

Predictive Modeling in 400-Metres Hurdles Races

Krzysztof Przednowek¹, Janusz Iskra² and Karolina H. Przednowek¹

¹Faculty of Physical Education, University of Rzeszów, Rzeszów, Poland

²Faculty of Physical Education, Academy of Physical Education in Katowice, Katowice, Poland

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Abstract: The paper presents the use of linear and nonlinear multivariable models as tools to predict the results of 400-metres hurdles races in two different time frames. The constructed models predict the results obtained by a competitor with suggested training loads for a selected training phase or for an annual training cycle. All the models were constructed using the training data of 21 athletes from the Polish National Team. The athletes were characterized by a high level of performance (score for 400 metre hurdles: 51.26±1.24 s). The linear methods of analysis include: classical model of ordinary least squares (OLS) regression and regularized methods such as ridge regression, LASSO regression. The nonlinear methods include: artificial neural networks as multilayer perceptron (MLP) and radial basis function (RBF) network. In order to compare and choose the best model leave-one-out cross-validation (LOOCV) is used. The outcome of the studies shows that Lasso shrinkage regression is the best linear model for predicting the results in both analysed time frames. The prediction error for a training period was at the level of 0.69 s, whereas for the annual training cycle was at the level of 0.39 s. Application of artificial neural network methods failed to correct the prediction error. The best neural network predicted the result with an error of 0.72 s for training periods and 0.74 for annual training cycle. Additionally, for both training frames the optimal set of predictors was calculated.

1 INTRODUCTION

Today we have a very high level of sport. The competitors and coaches have been looking for new solutions to optimize the training process. One of way is using the advances mathematical methods in planning the training loads. Advanced mathematical models include among others regularized linear models and intelligent computational methods. Those models facilitate description of training, which is a complex process, help to notice interrelations between the training load and the final result.

Sports prediction involves many aspects including predicting sporting talent (Papić et al., 2009, Rocznik et al., 2013) or the prediction of performance results (Maszczyk et al., 2011, Przednowek and Wiktorowicz, 2013). Models predicting sports scores, taking into account the seasonal statistics of each team, are also constructed (Haghighat et al., 2013). The present work focuses on predicting outcomes in terms of sports training.

The use of regression models in athletics was

described by Maszczyk et al., (2011), where the model implementing the prediction of results in a javelin throw was presented. The constructed model was used as a tool to support the choice and selection of prospective javelin throwers. On the basis of the selected set of input variables the distance of a javelin throw was predicted. The models presented were classic multiple regression models, and to select input variables Hellwig's method was used.

Another application used in walking races was regressions estimating the levels of the selected physiological parameters and the results over distances of 5, 10, 20 and 50 km (Drake and James, 2009). Calculated models were used to develop nomograms. The regressions applied were the classical OLS models, and the coefficient R^2 was chosen for the quality criterion. The study included 45 men and 23 women. The amount of registered data was changed depending on the implemented task and ranged from 21 to 68 models.

Chatterjee et al., (2009) have calculated a nonlinear regression equation to predict the maximal aerobic capacity of footballers. The data, on the

basis of which the models were calculated, came from 35 young players aged from 14 to 16. The experiment was to verify the use of the test of 20-m MST (Multi Stage Shuttle Run Test) in assessing the performance of VO_2max .

Roczniok et al., (2013) used a regression equation to identify the talent of young hockey players. The study involved 60 boys aged between 15 and 16, who participated in selection camps. The applied regression model classified individual candidates for future training based on selected parameters of the player. The classification method used was logistic regression.

A group of nonlinear predictive models used in sport also supplement the selected methods of 'data mining'. Among them a significant role is played by fuzzy expert systems. Practical application of such a system has been described in the work by Papić et al. (2009). The presented system used the knowledge of experts in the field of sport, as well as the data obtained as a result of a number of motor tests. The model based on the candidate's data suggested the most suitable sport. This tool was designed to help to search for prospective sports talents.

Previous studies also concern the widespread use of artificial neural networks in sports prediction (Haghighat et al., 2013). Artificial neural networks are used to predict sporting talent, to identify handball players' tactics or to analyze the effectiveness of the training of swimmers (Pfeiffer and Hohmann, 2012). Numerous studies show the application of neural networks in various aspects of sports training (Ryguła, 2005, Silva et al., 2007, Maszczyk et al., 2012). These models support the selection of sports, practice control or the planning of training loads.

The main purpose of the research was verification of artificial neural methods and regularized linear models (shrinkage regression) in prediction result in 400-metres hurdles for two different time frames. The verification was carried out based on training data of athletes running the 400-metres hurdles and featuring a very high level of sport abilities.

2 MATERIAL AND METHODS

The analysis included 21 Polish hurdlers aged 22.25 ± 1.96 years participating in competitions from 1989 to 2011. The athletes had a high sport level (the result over 400-metres hurdles: 51.26 ± 1.24 s). They were the part of the Polish National Athletic Team Association representing Poland at the

Olympic Games, World and European Championships in junior, youth and senior age categories. The best result over 400-metres hurdles in the examined group amounted to 48.19 s.

Table 1: Description of the variables used to construct the models.

Variable		Description
Training period	Annual cycle	
y	-	Expected 500 m sprint (s)
-	y	Expected result on 400-metres hurdles (s)
x_1	x_1	Age (years)
x_2	x_2	Body mass index
x_3	-	Current 500 m sprint (s)
-	x_3	Current result on 400-metres hurdles (s)
x_4	-	Period GPP*
x_5	-	Period SPP*
x_6	x_4	Maximal speed (m)
x_7	x_5	Technical speed (m)
x_8	x_6	Technical and speed exercises (m)
x_9	x_7	Speed endurance (m)
x_{10}	x_8	Specific hurdle endurance (m)
x_{11}	x_9	Pace runs (m)
x_{12}	x_{10}	Aerobic endurance (m)
x_{13}	x_{11}	Strength endurance I (m)
x_{14}	x_{12}	Strength endurance II (n)
x_{15}	x_{13}	General strength of lower limbs (kg)
x_{16}	x_{14}	Directed strength of lower limbs (kg)
x_{17}	x_{15}	Specific strength of lower limbs (kg)
x_{18}	x_{16}	Trunk strength (amount)
x_{19}	x_{17}	Upper body strength (kg)
x_{20}	x_{18}	Explosive strength of lower limbs (amount)
x_{21}	x_{19}	Explosive strength of upper limbs (amount)
x_{22}	x_{20}	Technical exercises – walking pace (min)
x_{23}	x_{21}	Technical exercises – running pace (min)
x_{24}	x_{22}	Runs over 1-3 hurdles (amount)
x_{25}	x_{23}	Runs over 4-7 hurdles (amount)
x_{26}	x_{24}	Runs over 8-12 hurdles (amount)
x_{27}	x_{25}	Hurdle runs in varied rhythm (amount)

*-in accordance with the rule of introducing a qualitative variable of a "training type" with the value of general preparation period, specific preparation period and competitive period was replaced with two variables x_4 and x_5 holding the value of 1 or 0.

The collected material allowed for the analysis of 144 training plans used in one of the three periods during the annual cycle of training, lasting three

months each. The annual training cycle is divided into three equal periods: general preparation, special preparation and the starting period. In the analysis of training periods, 28 variables were used, including 27 independent variables and 1 dependent variable (Table 1). Another examined time interval was the one-year training cycle, in which the training loads were considered as sums of the given training means used throughout the whole macrocycle. In the one-year training cycle, 25 variables were specified. In order to develop models for the one-year training cycle, a total of 48 standard training plans were used.

2.1 Regularized Linear Regression

We are considering the problem of constructing a multivariable (multiple) regression model for the set of multiple inputs $X_j, j = 1, \dots, p$, and the one output Y . The input variables X_j are called *predictors*, whereas the output variable Y – a *response*. We have assumed that it is a linear regression model in the parameters. In OLS regression a popular method of least squares is used (Hastie, et al. 2009; Bishop, 2006), in which weights are calculated by minimizing the sum of the squared errors. The criterion of performance $J(\mathbf{w})$ takes the form:

$$J(\mathbf{w}) = \sum_{i=1}^n (y_i - \sum_{j=1}^p x_{ij}w_j)^2 \quad (1)$$

where w_0, w_j , are unknown weights (parameters) of the model.

In ridge regression by Hoerl and Kennard (1970) the criterion of performance includes a penalty for increased weights and takes the form:

$$J(\mathbf{w}, \lambda) = \sum_{i=1}^n (y_i - \sum_{j=1}^p x_{ij}w_j)^2 + \lambda \sum_{j=1}^p w_j^2 \quad (2)$$

Parameter $\lambda \geq 0$ decides the size of the penalty: the greater the value λ the bigger the penalty; for $\lambda = 0$ ridge regression is reduced to OLS regression.

LASSO regression by Tibshirani (1996), similarly to ridge regression, adds to the criterion of performance penalty, where instead of L_2 the norm L_1 is used i.e. the sum of absolute values:

$$J(\mathbf{w}, \lambda) = \sum_{i=1}^n (y_i - \sum_{j=1}^p x_{ij}w_j)^2 + \lambda \sum_{j=1}^p |w_j| \quad (3)$$

To solve this regression an implementation of the popular LARS algorithm was used (least angle regression) (Efron et al., 2004). In the applied algorithm the penalty is decided by s parameter from the section from 0 to 1. The parameter is the fraction of the penalty used in the LASSO. This regression is also used for the selection of input variables.

Regularized linear models were implemented in GNU R software programming language with additional packets.

2.2 Artificial Neural Network

In order to build the predictive model, artificial neural networks (ANN) were also used. Two types of ANNs were applied: a multi-layer perceptron (MLP) and networks with radial basis functions (RBF) (Bishop, 2006).

The multi-layer perceptron is the most common type of neural network. In 3-layer multiple-input-one-output network the calculation of the output is performed in *feed-forward* architecture. Network teaching was implemented by the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm, which is a strong second-order algorithm. During MLP training exponential and hyperbolic tangent function were used as the activation functions of hidden neurons. All the analysed networks have only one hidden layer.

The problem with MLP network is that it can be *overtrained* which means good fitting to data, but poor predictive (generalization) ability. To avoid this the number m of hidden neurons, which is a free parameter, should be determined to give the best predictive performance.

In the RBF network we use the concept of radial basis function. The model of linear regression (5) is extended by considering linear combinations of nonlinear functions of the predictors in the form:

$$\hat{y}_i = \sum_{j=1}^p \phi_j(x_{ij})w_j \quad (4)$$

where $\phi = [\phi_1, \dots, \phi_p]^T$ is a vector of so called *basis functions*. If we use nonlinear basis functions, we get the nonlinear model which is, however, a linear function of parameters w_j . The feature of RBF network is the fact that the hidden neuron performs a radial basis function.

To implement MLP and RBF the Statistica 10 program was used along with the Automatic Statistica Neural Network.

2.3 Evaluation of Models

In order to select the best model the method of cross-validation (CV) (Arlot and Celisse, 2010) was applied. In this method, the data set is divided into two subsets: learning and testing (validation). The first of them is used to build the model, and the second to evaluate its quality. In this article, due to the small amount of data, leave-one-out cross-validation, (LOOCV) was chosen, in which a test set is composed of a selected pair of data (x_i, y_i) , and the number of tests is equal to the number of data n . As an indicator of the quality of the model the root of the mean square error was calculated from formula:

$$RMSE_{CV} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_{-i})^2} \quad (5)$$

where: n – total number of patterns, \hat{y}_{-i} – the output value of the model built in the i -th step of cross-validation based on a data set containing no testing pair (x_i, y_i) , $RMSE_{CV}$ – Root Mean Square Error.

3 RESULTS AND DISCUSSION

3.1 Prediction in Training Periods

Result prediction concerning the training period for 400-metres hurdle race involves a defined training status indicator, since it is technically impossible to use the running test in 400-metres hurdles within each of the analysed annual cycle periods. Therefore, as the training status indicator, the result of a race over a flat distance of 500 m in the particular periods was assumed. The correlation between the result of 500 m and 400-metres hurdles within the competition period is very strong ($r_{xy}=0,84$); apart from that, it demonstrates statistical significance at the level of $\alpha=0,001$, confirming the validity of assumption of the 500 m race result as a dependent variable in the course of prediction models development.

The development of a predictive model makes it possible to check how the suggested training affects the final result. The basic model is OLS regression, for which the cross-validation error was at the level of $RMSE_{CV} = 0.74$ s.

In ridge regression, the λ parameter is chosen; it determines the additional penalty associated with the regression coefficients. In the study, the dependency between the prediction error and the parameter λ changing from 0 to 20 in steps of 0.1 (Fig. 1a) was

determined. The smallest error is generated for the model in which the parameter $\lambda = 3$. The cross-validation error for the optimal ridge regression is 0,71 s. It was also noted that at the initial stage of model optimization, along with the increase of penalty parameter, the prediction error slightly decreases, and after reaching the minimum, it increases up to the level of approx. 0,8 s.

In the LASSO model, the s parameter is chosen; its value ranges from 0 to 1 and it determines the imposed penalty. A graph showing the relationship between the s parameter values and the prediction error $RMSE_{CV}$ was drafted (Fig. 1b).

The error generated by the optimal LASSO model ($s = 0,76$) was at the level of 0,67 s. From the determined coefficients (Tab. 2) it follows, that the variables $x_2, x_5, x_8, x_{11}, x_{15}, x_{16}, x_{23}, x_{25}$ are not taken into account in the prediction task in terms of training periods (coefficients equal to 0).

Table 2: Coefficients of linear models and error results - training periods.

Regression	OLS	Ridge	Lasso
Intercept	1,75e+01	2,20e+01	15,254
x_1	-6,43e-02	-8,12e-02	-0,058
x_2	-1,83e-02	-4,35e-02	0
x_3	7,50e-01	6,96e-01	0,776
x_4	4,85e-01	5,11e-01	0,562
x_5	-9,79e-02	-4,03e-02	0
x_6	1,28e-04	1,29e-04	1,86e-05
x_7	1,44e-04	1,44e-04	9,10e-05
x_8	-7,75e-05	-6,21e-05	0
x_9	2,43e-07	6,38e-08	6,21e-07
x_{10}	-9,04e-05	-8,98e-05	-8,33e-05
x_{11}	-2,67e-06	-2,39e-06	0
x_{12}	1,24e-06	1,25e-06	5,73e-07
x_{13}	-1,51e-05	-1,52e-05	-1,41e-05
x_{14}	-4,47e-05	-4,49e-05	-2,12e-05
x_{15}	5,88e-07	1,65e-07	0
x_{16}	4,77e-06	2,93e-06	0
x_{17}	1,31e-06	2,58e-06	1,26e-06
x_{18}	4,19e-06	3,48e-06	2,14e-06
x_{19}	-3,00e-05	-2,93e-05	-1,05e-05
x_{20}	-1,42e-03	-1,35e-03	-0,001
x_{21}	-3,28e-04	-4,23e-04	-0,0004
x_{22}	1,13e-03	1,34e-03	0,0006
x_{23}	3,94e-04	4,74e-04	0
x_{24}	-3,82e-03	-3,78e-03	-0,0019
x_{25}	-6,10e-04	-9,13e-04	0
x_{26}	-9,59e-04	-1,29e-03	-0,0007
x_{27}	5,68e-04	6,34e-04	0,0003
$RMSE_{CV}[s]$	0,74	0,71	0,67

The eliminated training means belong to the group of “targeted” ones. The results confirm thus the views prevailing among sport researchers, that in high-qualified training those exercises should be

restricted, and the coach should concentrate on special training (Iskra, 2013).

Calculation of the best neural model performing the task of result prediction in terms of training period amounts to determination of the number of neurons in the hidden layer and to selection of the

optimal function of hidden layer neurons activation.

Therefore, the dependency between prediction error and the number of neurons in the hidden layer for each of the analysed networks was determined (Fig. 1cd).

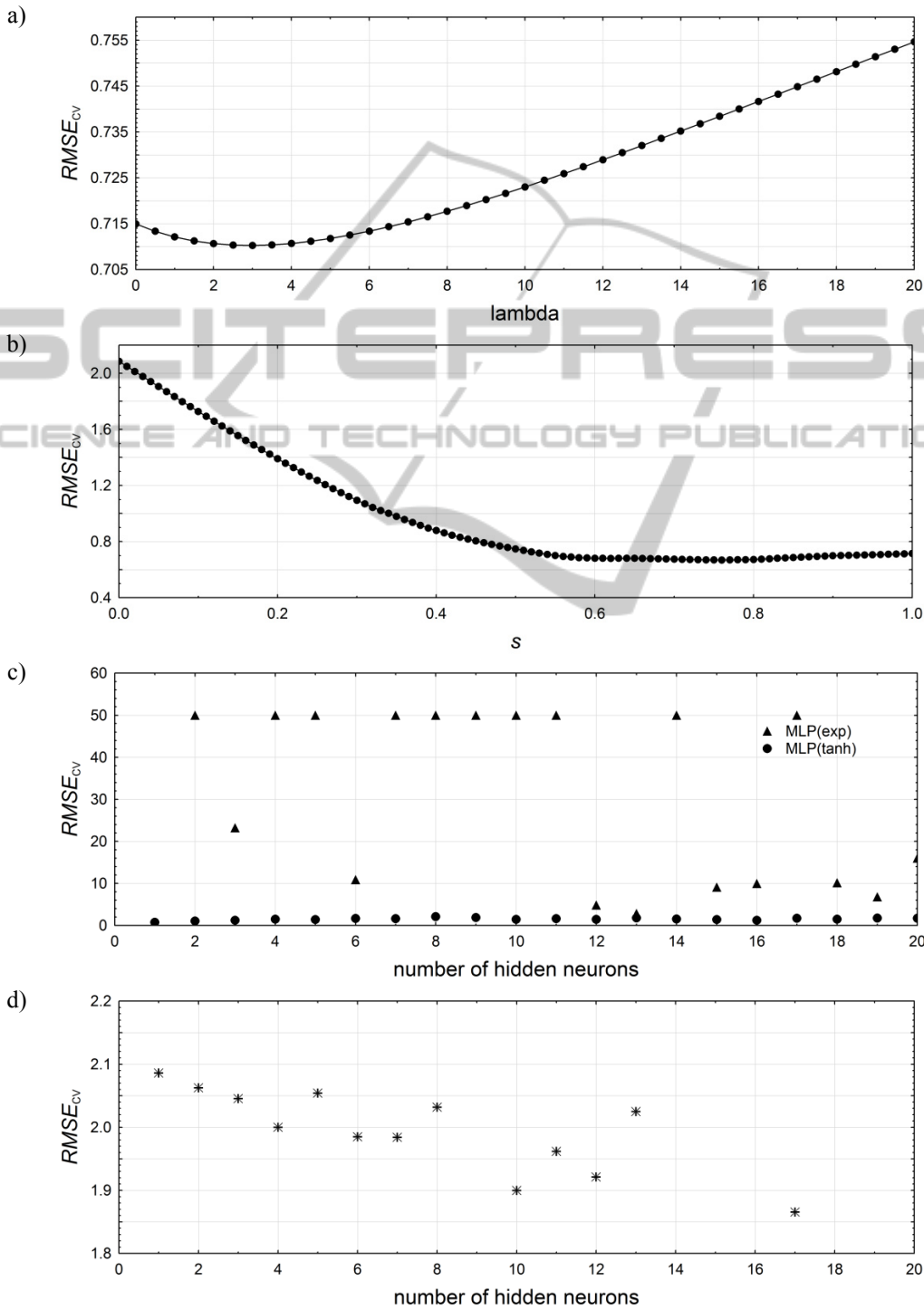


Figure 1: Predictive error for training period.

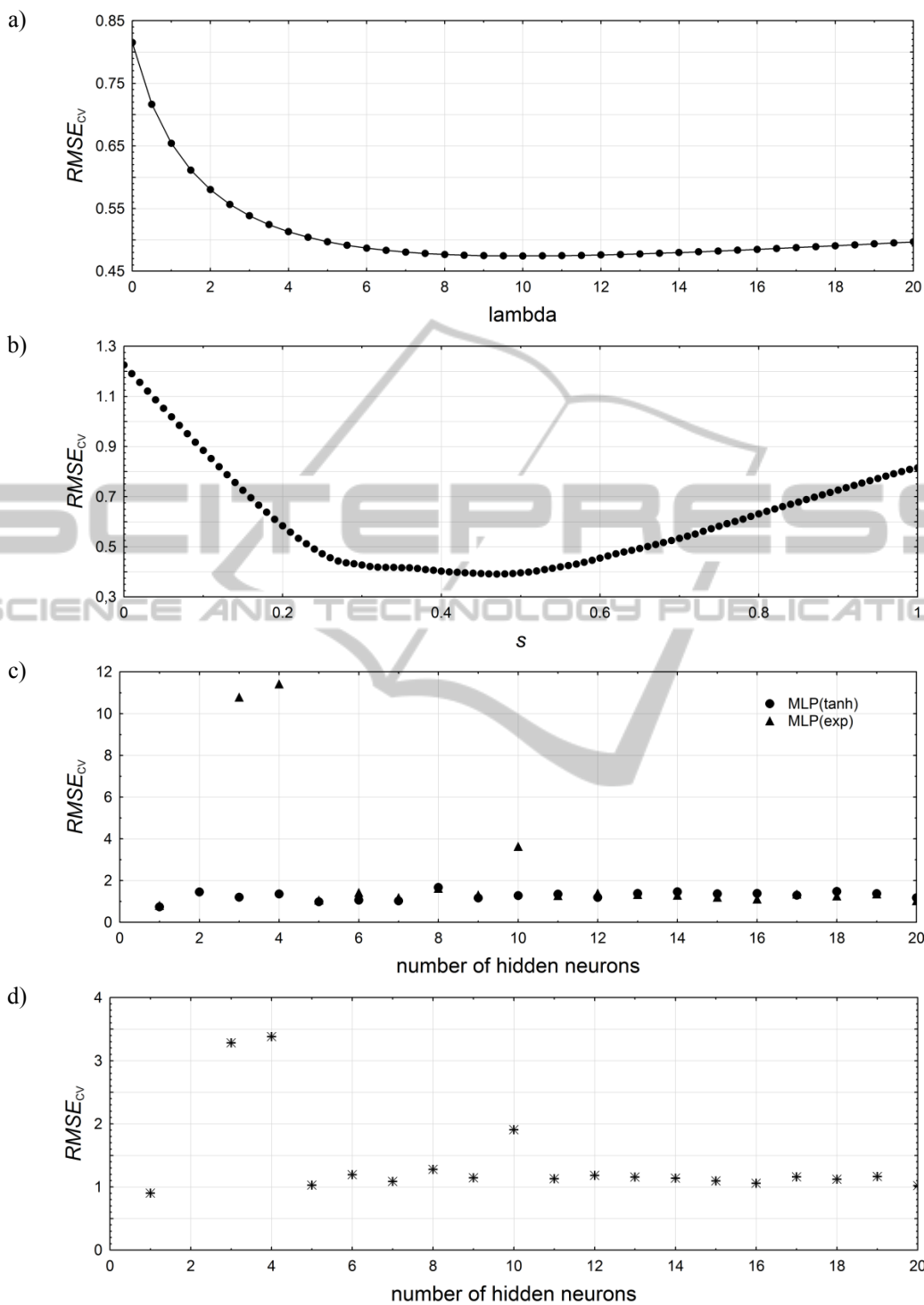


Figure 2: Predictive error for annual training cycle.

The first type of network was a multilayer perceptron with the function of hyperbolic tangent activation. Neural networks consisting of from 1 to 20 neurons in the hidden layer were subjected to

examination. It can be noted that the smallest prediction error is obtained for 1 neuron in the hidden layer (Fig. 1c). Prediction error for the optimal model is 0.73 s, and it does not improve the

result obtained by the LASSO regression model.

Another network is the MLP network with exponential function. The optimal model for exponential activation function includes also 1 neuron in the hidden layer. The prediction error ($RMSE_{CV} = 0.72$) is smaller in comparison to hyperbolic tangent function, but it is greater than the LASSO regression model. Similar to MLP networks, RBF network was also subjected to cross-validation. The results are presented in form of a graph, where the prediction error values are shown (Fig. 1d).

The optimal RBF model executing the considered task includes 12 neurons in the hidden layer and generates the error of $RMSE_{CV} = 1.9$ s. Errors generated by the RBF network are the greatest among the analysed models.

3.2 Prediction in Annual Training Cycle

OLS model is the basic method applied while seeking optimal solutions for predicting outcome in the annual cycle. The following regression performs this task with error $RMSE_{CV} = 0.81$ s, all the coefficients are different from zero (Table 3), which means that all the variables form a final result.

The analysed ridge models are regressions for the parameter λ equal from 1 to 20 (Fig. 2a). The best model was obtained for the parameter $\lambda = 10$. A prediction error generated by the best ridge regression is $RMSE_{CV} = 0.47$ s. Application of this method has improved by almost a half the capacity of prediction results compared to OLS regression. Ridge regression coefficients, as in the classical model, are different from zero (Table 3), so all the input variables are involved in the formation of the projected result.

Calculating the optimal Lasso model came down to analysing models for parameter s from 0 to 1 with a step of 0.01. The conducted analysis showed that the optimal model is the regression with a parameter $s = 0.47$ (Fig. 2b). This model generates an error of $RMSE_{CV} = 0.39$ s, which is the best result obtained by linear models. When using this method the selection of input variables becomes important. The Lasso model, apart from generating the smallest error, is characterized by a simpler structure, as many as 12 input variables ($x_3, x_4, x_5, x_6, x_8, x_{11}, x_{15}, x_{17}, x_{18}, x_{19}, x_{21}, x_{22}$) have been eliminated by assigning them a coefficient equal zero (Tab. 3).

The learning process of the network was done for the models with one hidden layer, which consisted of 1 to 20 neurons respectively. The analysis showed that the most accurate

perceptron predicting results in terms of the annual training cycle is the network with one neuron in the hidden layer and hyperbolic tangent activation function (Fig. 2c). The optimal perceptron generates an error $RMSE_{CV} = 0.74$ s. This result is better than the classical regression but it gives way to regularized linear models. Using the method of RBF network has not produced satisfactory results. RBF networks generate greater error than linear models and multilayer perceptrons. An optimal RBF network has one hidden neuron and prediction error $RMSE_{CV} = 0.90$ s (Fig. 2d).

Table 3: Coefficients of linear models and error results - annual training cycle.

Regression	OLS	Ridge	Lasso
Intercept	3.184e+01	4.02974e+01	14.469
x_1	4.858e-01	3.36950e-01	0.7165
x_2	-1.078e-01	-1.14145e-01	-0.0097
x_3	-1.291e-01	-1.42671e-01	0
x_4	4.170e-05	4.63483e-05	0
x_5	7.006e-05	1.61809e-05	0
x_6	-3.585e-06	1.35562e-05	0
x_7	1.862e-06	2.36310e-07	6.124e-07
x_8	-1.365e-05	-4.63184e-06	0
x_9	-6.451e-07	-3.3943e-07	3.912e-07
x_{10}	-1.307e-06	-7.26978e-07	-5.625e-07
x_{11}	1.417e-05	1.17982e-06	0
x_{12}	-1.491e-05	-1.8731e-05	-2.380e-06
x_{13}	-2.211e-06	-2.34129e-06	-7.572e-07
x_{14}	-6.315e-06	-6.23390e-06	-3.789e-06
x_{15}	-1.766e-06	-4.41342e-07	0
x_{16}	-2.816e-06	-2.34715e-06	-4.561e-07
x_{17}	1.223e-05	8.26353e-06	0
x_{18}	-9.097e-05	-1.90436e-04	0
x_{19}	2.011e-04	-5.91045e-05	0
x_{20}	1.080e-03	1.0042e-03	0.000516
x_{21}	1.848e-04	1.56069e-04	0
x_{22}	1.793e-03	-1.51247e-03	0
x_{23}	3.782e-03	2.19824e-03	0.00214
x_{24}	-3.560e-03	-2.05897e-03	-0.00137
x_{25}	-3.822e-04	-2.47327e-04	-8.057e-05
$RMSE_{CV}$ [s]	0,81	0,47	0,39

4 CONCLUSIONS

In the following paper the effectiveness of the use of regularized linear regression and artificial neural

networks in predicting the outcome of competitors training for the 400-metres hurdles was verified. In both analysed time intervals, the LASSO regression proved to be the most precise model. Prediction in terms of the one-year cycle, where 400m hurdles result was predicted featured a smaller error. The prediction error for a training period was at the level of 0.69 s, whereas for the annual training cycle was at the level of 0.39 s. Additionally, for both training frames the optimal set of predictors was calculated. In terms of training periods, the LASSO model eliminated 8 variables, whereas in terms of the one-year training cycle, 12 variables were eliminated.

In every time frame (training period, 1-year cycle), similar sets of training means in modelling the predicted result are used. Common predictors in both analysed tasks are: age, speed endurance, aerobic endurance, strength endurance II, trunk strength, technical exercises - walking pace, runs over 8-12 hurdles and hurdle run in a varied rhythm.

The outcome of the studies shows that Lasso shrinkage regression is the best method for predicting the results in 400-metres hurdles.

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