoretical speedups and efficiencies of parallel B&B algorithm which have been based on the number of parallel iterations rather than running-times. Reporting on the real speedups and efficiencies based on the running-times and running the programs on large transportation networks with more than 12 projects and with higher number of processors can all be helpful to extend the results of this paper.

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