# Reordering Variables using ‘Contribution Number’ Strategy to Neutralize Sudoku Sets

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Keywords: Sudoku, Contribution Number, Neutralized Set, Search Algorithm, Search Strategy.

Abstract: Humans tend to form decisions intuitively, often based on experience, and without considering optimality; sometimes, search algorithms and their strategies apply the same approach. For example, the minimum remaining values (MRV) strategy selects Sudoku squares based on their remaining values; squares with less number of values are selected first, and the search algorithm continues solving squares until the Sudoku rule is violated. Then, the algorithm reverses the steps and attempts different values. The MRV strategy reduces the backtracking rate; however, when there are two or more blank squares with the same number of minimum values, such strategy selects any of these blank squares randomly. In addition, MRV continues to target squares with minimum values, ignoring that some of those squares could be considered ‘solved’ when they have no influence on other squares. Hence, we aim to introduce a new strategy called Contribution Number (CtN) with the ability to evaluate squares based on their influence on others candidates to reduce squares explorations and the backtracking rate. The results show that the CtN strategy behaves in a more disciplined manner and outperforms MRV in most cases.

## 1 INTRODUCTION

Fundamentally, Sudoku puzzles are among the difficult problems in computer science (Aaronson, 2006; Jilg & Carter, 2009) that have been categorized as constraint satisfaction problems (CSPs) (Ercsey-Ravasz & Toroczkai, 2012; Moraglio, Togelius, & Lucas, 2006), and that are also commonly known as nondeterministic polynomial time (NP)-complete problems (Edelkamp & Schrodl, 2012; Eppstein, 2012; Ercsey-Ravasz & Toroczkai, 2012; Klingner & Kanakia, 2013; Moraglio et al., 2006). Therefore, in order to solve this type of difficult problems, a set of values has to be examined in each single blank square in specific order until the correct value is assigned. The ultimate objective of Sudoku agent solvers is to find valid values for the remaining squares to satisfy Sudoku constraints. The puzzle is considered solved when all remaining squares are completed with valid values (Edelkamp & Schrodl, 2012; Eppstein, 2012).

The algorithms used to solve Sudoku puzzles fall into two main categories (Eppstein, 2012; Norvig, 2010):

1- Deductive algorithms: this approach searches for patterns to eliminate invalid candidates; no estimation is performed.

2- Search algorithms: these are brute-force type searches through predefined sets of potential candidates using the ‘*trial and error’* approach.

Deductive algorithms cannot solve Sudoku puzzles when the information provided (clues) is not sufficient to recognize deductive patterns (Eppstein, 2012; Norvig, 2010). On the other hand, search algorithms always find solutions (when there is one) because they attempt all possible values on all available variables until a solution is found. Consequently, solvers from both categories have to work iteratively through all remaining squares at least $n$ times, where $n$ is the total number of blank squares to assign their values. We can afford to design computational components based on optimality principles that eventually lead to a reduction of the search space. By altering the solver’s objective from *solving* the puzzle to *neutralizing* it, we can promote a different approach for identifying the most optimal square among those that have the same number of minimum remaining

Abuluaih S., Mohamed A., Muthukkaruppan A. and Iida H..
Reordering Variables using ‘Contribution Number’ Strategy to Neutralize Sudoku Sets.
DOI: 10.5220/0005188803250333
In *Proceedings of the International Conference on Agents and Artificial Intelligence* (ICAART-2015), pages 325-333
ISBN: 978-989-758-074-1

candidates, and that simultaneously has the most impact on other square candidates before they are selected and processed. This leads to a faster reduction of the total average of candidate numbers in the puzzle; Contribution Number (CtN) can play a major role in this.

CtN is a strategy works with the Backtracking (BT) search algorithm that is dedicated to marking each square with a weight that indicates how solving a specific square can influence other square candidates. Furthermore, CtN strategy indicates the comparative benefits of solving a specific puzzle square compared to another. The results obtained from preliminary experiments show that the game-depth of Sudoku puzzles is greatly reduced after implementing the concept of neutralization and CtN. In this paper, we favour two claims. First, there are Sudoku configurations that do not require any type of search algorithms to be considered solved; we call these 'neutralized sets'. Second, to increase MRV efficiency, assessing all squares with the minimum remaining values is required in order to identify the most optimal square and avoid random selections.

## 2 SUDOKU PUZZLE AND SUDOKU SOLVERS

Sudoku is a grid with $a \times b$ rows and $a \times b$ columns, where $a$ and $b$ are natural numbers, and the grid consists of $(a \times b)(a \times b)$ total squares. The container that holds the assembled squares is called the 'main grid', and it consists of $a \times b$ sub-grids (also known as 'Boxes'), each sub-grid is $a$ squares on wide and $b$ squares on high (Eppstein, 2012). Initially, the puzzle grid is pre-assigned with numbers in order to make the puzzle consist of only one valid and unique solution; those numbers are called 'Clues' or 'Given Numbers'. The rest of the squares are empty, and they are called 'Blank Squares', or 'Remaining Squares'. Figure 1 shows one of the most common non-regular Sudoku grids (Crook, 2007), which consists of six rows, six columns, six sub-grids, 17 clues, 19 blank squares, and 36 squares in total. In this paper, we consider the classic and most common regular Sudoku size, 9 × 9. The 9 × 9 Sudoku puzzle consists of 81 squares arranged into nine rows (denoted with the letters A to I) and nine columns (denoted with the numbers one to nine). The main grid is divided into nine sub-grids, each one of which has a size of 3 × 3 squares.

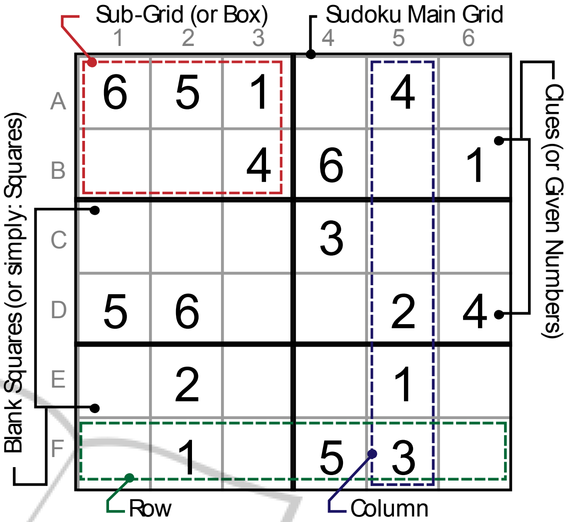
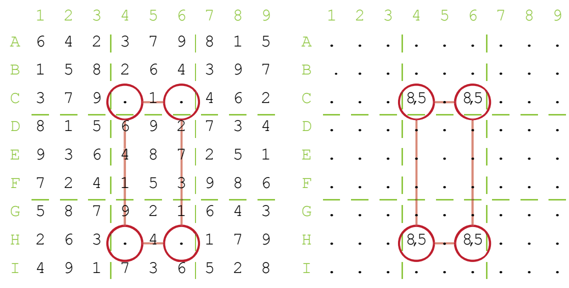



Figure 1: A 6×6 non-regular Sudoku grid.

Without clues, this grid can exceed $6{,}670 \times 10^{18}$ valid completed $9 \times 9$ Sudoku configurations (Edelkamp & Schrodl, 2012; Jiang, Xue, Li, & Yan, 2009; Klingner & Kanakia, 2013; Mcguire, Tugemann, & Civario, 2014). And because the puzzle is considered convenient only if it has one completion (Eppstein, 2012); a 9 × 9 Sudoku puzzle requires at least 17 given numbers to have a valid unique solution. Numerous studies have concluded that no 16-clues puzzle has been solved using a single solution (Jiang et al., 2009; Jilg & Carter, 2009; Mcguire et al., 2014). However, there is no guarantee that puzzles with more than 16 clues will have a unique solutions. For example, it is possible to be given 77 clues still not have a unique solution (Mcguire et al., 2014) (see Figure 2).



(A) Given Numbers (B) Potential Candidates

Figure 2: Sudoku set contains 'Forbidden Rectangle'.

Although the summation of any row, column, or even sub-grid squares' value in a $9 \times 9$ puzzle equals to 45, the solving process mainly relies on pure logic (Crook, 2007); and, no estimations are required. The rule is to complete the blank squares with the numbers of the set [1,2,3, … ,9] in such a way that each number appears once, and *only* once in a row, column, and sub-grid (Ercsey-Ravasz & Toroczkai, 2012). The rule implies each square of the puzzle is tightly associated with other squares located on the same row, column, and sub-grid. These squares are called '*Peers*', and their count number is calculated as follows:

$$PN = 3ab - (a + b) - 1 \quad (1)$$

where $PN$ refers to the peers number of any square in the puzzle. In a $9 \times 9$ Sudoku puzzle (where $a$ and $b$ equals to 3); there are 20 peers for each square (see Figure 3).

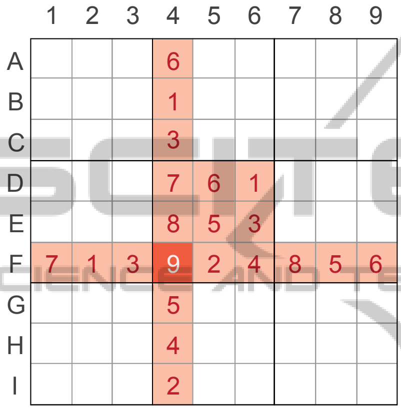



Figure 3: 'Peers' (light red) of [F,4] square (dark red).

In order to solve a Sudoku puzzle, blank squares have to be completed with valid candidates until the correct number is found; each square contains what are called 'Potential candidates', or simply 'Candidates'. The potential candidates are the possible valid values of the set of integers: one to nine, and each square has an exclusive set of candidates while solving the puzzle. The set of valid candidates can be described as follows (Crook, 2007; Edelkamp & Schrodl, 2012; Klingner & Kanakia, 2013):

$$C_k = \{1,2,..,9\} \backslash \{AR_k \cup AC_k \cup AS_k\} \quad (2)$$

where $C$ denotes the valid candidates set of the current square $k$, and $1 \leq |C_k| \leq a \times b$. $AR$, $AC$, and $AS$ are assigned values sets of $k$'s peers located on row, column and sub-grid, respectively.

As previously mentioned, Sudoku solvers are categorized to two main types, deductive and search algorithms. Deductive algorithms are remarkably slower and more difficult to develop because immense coding effort is required (Norvig, 2010). Each pattern requires a strategy to be recognized by these algorithms, for instance, the 'forbidden rectangle'. A forbidden rectangle, as shown in Figure 2, is a virtual rectangle that appears in the Sudoku main grid, and all its corners have the same candidates. This phenomenon prevents the puzzle from having a unique solution. Thus, unless the deductive algorithm is provided with sufficient tools to manage this pattern (which is usually caused by poor puzzle design), the algorithm ends without solving the puzzle.

On the other hand, search algorithms, such as BT, do not encounter problems when solving the Sudoku set with forbidden rectangle. For example, while solving the puzzle shown in Figure 2 (A), the solver could assume that the correct answer for the [C,4] square is the value five. This makes it imperative for [C,6] to take the value eight, [H,4] to also take eight, and [H,6] to take five because these values are the only valid remaining ones. Furthermore, the solver could make a different decision by assuming that the correct answer for [C,4] is eight. In this context, the assignment value chain varies to fulfil the Sudoku rule, and the final result is determined by the first assumption made. This is attributed to the algorithm's ability to backtrack when a conflict occurs, and to attempt other values.

In practice, the BT search algorithm goes into iterative recursion calls called 'labelling' or 'assignment' process (Kumar, 1992), where one of the candidates is placed in a square, while the others are stored locally in case the chosen value fails to be part of the solution. The algorithm continues assigning values to new variables provided that the values do not violate the Sudoku rule. However, if they do, a conflict is declared and the algorithm aborts the current labelling process in order to backtrack. After reversing several steps (depending on availability and the validity of the square candidates), the algorithm tries other candidates until the conflict is resolved. This is the basic principle of backtracking, which is most likely to be a '*trial and error*' procedure (Eppstein, 2012; Moraglio et al., 2006). In this paper, we consider this type of search algorithm without involving any type of deductive techniques to enhance efficiency.

BT is one of the most classical brute-force, depth-first search algorithms (Kumar, 1992; Moraglio et al., 2006) that guarantee finding a solution for any Sudoku set (when there is one) because all potential candidates are examined with respect to the puzzle rule. Forward Checking (FC) is considered an important improvement technique for the BT algorithm, and it has the ability of maintaining a list of valid values for each variable to be examined. However, it does not follow a specific strategy for selecting squares. Thus, the square selection (technically: node expansions) will take the form of systematic order of selecting squares, for instance, if the algorithm starts with the [A,1] square; [A,2] is selected next unless it is occupied; then [A,3] is selected, followed by [A,4], and so on, until the last square [I,9]. Hence, if the algorithm selects a square with many candidates at the beginning, the probabilities of choosing incorrect values are high. And, the solving process will have to iterate through a wide search space before it realizes the error, and the previously assigned values are rendered useless.

Fortunately, the BT algorithm can exploit the advantage of the MRV strategy. MRV is a "fail-first" heuristic strategy that prioritizes and selects squares based on the number of candidates that a given square holds, i.e., the least candidates the square has, the higher priority it receives (Russell & Norvig, 2010). This does not prevent backtracking from occurring, but is certain to reduce it. Nonetheless, the MRV strategy still selects squares randomly if there are two or more squares with the same number of minimum values.

The CtN strategy is designed to select the most promising square among those with the minimum values in order to reduce the backtracking rate further and to accelerate the solving process.

## 3 NEUTRALIZATION AND SUDOKU NEUTRALIZED SET

People enjoy completing Sudoku puzzle squares with numbers because they consider such puzzles as mentally challenging activities and as 'time killers' (Crook, 2007). To such individuals, each square has a solution and the puzzle is considered solved when the last blank square is solved; however, algorithms should not experience the same solving process. Sudoku solvers reinforce the notion of maintaining the algorithm engaged in searching process for as long as there is at least one blank square without an assigned value, and if there is any similarity between them, it is their objective. As a result, the explored squares to solve any Sudoku puzzle are at least equal to the number of blank squares (variables) at the initial configuration in the best-case scenario (assuming no backtracking occurs).

The MRV strategy is attracted to squares with one candidate because they guarantee that no backtracking occurs given this selection process. However, solving squares of this type does not always improve the progress of solving the puzzle given that the square value is not a candidate in any of its peers. In this case, this square can be to be treated as solved, and the algorithm can exclude it from its search space. We call this a 'Neutralized Square'. If all remaining squares are neutralized, the puzzle is considered solved; and so, the solving process can. We call this configuration the 'Neutralized Set'.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 4 | 6 | 7 | 8 | | | |
| 6 | 7 | 2 | 1 | 9 | 5 | | | |
| 1 | 9 | 8 | 3 | 4 | 2 | | | |
| 8 | 5 | 9 | | | | 4 | 2 | 3 |
| 4 | 2 | 6 | | | | 7 | 9 | 1 |
| 7 | 1 | 3 | | | | 8 | 5 | 6 |
| | | | 5 | 3 | 7 | 2 | 8 | 4 |
| | | | 4 | 1 | 9 | 6 | 3 | 5 |
| | | | 2 | 8 | 6 | 1 | 7 | 9 |

Figure 4: Neutralized Sudoku puzzle set.

The Sudoku puzzle shown in Figure 4 has 27 missing numbers that can be considered solved using the neutralization concept. Search algorithms need not be engaged in solving what it considers a 'neutralized set'. With regard to the blank squares, their values can be revealed through a validator to confirm whether the only available solution is valid.

The neutralization concept covers two different levels:

- Neutralized Square: a Sudoku blank square with only one candidate, and all its peers are not affected by solving the square. We consider any engagement with this square as a redundant iterative process for the solver.
- Neutralized Set: a Sudoku set where all the blank squares are neutralized. In this case, the solver has to declare the puzzle as 'solved', and all searching activities are terminated.

A Sudoku neutralized configuration can be mathematically described as follows:

$$NtN = \frac{TBS}{TRC} \tag{1}$$

where $NtN$ (Neutralization Number) is the result of dividing $TBS$ (Total number of Blank Squares) over $TRC$ (Total number of Remaining Candidates). The puzzle is considered neutralized if $NtN = 1$. In other words, if the total number of all remaining squares is equal to the total number of all potential candidates, the puzzle is considered solved. Moreover, neutralizing a solved puzzle is impossible because both $TBS$ and $TRC$ are equal to zero, which results in $NtN = \infty$.

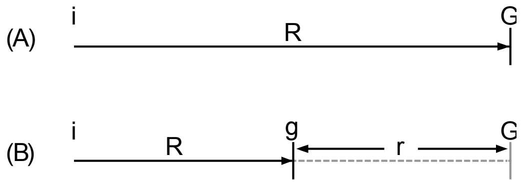



Figure 5: (A) Algorithm lifecycle, (B) Algorithm lifecycle with applications of the neutralization concept.

Hence, redefining the purpose the objective that algorithms attempt to achieve is crucial; let i be the initial configuration of a Sudoku puzzle to be solved as illustrated in Figure 5. G is the goal of a search algorithm delegated to solving the puzzle; the algorithm requires iterative square explorations (in a technical term: search recursion calls) to assign values and achieve the goal, and these are denoted R. Search algorithms such as BT with MRV evaluate square candidates to identify the one to select first, and then iterate through all the squares recursively to assign them values in a labelling process. The process continues to the last square, unless a conflict occurs as mentioned earlier in this paper. In this case, R is equal to the total number of blank squares, plus any occurring backtracking. On the contrary, the neutralization approach imposes a sub-goal (denoted g) as shown in Figure 5 (B); the search algorithm has to reach the sub-goal of ‘neutralizing the puzzle’ and decrease R by increasing the number of neutralized squares (denoted r).

In other words, the lifecycle of the BT search algorithm with neutralization concept implemented equals R (the number of blank squares + occurring backtracking) − r (the neutralized squares). This approach improves solving performance and maintains resource consumption. The following example illustrates a simple Sudoku puzzle: Figure 6 (A) shows a Sudoku set with 23 blank squares, most of which have only one candidate as shown by the grid in Figure 6 (B). By excluding the [B,7] and [B,9] squares, selecting any square located in the last six columns (4, 5, 6, 7, 8, 9) does not improve the solving process because the squares have only one candidate, *and* none of their peers consider their values as potential candidates. In this case, MRV is not the best strategy to use with the BT algorithm because MRV cannot differentiate between the competitive advantages of the puzzle squares, and therefore, one of the squares will be selected randomly; however, most of the squares are already neutralized.

Evaluation of the Figure 6 (A) Sudoku set based on neutralization principles reveals two optimal squares located on [A,3] and ([C,1] or [B,7]); solving the squares in this sequence leads to the neutralization of all the remaining puzzle squares. Thereafter, the puzzle is declared neutralized, the algorithm terminates, and all resources reserved for the solving process are released.

Finally, $NtN$ can function as an indicator of Sudoku puzzle complexity because its value could represent a reliable measurement equals to the ‘gap’ between the blank squares and their candidates, and is limited to the following range:

$$^{1}/_{9} \leq NtN \leq 1 \quad (2)$$

If the $NtN$ value of a Sudoku set is close to one and the set has many blank squares, the set difficulty level can be considered easy, and vice versa.



## 4 CONTRIBUTION NUMBERS

The basic core of neutralization is to rely on altering the algorithm objective from solving the problem to neutralizing it; however, the manner in which the existing Sudoku algorithms work does not help to neutralize a puzzle. Any new strategy designed for neutralizing Sudoku puzzles has to allow algorithms to neutralize as many squares as possible per assignment during the labelling process. The strategy that we developed has the ability of identifying the optimal square among those with the minimum remaining values to escalate eliminating other square candidates. Furthermore, our perception towards optimality in the domain of solving Sudoku relies on finding a square with the minimum number of candidates and the maximum number of similar candidates that exist among square peers. This is because $TRC$ is reduced faster and neutralization is accelerated. In other words, the optimal square considers the following two criteria:

- Number of potential candidates.
- Ability to deduct candidates from square peers.

At first, the CtN strategy selects squares with minimum remaining values, and then assesses them (if there is more than one) by assigning weights based on the criteria indicated above. The square with the highest CtN is selected first as a new frontier of the progressive labelling process. The weights produced by this strategy can be mathematically described as follows:

$$CtN_k = \frac{\sum_{i=1}^{n_k} \sum_{j=1}^{|P_i|} (d_j \in C_k)}{|C_k|} \qquad (1)$$

Because the objective is to eliminate as many candidates as possible per value assignment, the process starts computing the Contribution Number ($CtN$) of the current evaluated square $k$ by counting similar candidates within the square *blank* peers to determine the square that has the most influence on the others. As previously mentioned, there are $PN$ number of peers for each square (see Section 2, Equation 1), we need to visit all except those with assigned values. In this case, $n$ (which denotes the count number of the unassigned peers of the current square$k$) is equal to:

$$n_k = PN - |AR_k| + |AC_k| + |AS_k| \qquad (2)$$

where $AR_k$ is a set of all assigned squares located on the row of square $k$, $AC_k$ is a set of all assigned squares located on the column of square $k$, and $AS_k$ is a set of all assigned squares located in the sub-grid of square $k$. Thus, by computing $n_k$ (where $n$ is always limited to $1 \leq n \leq PN$), the number of blank peers of the current square $k$ is identified.

The next step is to select one of those blank squares $P$ and iterate through all its candidates $d$ to determine whether one is a member of the current square candidate set $C_k$; if such is the case, the counter is increased by one. The total counting of similar candidates will be then divided over the size of square $k$'s candidate set $|C_k|$. This ensures the squares with minimum number of candidates will get higher weights. The following paragraphs elaborate Equation (1) in detail.

To demonstrate the efficiency of the proposed strategy, we consider solving the Sudoku set from Figure 6 (A) using BT with FC technique, MRV, and CtN strategies. All of them are subjected to the sub-goal 'Neutralization'. Starting with BT, the algorithm selects frontiers in a systematic order. In the worst-case scenario (as shown in Figure 7 (A-1)), the algorithm selects invalid values to be examined at the beginning. This justifies backtracking because wrong values are selected. As a result, the algorithm must go through eight explored squares and three backtrackings; the performance can be improved slightly if the algorithm selects all the correct values from the beginning. In this case, the results are five explored squares without backtracking (see Figure 7 (A-2)). Furthermore, the worst-case scenario for MRV is not better than the worst-case scenario for FC. MRV successfully avoids backtracking because squares with minimum values are selected first. However, this also prevents MRV from becoming neutralized earlier (see Figure 7 (B-1)). MRV results in 17 explored squares, though no backtracking occurs. On the other hand, MRV can perform exactly as CtN if optimal squares are selected first. However, the probabilities of that are rather low. The results of the solving process are two explored squares (see Figure 7 (B-2)).

The ability of the CtN strategy to identify the most promising squares protects it from having worst-case scenarios (at least, in this example). Figure 6(C) shows the calculated CtN of the blank squares from Figure 6(A). The [A,3] square from the puzzle has only one candidate, just like the other unassigned squares; however, solving it first leads to a reduction in the total candidate average by eliminating the value seven from [A,2] and [B,1]. Thus, by computing CtNs of the puzzle squares, the [A,3] square receives the highest weight for algorithm selection for the first iterative recursion of the solving process. At the second recursion, both [C,1] and [B,7] squares have one candidate, but solving either eliminates the value four from [B,1]. This makes [C,1] and [B,7] valuable for choosing; the calculated weights at the second recursion are

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 9 | . | . | 8 | . | . | 2 | 5 | 3 |
| B | . | 8 | 3 | . | 9 | 5 | . | 1 | . |
| C | . | 1 | 5 | 3 | 7 | 2 | 6 | 8 | 9 |
| D | 1 | 2 | . | 7 | 6 | . | 3 | . | 5 |
| E | . | 5 | 6 | 4 | 8 | 3 | . | 2 | . |
| F | . | 9 | 4 | . | 5 | 1 | 8 | 7 | 6 |
| G | 6 | . | 9 | . | 4 | 8 | 1 | . | 2 |
| H | 8 | 3 | 1 | 7 | 2 | . | 5 | . | 4 |
| I | . | 4 | 2 | 1 | 3 | 6 | 7 | 9 | 8 |

(A) Given Numbers

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | . | 7,6 | 7 | . | 1 | 4 | . | . | . |
| B | 7,4,2 | . | . | 6 | . | . | 4 | . | 7 |
| C | 4 | . | . | . | . | . | . | . | . |
| D | . | . | 8 | . | . | 9 | . | 4 | . |
| E | 7 | . | . | . | . | . | 9 | . | 1 |
| F | 3 | . | . | 2 | . | . | . | . | . |
| G | . | 7 | . | 5 | . | . | . | 3 | . |
| H | . | . | . | . | . | 9 | . | 6 | . |
| I | 5 | . | . | . | . | . | . | . | . |

(B) Potential Candidates

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | . | -1.6 | 3 | . | 1 | 1 | . | . | . |
| B | -2.8 | . | . | 1 | . | . | 2 | . | 2 |
| C | 2 | . | . | . | . | . | . | . | . |
| D | . | . | 1 | . | . | 1 | . | 1 | . |
| E | 2 | . | . | . | . | . | 1 | . | 1 |
| F | 1 | . | . | 1 | . | . | . | . | . |
| G | . | 2 | . | 1 | . | . | . | 1 | . |
| H | . | . | . | . | . | 1 | . | 1 | . |
| I | 1 | . | . | . | . | . | . | . | . |

(C) CtNs weights at the first recursion

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | . | 1 | . | . | 1 | 1 | . | . | . |
| B | -2.5 | . | . | 1 | . | . | 2 | . | 1 |
| C | 2 | . | . | . | . | . | . | . | . |
| D | . | . | 1 | . | . | 1 | . | 1 | . |
| E | 1 | . | . | . | . | . | 1 | . | 1 |
| F | 1 | . | . | 1 | . | . | . | . | . |
| G | . | 1 | . | 1 | . | . | . | 1 | . |
| H | . | . | . | . | . | 1 | . | 1 | . |
| I | 1 | . | . | . | . | . | . | . | . |

(D) CtNs weights at the second recursion

Figure 6: Easy Sudoku puzzle set with candidates and CtN weights within two recursions.

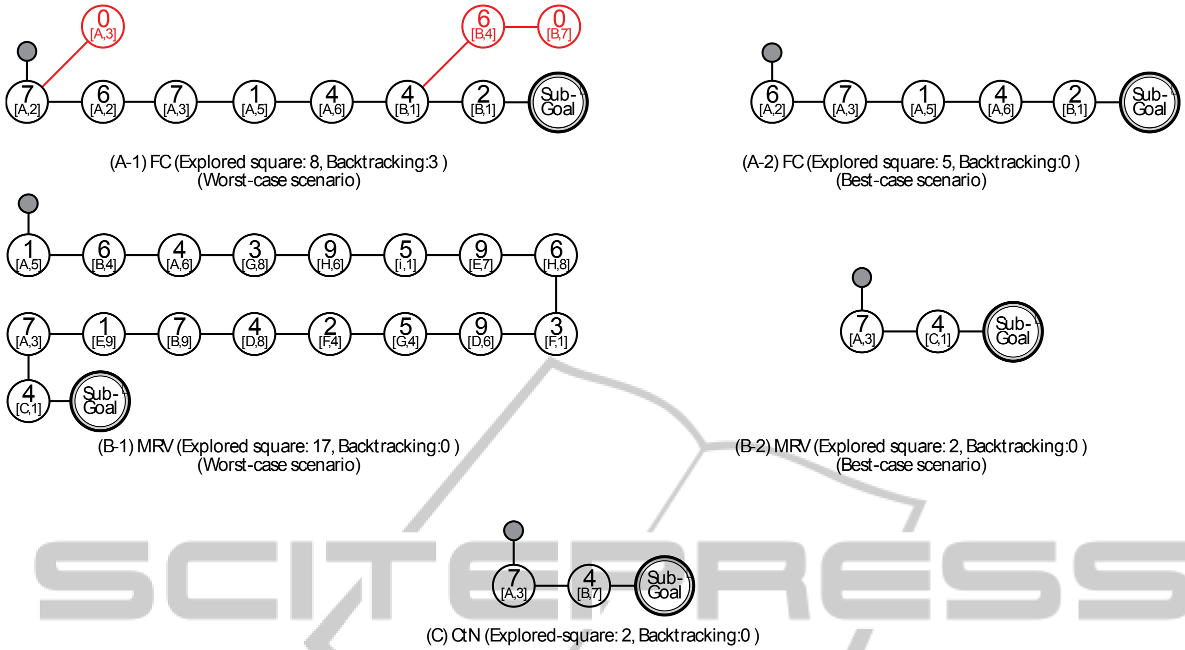



Figure 7: Search trees compression.



illustrated in Figure 6(D). The puzzle is declared neutralized immediately following two square assignments, and the algorithm is terminated at that moment. It is noticeable that the CtN strategy produces negative values, as seen in Figure 6(C) and (D); this occurs because the remaining candidate numbers for the squares with negative weights are larger than some non-neutralized squares. The strategy considers such squares undesirable choices and multiplies them with -1 to ensure they are never selected at this stage.

As part of our empirical experiments, we developed the components, strategies, and core of the search algorithm using the C# language to evaluate strategy performance. The BT algorithm and MRV strategy were adopted from a Python program (Norvig, 2010) (but the deductive part ‘constraint propagation’ was excluded). The core of the BT algorithm was developed as an independent component and extensively shared for use by the tested strategies and techniques to standardize the algorithm performance.

## 5 RESULTS AND DISCUSSION

For the purpose of assessing the strategies, we generated approximately 900 valid Sudoku puzzles with three difficulty levels: easy, medium, and difficult. The criteria for classifying Sudoku difficulty levels were adopted from *Sudoku Puzzles Generating: from Easy to Evil* (Jiang et al., 2009). The results show that BT with FC requires more iterative recursions because it continues to choose incorrect squares with incorrect values when following a systematic order for selecting squares. The search algorithm uses a reasonable number of iterative recursions to solve easy Sudoku sets, but this number increases tremendously as the puzzles become more difficult. However, MRV selects squares based on their values (squares with minimum candidates are solved first), which shows great improvement on the number of explored squares; this is caused by a significant reduction in backtracking as the strategy targets squares with minimum values. The CtN strategy shows an even more disciplined behaviour for selecting squares among those with fewer candidates, and the results reflect a greater reduction in iterative recursions, in particular for easy and medium difficulty sets. However, difficult Sudoku puzzles represent a challenge for the solver because squares with a similar number of candidates are fewer than expected. Sometimes, CtN acts exactly as MRV when solving difficult Sudoku puzzles. Nevertheless, the results show an improvement on algorithm performance compared with MRV. Tables 1 and 2 list the recursions required to solve the 900 different Sudoku sets, and the backtracking occurrences during the solving process.

Accordingly, CtN requires exploring nearly 1/3 fewer squares than MRV, but not for difficult puzzles. The number of squares with minimum remaining candidates is limited for difficult sets, which means that the strategy has fewer squares to evaluate. This leads CtN to behave similarly to MRV at that difficulty level. As the solving process advances, the number of squares with the same minimum number of candidate increases, and their influence on their peers is more distinct.

Table 1: The average of recursions required to neutralize 900 Sudoku sets.

| Strategies/ techniques | Difficulty level | | |
|---|---|---|---|
| | Easy<br>Clues: 41-53 | Medium<br>Clues: 30-40 | Difficult<br>Clues: 22-29 |
| FC | 45 | 394 | 84,594 |
| MRV | 33 | 48 | 215 |
| CtN | 10 | 26 | 171 |

Table 2: The average of backtracking that occurs when solving Table 1 Sudoku sets.

| Strategies/ techniques | Difficulty level | | |
|---|---|---|---|
| | Easy<br>Clues: 41-53 | Medium<br>Clues: 30-40 | Difficult<br>Clues: 22-29 |
| FC | 12 | 341 | 84,539 |
| MRV | 1 | 4 | 161 |
| CtN | 1 | 1 | 132 |

Overall, the achievement to be highlighted is the ability of the BT algorithm that uses the CtN strategy to neutralize Sudoku puzzles with minimum iterative recursions. Figure 8 reflects the results of neutralizing 900 Sudoku puzzle using MRV and CtN. The figure shows the efficiency of CtN to neutralize easy and medium Sudoku sets; FC is excluded because its values cannot be represented on the chart as its results are extremely greater than the graph scale.

# 6 CONCLUSIONS

In this paper, we presented a new strategy for Sudoku algorithms that can accelerate the solving process, reduce the number of required explored squares, and minimize the number of backtracking occurrences. The puzzle is declared ‘neutralized’ once the sub-goal is achieved. Moreover, the concept of achieving a sub-goal relies on re-ordering and prioritizing the puzzle's blank squares as the solving process progresses based on the influences on their pairs and the number of candidates. In order to do so, an evaluation method assesses all existing blank squares in a puzzle and assigns their weights; we called this strategy the Contribution Number (CtN) strategy.


# ACKNOWLEDGEMENTS

We gratefully acknowledge the support provided by Universiti Teknologi MARA (UiTM) and the Japanese Student Service Organization (JASSO). This work was supported by JSPS KAKENHI Grant Number 23300056.

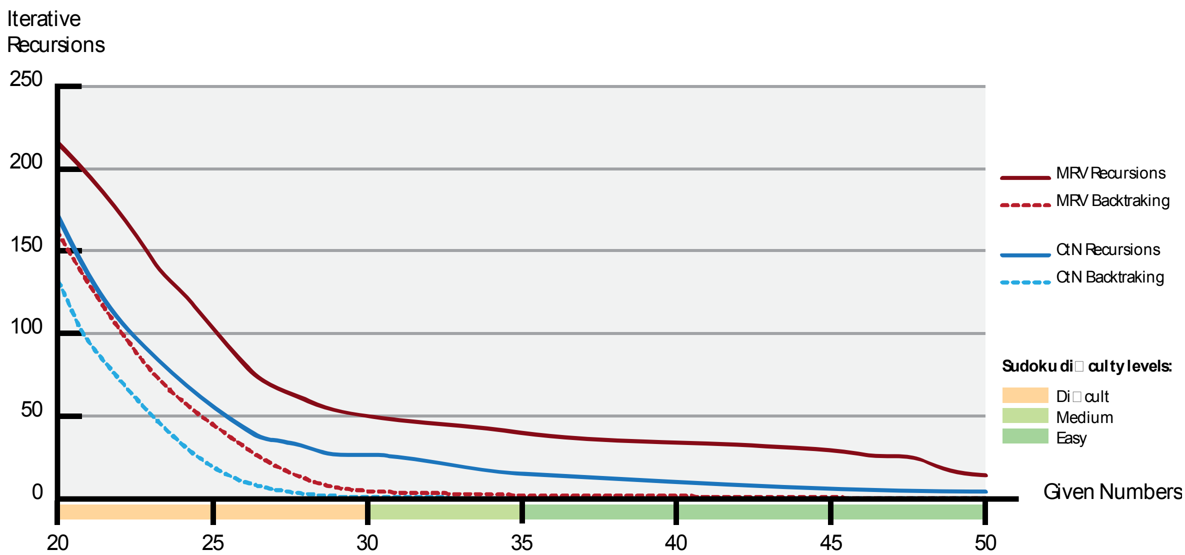




Figure 8: Performance of neutralizing 900 Sudoku puzzles using MRV and CtN.

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