A Network Model for the Hospital Routing Problem

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Abstract: We consider the problem of routing samples taken from patients to laboratories for testing. These samples are taken from patients housed in hospitals, and are sent to laboratories in other hospitals for testing. The hospitals are distributed in a geographical area, such as a city. Each sample has a deadline, and all samples have to be transported within their deadlines. We have a fixed number of vehicles as well as an unlimited number of taxis available to transport the samples. The objective is to minimize a linear function of the total distance travelled by the vehicles and the taxis. We provide a mathematical programming formulation for the problem using the multi-commodity network flow model, and solve the formulation using CPLEX, a general-purpose MIP solver. We also provide a computational study to evaluate the solution procedure.

1 INTRODUCTION

Every metropolitan city has hospitals of varying sizes, each cost-effective in serving the healthcare needs of its surrounding population. Most hospitals include a laboratory that can perform a variety of tests on samples collected from its patients. Since laboratories in smaller hospitals are often not equipped to perform all tests on its samples, these samples have to be sent to laboratories in larger hospitals for testing. This paper addresses the problem of routing samples to hospitals, called the Hospital Laboratory Courier Routing Problem (HLCRP). This is the pickup and delivery problem with time windows without capacity constraints and with transshipments allowed. Even though we address the specific problem of routing test samples between hospitals, our model and solution procedure can be applied to other problems.

The test samples are collected from the patients in the hospitals during a day. The samples include blood, urine, sputum, or tissue, and each sample has a deadline before which the test should be conducted. The hospitals are located in a given geographical area, such as the metropolitan area of a city. Each hospital is equipped with a laboratory of a given capability. Some samples can be tested at the hospital where it was collected, while others have to be transported to another hospital with better equiped laboratories.

The transportation of samples is done by a fleet of vehicles of a fixed size. In addition, the use of taxis is also allowed to transport samples with impending deadlines. For the fleet of vehicles, there is no depot, and each vehicle can start its route at any hospital and finish it at any other hospital. (This characteristic of the problem also comes from the situation observed in practice where the cost of the route does not include the cost of getting the vehicle to the first hospital in the route.)

The Hospital Laboratory Courier Routing Problem (HLCRP) deals with finding the routes and schedules for the vehicles such that all the samples are transported within their deadlines, and a linear function of the total distance travelled by the vehicles and taxis is minimized. Since the cost of using a taxi is several times higher than the standard vehicle, the optimizer should also reduce the number of taxi calls.

Thus, the HLCRP is a variation of the pickup and delivery problem with time windows, without capacity constraints, and with transshipment ((Savelsbergh and Sol, 1995), (Minic, 1998), (Minic and Laporte, 2006), (Cortés et al., 2010)).

The HLCRP may either be modeled as a vehicle routing problem (VRP), or as a multi-commodity network flow problem (MCNFP). Typically, the number of locations in the VRP is large, and each location has to be visited once. In contrast, in the MCNFP, the number of locations is smaller, though the num-

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ber of requests originating at each location is large, with multiple visits to the same location during the day.

There is a large body of research on the VRP, with many surveys, including (Cordeau et al., 2007), (Golden et al., 2008), (Laporte, 1992), (Laporte et al., 2000), (Toth and Vigo, 2002). There are also many surveys on the time-constrained version of the VRP - the Vehicle Routing Problem with Time Windows (VRPTW) - including (Brysy and Gendreau, 2005) and (Kallehauge et al., 2005). Examples of the methods for optimally solving the VRPTW include Desrochers et. al (Desrochers et al., 1992), who pioneered the column-generation approach for the vehicle routing problem. They decomposed the problem into a master problem and a subproblem, and solved the master problem using column generation. Kohl et. al (Kohl et al., 1999) introduced cuts to the decomposition-based approach, and Kohl and Madsen (Kohl and Madsen, 1997) develop a Lagrangean relaxation approach to solve the VRPTW exactly. For a comprehensive review of the column generation method to solve the VRPTW, see (Kallehauge et al., 2005).

We model the HLCRP as an MCNFP. Each node in the network is a hospital at a particular time instant, and each arc between two nodes is the route between the corresponding hospitals. Each set of boxes that are carried together by a vehicle is a commodity that flows through the network. Such models have been used to design networks and routes for public transportion (Ceder, 2003), to solve ship routing and scheduling problems (Christiansen et al., 2004), in maritime transportation (Brønmo et al., 2007), airline schedule planning (Gopalan and Talluri, 1998), and ferry scheduling (Karapetyan and Punnen, 2013; Minic and Punnen, 2011).

In related work, heuristics using genetic algorithms have been used to solve the problem of routing blood samples collected from hospitals and health care centres to two central laboratories in Spain (Grasas et al., 2014). In this problem, in addition to imposing time windows on samples, vehicles also have capacity restrictions. Finally, (Rais and Viana, 2010) provides a comprehensive survey of operations research methods used in the healthcare industry. The applications listed in the survey are far too many to list here.

2 MODEL

In the model we use for HLCRP, the time horizon is divided into intervals of size δ (where δ is a suitably chosen constant), and each hospital is represented by multiple nodes, one for each time instant (the multiple of δ). Representing each node by multiple nodes, one for each time instant, is a standard modelling approach, usually called time expanded network. This approach has been successfully used to solve many practical instances of similar routing and scheduling problems.

A directed arc exists between two nodes a and b, if it is feasible to travel from node a to node b within the corresponding time. The movement of the packages and the vehicles represent the flow through the network.

The solution to our problem consists of a set of routes and schedules. Each route is a sequence of hospital locations, each of which has the arrival time and the departure time. We assume that each vehicle starts immediately from the first pick-up point, thus there is no travel cost from and to the depot. We also consider a second set of vehicles - the taxis. There are no limits on the number of taxi trips. However, using a taxi to travel between two nodes costs ρ times more than using a vehicle.

We provide details of the network construction for the model in Section 2.1, and the mathematical programming formulation for the model in Section 2.2.

2.1 Network Construction

Time Discretization: The size of the network is a function of the discretization time, denoted Δt. Δt is specified in minutes. T is the set of all discrete time instants/stamps, M is the set of all packages, and l_j^p, $l_j^d \in N$ denote the pickup, delivery locations of package j, $\forall j \in M$. Furthermore, t_j^p, t_j^d denote the earliest pickup time, latest delivery time of package j, $\forall j \in M$, and $h^b = \min_{j\in M} t_j^p$, $h^e = \max_{j\in M} t_j^d$ denote the beginning, end of the horizon for our problem.

$|T|$ is given by $|T| = \lfloor \frac{h^e - h^b}{\Delta t} \rfloor + 1$. The earliest pickup time stamp of package $j \in M$ is given by $\tau_j^p = \lceil \frac{t_j^p - h^b}{\Delta t} \rceil$. The latest delivery time stamp of package $j \in M$ is given by $\tau_j^d = \lfloor \frac{t_j^d - h^b}{\Delta t} \rfloor$. The discretized cost (distance) from $u \in N$ to $v \in N$ (measured in time stamps), denoted $\delta_{u,v}$, is given by $\delta_{u,v} = \lceil \frac{d_{u,v}}{\Delta t} \rceil$. Thus $\delta_{u,u} = 1$ denotes waiting for one time stamp at node $u \in N$. We assume the graph G is not complete, so the distance $d_{u,v}$ (as well as the discretized distance $\delta_{u,v}$) is ∞ if there is no direct route from node u to node v. We let $\sigma_{u,v}$ denote the shortest path distance from u to v in the network (computed in units of δ).

Nodes and Arcs in the Network:

We are given a set N of site nodes (each site node denotes a hospital or test site). Corresponding to each site node $u \in N$, we construct q copy nodes $(u,1),(u,2),\ldots,(u,q)$, where (u,l) represents the copy of site node u at time stamp l. We also add a start node s and a destination node f to the set of copy nodes. We now describe the set of arcs comprising the network.

We have three types of arcs in the network, the set of package arcs A_p, the set of vehicle arcs A_c, and the set of taxi arcs A_t. We provide the ability to use additional problem-specific information to reduce the number of arcs (and therefore the size) of our model. Thus, if no package travels from node (u,q) to node (v,r) in any optimal route, then there is no arc between nodes (u,q) and node (v,r). Arcs may also not be present if routing a package through the arc violates feasibility.

A package arc between two copy nodes indicates that a package can travel between the corresponding site nodes feasibly in time. Thus, the package arc $e^j_{(u,q),(v,r)}$ is in set A_p if package j can arrive at site node u before time q, can leave site node v at or after time r, and be feasibly delivered at its destination node before its deadline. Moreover, r is the earliest possible time during which the package may arrive at site node v after departing from site node u at time p. Similarly, a vehicle arc $e^c_{(u,q),(v,r)}$ (taxi arc $e^t_{(u,q),(v,r)}$) exists if a vehicle (taxi) can feasibly travel from copy node (u,q) to copy node (v,r).

We describe below the conditions that have to be fulfilled for the existence of package arcs, vehicle arcs, and taxi arcs. We define a boolean variable $b_{u,v}$ which is set to 1 if a package originating at copy node (u,q) is allowed to travel through copy node (v,r) (it is set to 0 otherwise). This boolean variable is used to specify the conditions for the existence of package arcs.

Conditions for the existence of package arcs:
$\forall j \in M, \forall u,v \in N \quad u \neq l^d_j \quad v \neq l^p_j, \forall q \in T$, package arc $e^j_{(u,q),(v,r)} \in A_p$ if each of the conditions below hold:

$$b_{l^p_j,u} = 1 \wedge b_{l^p_j,v} = 1 \tag{1}$$

$$q \geq \tau^p_j + \sigma_{l^p_j,u} \tag{2}$$

$$r = q + \delta_{u,v} \leq \tau^d_j - \sigma_{v,l^d_j} \tag{3}$$

Here, Equation (2) (respectively Equation (3)) determines the earliest possible departure time of the package from u (respectively the latest possible arrival time at v). Note that there can be more than one package arc (for different packages) between copy nodes (u,q) and (v,r).

Conditions for the existence of vehicle and taxi arcs:
$\forall u,v \in N \quad \forall q \in T \quad e^c_{(u,q),(v,r)} \in A_c \; (e^t_{(u,q),(v,r)} \in A_c)$ if $r = q + \delta_{u,v} \leq |T| - 1$
$\forall v \in N \quad \forall r \in T \quad e^c_{s,(v,r)} \in A_c$
$\forall u \in N \quad \forall q \in T \quad e^c_{(u,q),f} \in A_c$

We add a vehicle arc from the start node s to every copy node (u,q) and from every copy node (v,r) to the end node f.

2.2 Mathematical Programming Formulation

The decision variables are the boolean variables $x_{j,u,q,v,r}$, $y_{u,q,v,r}$, and $z_{u,q,v,r}$, that indicate whether a package, vehicle, or taxi travels along arc $e^j_{(u,q),(v,r)}$, $e^c_{(u,q),(v,r)}$, or $e^t_{(u,q),(v,r)}$, respectively. The number of vehicles used is modeled using integer variables $s_{v,r}$ and $f_{u,q}$.

$$\min \sum_{u,q,v,r:\, e^t_{(u,q),(v,r)} \in A_t} d_{u,v}(y_{u,q,v,r} + \rho z_{u,q,v,r}) \tag{4}$$

Subject to

$$\sum_{q,v,r:\, e^j_{(l^p_j,q),(v,r)} \in A_p} x_{j,l^p_j,q,v,r} = 1 \qquad \forall j \in M \tag{5}$$

$$\sum_{u,q,r:\, e^j_{(u,q),(l^d_j,r)} \in A_p} x_{j,u,q,l^d_j,r} = 1 \qquad \forall j \in M \tag{6}$$

$$\sum_{u,q:\, e^j_{(u,q),(v,r)} \in A_p} x_{j,u,q,v,r} = \sum_{w,s:\, e^j_{(v,r),(w,s)} \in A_p} x_{j,v,r,w,s} \quad \forall j \in M, \forall v \in N \setminus \{l^p_j, l^d_j\}, \forall r \in T \tag{7}$$

$$y_{u,q,v,r} + z_{u,q,v,r} \geq x_{j,u,q,v,r} \quad \forall j,u,q,v,r:\, u \neq v \wedge e^j_{(u,q),(v,r)} \in A_p \tag{8}$$

$$\sum_{v,r:\, e_{s,(v,r)} \in A_c} s_{v,r} = k \tag{9}$$

$$\sum_{u,q:\, e_{(u,q),f} \in A_c} f_{u,q} = k \tag{10}$$

$$s_{v,r} + \sum_{u,q:\, e_{(u,q),(v,r)} \in A_c} y_{u,q,v,r} = f_{v,r} + \sum_{w,s:\, e_{(v,r),(w,s)} \in A_c} y_{v,r,w,s} \quad \forall v \in N, \forall r \in T \tag{11}$$

$$x, z \in \{0,1\} \quad y, s, f \in \{0,1,\ldots,k\} \tag{12}$$

Here, Equations (5) and (6) are package pickup and delivery contraints. Equation (7) are package transit contraints to ensure that if a package enters a node, it also exits the node. Equation 8 are package carry constraints to ensure that packages are carried by vehicles or taxis. Equations (9) and (10) ensure that k vehicles start and finish the tours. Finally, Equation (11) ensures that flow conservation constraints are met for the vehicles.

3 IMPLEMENTATION AND EXPERIMENTAL RESULTS

We solve the mathematical programming model described above using the general-purpose mixed-integer program solver CPLEX. Since the time-space network can be too large, we apply arc reduction procedures prior to the integer program construction. Moreover, we remove some of the arcs in the network based on the available heuristic information about the structure of possible routes. For example, in the graph networks corresponding to city maps, there are tendencies for routes to extensively use arterial roads. Another type of arc reduction can come from the fact that there is rarely a need for a package, that has its pickup and delivery in the same local area, to be travelling through another distant part of the map. All this extra information can easily be incorporated into our model using conditions for the existence of the package arcs ($b_{u,v}$ in Sec. 2.1).

We generate forty problem instances in total, comprised of four sets, each with ten problem instances. The instances we generate are based on the geographical location of hospitals in a metropolitan city in Canada, and publicly available data on the population these hospitals serve.

In the extreme case, the number of packages we have is around 140. This, together with the number of hospitals (at most 20), determines the size of the input. Each instance was run on the Simon Fraser University RCG Colony, a cluster of 64-bit Linux computers (each run is set to use exactly one core of one processor). We specify the details of our computational study below.

Comparing Solution Quality Across Problem Instances. We use the relative MIP gap, the ratio of the difference between the solution value (obtained by the MIP solver) and either the optimal, or a bound on the optimal, as a measure of the solution quality. We examine its dependence on three parameters: the sparsity of the input graph, the number of vehicles in the fleet, and the discretization time δ used to construct the network. The input graph is either sparse or complete, the number of vehicles ranges from 0 to 25 (in steps of 5), and the discretization time, in minutes, ranges from 5 to 30 (in steps of 5).

We also measure the running time of our model to reach the relative gap of 10% for discretization time steps of 5 minutes and 10 minutes, using 10 vehicles. When δ is 10 minutes (5 minutes), the average time to reach the gap is 1244 seconds (4124 seconds). We set a CPU time limit of 2 hours.

Sparse vs Complete Graph. Our model permits us to specify and exploit the sparsity of the input graph. It is clear from Figure 1 that our model requires much less CPU time for sparse graphs. Intuitively, there are more options available in a denser graph. The larger solution space that results slows down the MIP solver.

Missing edges between pairs of nodes may be replaced by 'edges' with shortest path distances between corresponding nodes. Adding such missing edges may provide feasible solutions, where none may exist in the sparse graph, due to the fact that we discretize time windows and distances. We evaluate our solutions on the sparse graph in the rest of the paper.

Number of Vehicles and Discretization Time Steps. Figure 2 displays how the relative gap changes with the number of vehicles allowed and the discretization time. We note that solving the problem for the case when the packages have to be delivered by using both vehicles as well as taxis is harder than for the case when the packages have to be delivered either entirely by vehicles or entirely by taxis.

Figure 3 presents a table that displays how the relative MIP gap, the objective function value, and the number of taxis, depend on the number of vehicles used and the discretization time step. As can be observed, both the objective function value and the number of taxis decrease with the number of vehicles allowed. The flexibility of our model in allowing taxis becomes apparent when fewer vehicles are present. In these cases, instead of obtaining infeasible solutions, we get solutions with larger objective function value. Similarly, in the real-life scenario that motivated our work, taxis were used whenever it was impossible to transport a sample within its deadline using the scheduled vehicles.

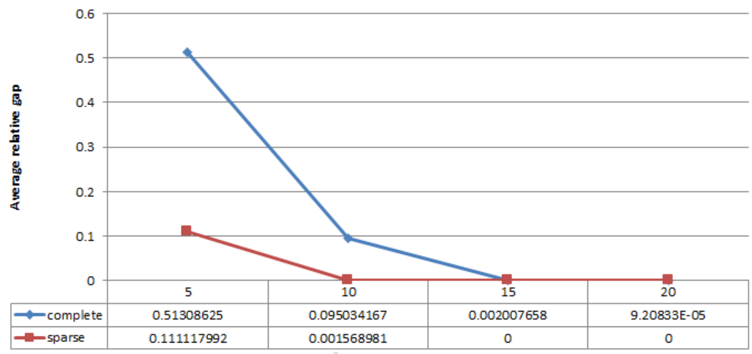

Figure 1: Dependence of Average Relative Gap on Number of Vehicles and Discretization Time Steps.

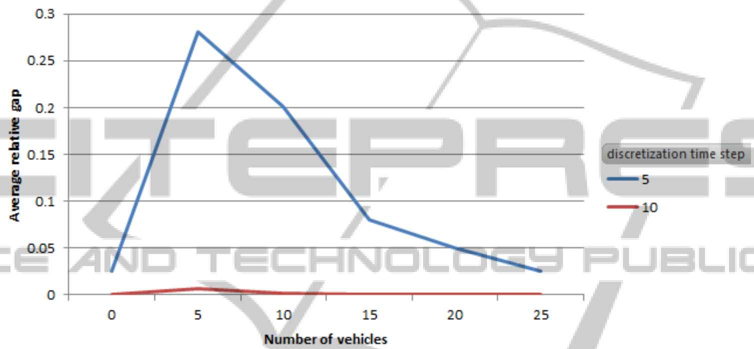

Figure 2: Dependence of Average Relative Gap on Number of Vehicles and Discretization Time Steps.

		δ					
		5			10		
		o.v.	gap	taxi calls	o.v.	gap	taxi calls
vehicles	0	22467.38	0.0252	217.175	4519.667	0	46.9444
	5	146686	0.2812	1378.15	1922.944	0.0067	8.3333
	10	128120.1	0.2015	1200	1350.583	0.0023	0.4444
	15	58380.45	0.0805	541.925	1186.167	0.0004	0.3333
	20	38921.08	0.0506	357.425	1135.056	0	0.0556
	25	19329.87	0.0256	171.4359	1129.917	0	0

Figure 3: Dependence of Average Objective Function Value, Relative Gap and Number of Taxis on Number of Vehicles and Discretization Time Steps.

4 CONCLUSION

In this paper, we address an important problem that arises in the healthcare industry, that of transporting laboratory samples between hospitals. This problem arises because the laboratories within a hospital may not be equipped to perform the required tests on a sample. We present a mathematical programming formulation of the problem, a solution procedure using CPLEX, and a set of experiments to evaluate the solution procedure. Even though we test our solution procedure on generated data, we believe our solution procedure can be used to solve the real-world problem that motivated this exercise in the first place. The approach outlined in this paper can be applied to solve problems of comparable size that arise in the healthcare industry.

Future work may include a model-based heuristic that will provide good solutions for larger problem instances. In addition, the geographical area may be partitioned into zones, and the size of the flow network reduced by removing arcs unlikely to be used in an optimal solution. This may permit us to solve much larger instances of the problem.

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