

# A Petri Net Model for an Open Path Multi-AGV System

Davide Giglio

*Department of Informatics, Bioengineering, Robotics and Systems Engineering, University of Genova, Genova, Italy*

**Keywords:** Coloured Petri Nets, Automated Distribution Warehouses, Autonomous Guided Vehicles, Forklift AGVs.

**Abstract:** Automated distribution warehouses in which pallet and roll pallet loads are transported by means of forklift AGVs are considered in this work, with the objective of defining a mathematical model which accurately represents the behaviour of AGVs in the system. AGVs can move freely in the warehouse (an open path AGV system is adopted), and their transportation activities can be modelled as a sequence of elementary or basic actions. In the paper, a coloured Petri net (CPN) model is proposed. It allows representing any sequence of elementary actions of AGVs (including pick-up and drop-off activities), and accurately models the interactions among AGVs, in order to guarantee the safety during the execution of activities. The CPN model can be used to analyse and implement deadlock prevention and deadlock recovery strategies, and it has been adopted in the building of a discrete-event simulator which is employed to analyse the system's performance and to evaluate scheduling policies for transportation tasks.

## 1 INTRODUCTION

In the last century, the process of automating manufacturing systems and other business activities, aimed at increasing the performance of the systems, has involved both production tasks and transportation operations. In this connection, autonomous guided vehicles (AGVs) have been used to automate the transportation of parts and materials within manufacturing systems and other indoor or outdoor facilities (Vis, 2006; Le-Ahn and De Koster, 2006). The definition of an AGV system includes several activities that can be related to the three standard decision-levels, namely, the strategic level (layout design), the tactical level (determination of the number of AGVs, deadlock prevention, planning of transportation activities), and the operational/real-time control level (dispatching of AGVs, scheduling of transportation activities, vehicle routing, deadlock avoidance). In this paper, a mathematical model based on the graphical formalism of the Petri nets (Petri, 1962; Murata, 1989) is presented; it can be used to solve some of the problems to be dealt with at the tactical and operational decision-levels.

In this work, automated distribution warehouses are taken into account. In the considered class of systems, the storage area is laid with pallet and roll pallet loads that must be moved to the gate area when the trucks which will deliver the items to their final destinations arrive at the warehouse. All transportation

activities are carried out by forklift AGVs (Seelinger and Yoder, 2006; Martínez-Barberá and Herrero-Pérez, 2010), which are not constrained to follow any guide path as they can move freely along the warehouse's aisles; then, a free-ranging or open path model (Sen et al., 1991; Duinkerken et al., 2006) is adopted. In the system, several AGVs work simultaneously (multi-AGV system) and cooperate to parallelize the execution of truck loading activities, thus minimizing the overall completion time.

The management of an open path multi-AGV system is of a crucial importance, in particular to guarantee the safety during the execution of transportation activities. Even if vehicles are nowadays equipped with devices (e.g., lasers) to detect obstacles and other vehicles, it is necessary to accurately model the movements of AGVs in order to dispatch AGVs, schedule their activities, and route them in an efficient and safe way. In this paper, a coloured Petri net (Jensen and Kristensen, 2009) model, representing all elementary actions of AGVs (including pick-up and drop-off activities) and precisely modelling the interactions among AGVs, is proposed.

The Petri net formalism has been adopted to model AGVs from the nineties (Holloway and Krogh, 1990; Lee and DiCesare, 1994), and since then Petri nets have been used to analyse the system's performance, to dispatch AGVs and schedule their activities, and to analyse and implement deadlock prevention strategies. In (Castillo et al., 2001), the perfor-

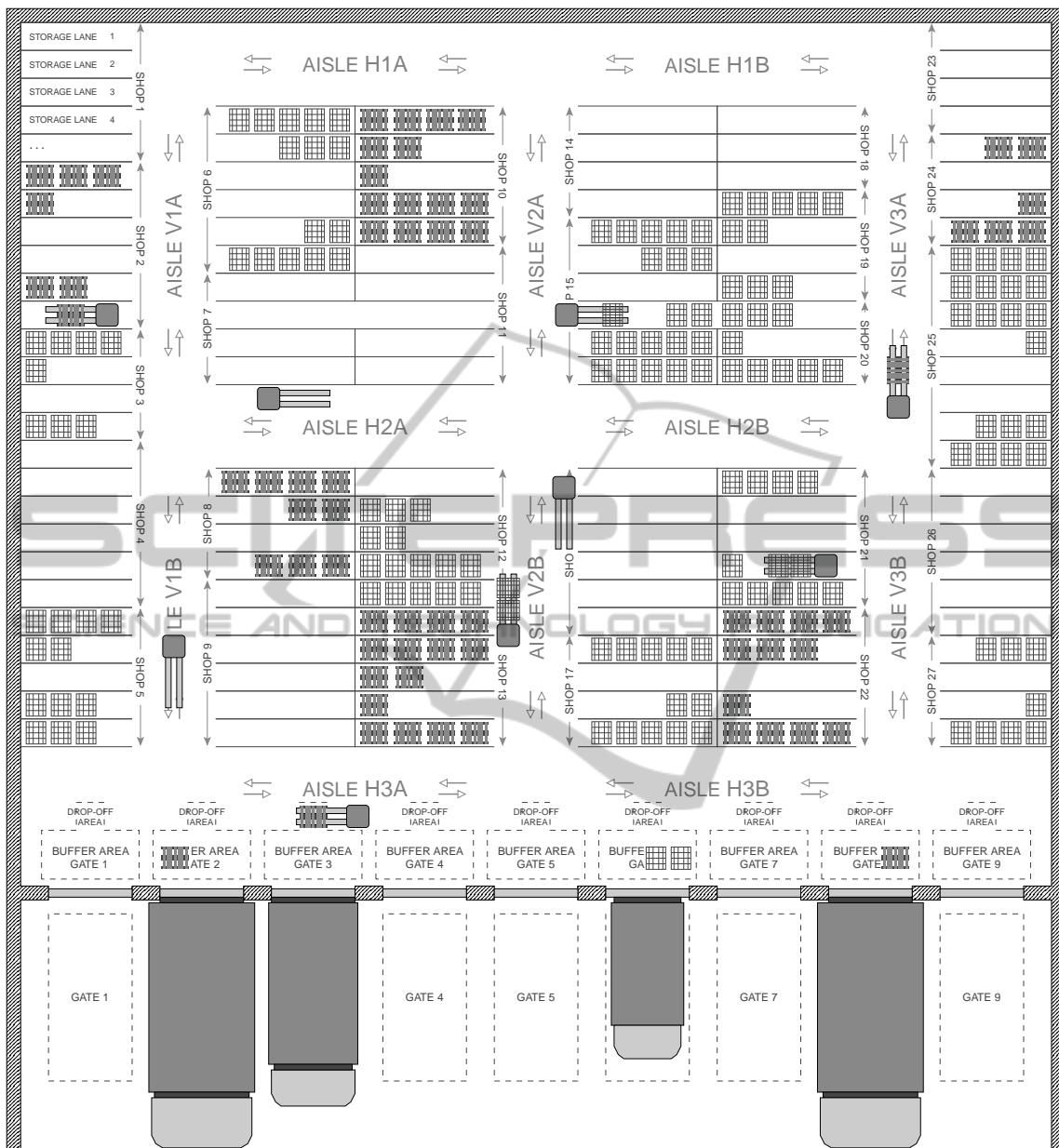


Figure 1: The distribution warehouse.

mance of a tandem AGV system is analysed through a generalized stochastic Petri net (GSPN) model (Ajmone Marsan et al., 1984); deadlock issues are considered in (Wu and Zhou, 2005), where a maximally permissive deadlock avoidance policy is proposed; recently, in (Nishi and Maeno, 2010) and (Nishi and Tanaka, 2012), Petri nets have been used to dispatch AGVs and to route them within an AGV system. The coloured Petri nets has been adopted in (Hsieh, 1998; Hsieh and Chen, 1999; Dotoli and Fanti, 2004; Aized, 2009) to model, analyse, and simulate AGV systems. However, in all these works, the considered AGV

systems are guided path, in which AGVs are constrained to follow the arcs (generally unidirectional) of a network representing the available routes/paths for AGVs in the system.

Instead, in this paper, an open path AGV system is adopted, since it allows more flexibility in managing the time-variant AGVs' tasks that have to be accomplished in the considered class of systems. As it will be described in the following section, AGVs are required to travel paths which change with time, as the pick-up and drop-off points change dynamically on the basis of the trucks' arrivals and of the assign-

ment of the gates to the trucks; moreover, the vertical aisles must be traveled by AGVs both in the left side and in the right side, since the loads to be picked-up are placed next to both sides of the aisles (see Figure 1). A guided path system for the considered class of warehouses would require a very large number of paths, thus making that kind of approach very complex. Therefore, an open path model is preferred.

This paper is organized as follows. In section 2 the class of distribution warehouses and the adopted system model are respectively introduced. The coloured Petri net model is proposed in section 3. Issues about deadlock prevention and deadlock recovery are dealt with in section 4, whereas in section 5 some concluding remarks are reported.

## 2 THE DISTRIBUTION WAREHOUSE

In the considered class of warehouses, items to be delivered are stacked and stored on pallet and roll pallet loads (pallets and rolls, simply), which are placed on the floor in short lanes that are physically defined among the warehouse's aisles (storage lanes). An example of distribution warehouse is illustrated in Figure 1. Each lane may contain a variable number of pallets (usually 3 or 4) or rolls (usually 4 or 5), which are lined up in a row since a lane is about 1 meter wide. Each storage lane is associated with a "shop" (e.g., a store or a supermarket) and each shop consists of one or more consecutive lanes. In general, each shop holds either pallets or rolls. Shops/lanes are accessible only through the vertical aisles and only one side of a storage lane can be approached by the forklift AGVs to get the loads, as illustrated in Figure 1. Both the horizontal and the vertical aisles are about 3 meters wide, but the horizontal aisle adjacent to the gate zone, which is wider as it includes the buffer areas where pallets and rolls, just dropped off by the forklift AGVs, are temporarily left. Bidirectional traffic of transportation resources is allowed in any aisle, and AGVs are not required to respect any right-hand or left-hand rule.

Warehouse activities start each day in the afternoon and conclude in the morning of the next day. In the afternoon the orders of stores and supermarkets are processed, pallets and rolls are prepared and moved to the storage lanes corresponding to the shop to which items have to be delivered (the warehouse is initially empty). These operations are man-made and can last up to the evening. In the early morning of the following day (from about 4 a.m.), trucks arrive at the distribution warehouse (in accordance with a pre-

defined schedule); each truck has to deliver items to one or more shops and then pallets and/or rolls are transported from the storage lanes relevant to those shops to the buffer area of the gate at which the truck is parked. A forklift AGV moves to the open side of the lane, enters the lane horizontally, picks either 1 pallet or 1 roll or 2 rolls up, exits the lane, travels towards the relevant gate where it drops the pallet/roll(s) off in a dedicated zone next to the buffer area of the gate (drop-off area). This sequence of operations made by an AGV (empty transfer, pick-up, transportation, drop-off) represents a single task to be accomplished by the transportation resource. Activities end when all pallets and rolls have been delivered.

### 2.1 The System Model

In order to obtain a system model which allows representing the behaviour of AGVs in the distribution warehouse, the layout (in particular, the horizontal and the vertical aisles) is divided into cells of approximately the same size ( $0.8\text{-}1\text{ m} \times 0.8\text{-}1\text{ m}$ ) so that a forklift AGV occupies 3 cells when moving in the warehouse, as illustrated in Figure 2. The position of a forklift AGV is defined by the cell which is occupied by the extremities of the forks, and by the direction of the AGV which is specified by the "truck" part of it (then, an AGV has north direction when its forks are down-side with respect to the truck, has east direction when its forks are left-side, and so on, for example, the AGV which is in the aisle V1B has north direction). The state of a forklift AGV consists of the position and the load which is transported (no load, 1 pallet, 1 roll, 2 rolls).

An AGV is allowed to execute three kinds of elementary movements: move straight, rotate  $90^\circ$ , and turn; they lead to eight "basic actions" which are: move north, move east, move south, move west, rotate  $90^\circ$  clockwise, rotate  $90^\circ$  counterclockwise, turn left, and turn right. A forklift AGV with a certain position is allowed to perform one or more basic actions (for example, with reference to Figures 1 and 2, the AGV which is in the aisle H2A can either move west or rotate counterclockwise or turn left; turn right is not allowed since the presence of the corner of the storage lane). All trips, from whichever cell to another different cell, can be defined as a sequence of basic actions. Other movements are relevant to the loading and unloading activities; in this case, a loading action includes all the basic movements which are necessary to enter a lane, to place the AGV in the right position, to pick the pallet or the roll(s) up, and to exit the lane; in the same way, an unloading action includes all the basic movements which are necessary to place

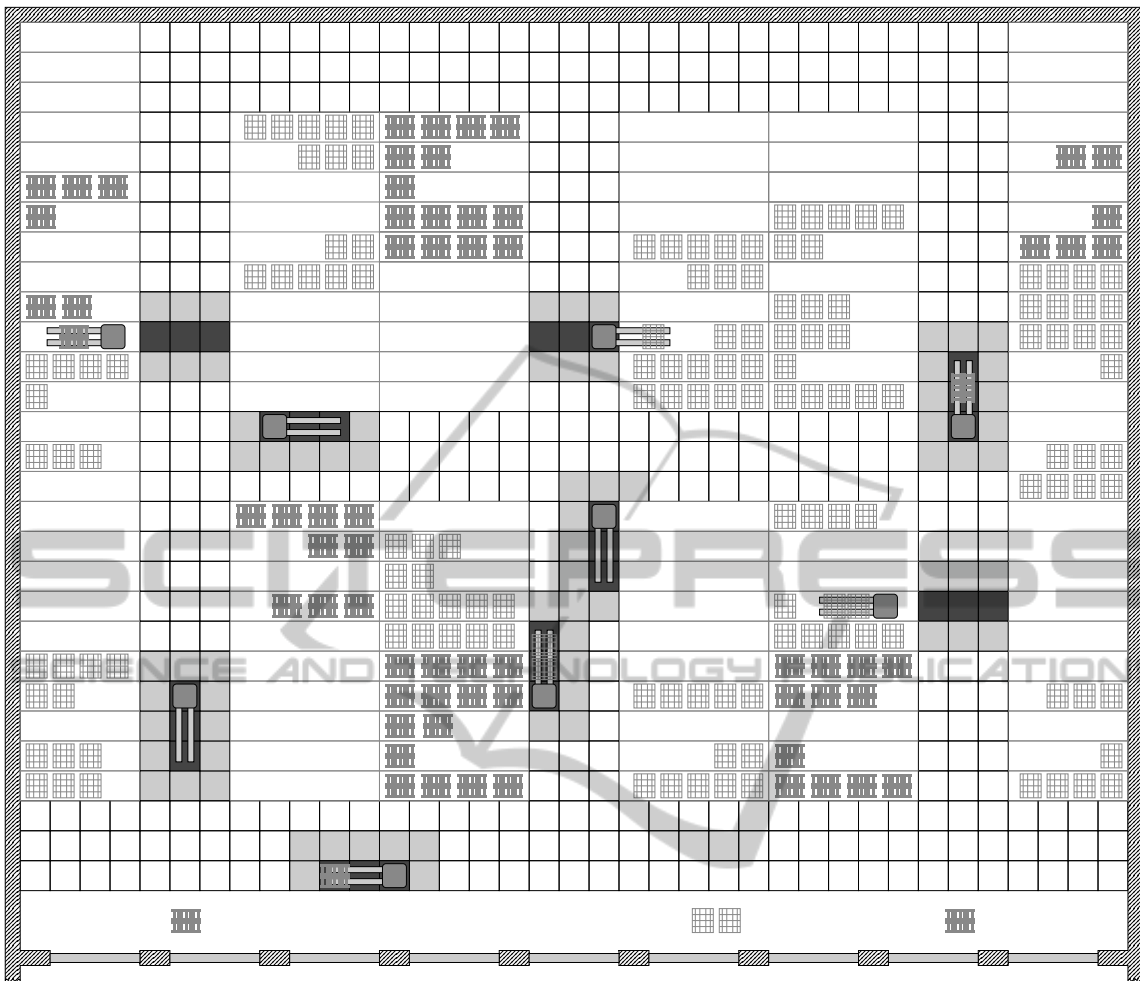


Figure 2: The system model.

the AGV in the right position and to drop the pallet or the roll(s) off.

It is worth remarking that the proposed model can be quite easily enhanced by adding further basic movements, such as, for example, a diagonal movement to be used to change lane when travelling an aisle; the new movements would be then modelled within the CPN representation by following the same reasoning lines used to model the three kinds of elementary movements here taken into account (described in section 3). In any case, the considered basic movements allows the AGVs to freely travel in the warehouse, that is, the AGVs can run through an aisle in any direction and to do that they can stay in the middle or in any of the side parts of the aisle.

## 2.2 On Representing the System Model by means of Coloured Petri Nets

The system model is represented in this paper by

means of coloured Petri nets (CPNs), with the aim of providing a formal model to be used to analyse the performance of the system, to define deadlock prevention and recovery strategies, and to simulate AGVs tasks. The choice of using CPNs, instead of (classic) Petri nets, is due to the need of representing different types of loads (pallets and rolls), different states of the AGV (empty, loaded with 1 pallet, loaded with 1 roll, or loaded with 2 rolls), and different states of the cells which form the layout. The adopted class of CPN is the one proposed in (Jensen and Kristensen, 2009), whose definition is the following.

**Definition 1.** A (non-hierarchical) Coloured Petri Net is a nine-tuple  $CPN = (P, T, A, \Sigma, V, C, G, E, I)$ , where:

1.  $P$  is a finite set of places;
2.  $T$  is a finite set of transitions ( $P \cap T = \emptyset$ );
3.  $A \subseteq P \times T \cup T \times P$  is a set of directed arcs;
4.  $\Sigma$  is a finite set of non empty colour sets;

5.  $V$  is a finite set of typed variables such that  $\text{Type}[v] \in \Sigma$  for all variables  $v \in V$ ;
6.  $C : P \rightarrow \Sigma$  is a colour set function that assigns a colour set to each place;
7.  $G : T \rightarrow \text{Expr}_V$  is a guard function that assigns a guard to each transition  $t$  such that  $\text{Type}[G(t)] = \text{Bool}$ ;
8.  $E : A \rightarrow \text{Expr}_V$  is an arc expression function that assigns an arc expression to each arc  $a$  such that  $\text{Type}[E(a)] = C(p)_{MS}$ , where  $p$  is the place connected to the arc  $a$ ;
9.  $I : P \rightarrow \text{Expr}_0$  is an initialization function that assigns an initialization expression to each place  $p$  such that  $\text{Type}[I(p)] = C(p)_{MS}$ .

In the proposed model, the  $i$ -th cell of the layout is modelled through two places, namely,  $p_i$  and  $s_i$ . Place  $p_i$  models the presence of a forklift AGV in the cell, in particular, the presence of the extremities of the forks (then, each AGV, even if it physically occupies 3 cells, is represented by a single token moving through places  $p_i$ ). Place  $s_i$  represents the state of the cell, which is defined on the basis of the following safety-related considerations.

In an open path multi-AGV system, in which AGVs can move freely in the system layout, safety is of primary importance. In the proposed system, safety is guaranteed by assuming that no AGV can be in any of the cells that are adjacent to the three occupied by another AGV. In this way, there is always 0.8-1 meter (at least) between two AGVs. In Figure 2, the light grey cells surrounding each AGV are those that cannot be occupied by another AGV. In other words, an AGV *physically occupies* three cells (the dark grey cells) and *virtually occupies* a maximum of 12 cells (the light grey ones). Then, before executing any action or movement, it is always necessary to verify if such a safety constraint is satisfied or not (in this connection, note that cells that are virtually occupied by an AGV can be also virtually occupied by another AGV). In the rest of the paper, a cell is:

- “O” (occupied) when it is physically occupied;
- “V” (virtual) when it is virtually occupied;
- “A” (available) if it is neither physically nor virtually occupied.

Further places of the CPN model are relevant to the storage lanes and to the drop-off areas. Place  $l_a$  represents the  $a$ -th storage lanes (the marking of  $l_a$  models the presence of pallets and/or rolls to be transported), whereas place  $d_b$  represents the  $b$ -th drop-off area (the marking of  $d_b$  models the presence of pallets and/or rolls that are ready to be loaded on the truck).

On the basis of such considerations, the following sets  $P, \Sigma, C, V$  characterize the proposed CPN model.

$$P = \{p_i, s_i, l_a, d_b; \quad i = 1, \dots, N, a = 1, \dots, L, b = 1, \dots, D\} \quad (1)$$

being  $N, L$ , and  $D$ , the number of cells which form the layout, the number of storage lanes and the number of drop-off areas/gates, respectively;

$$\Sigma = \{\text{CELL}, \text{LOAD}, \text{LTYPE}, \text{NO}, \text{DIR}, \quad \text{LTYPE} \times \text{NO} \times \text{DIR}\} \quad (2)$$

where the colour sets in (2) are defined as

$$\text{CELL} = \{\text{O}, \text{V}, \text{A}\} \quad (3)$$

$$\text{LOAD} = \{\text{P}, \text{R}\} \quad (4)$$

$$\text{LTYPE} = \{\text{none}, \text{P}, \text{R}\} \quad (5)$$

$$\text{NO} = \{0, 1, 2\} \quad (6)$$

$$\text{DIR} = \{\text{N}, \text{E}, \text{S}, \text{W}\} \quad (7)$$

(P and R stand for pallet and roll, respectively; N, E, S, and W stand for north, east, south, and west, respectively);

$$C(p_i) = \text{LTYPE} \times \text{NO} \times \text{DIR} \quad i = 1, \dots, N \quad (8)$$

$$C(s_i) = \text{CELL} \quad i = 1, \dots, N \quad (9)$$

$$C(l_a) = \text{LOAD} \quad a = 1, \dots, L \quad (10)$$

$$C(d_b) = \text{LOAD} \quad b = 1, \dots, D \quad (11)$$

$$V = \{c : \text{CELL}; q : \text{LOAD}; l : \text{LTYPE}; n : \text{NO}; \quad d : \text{DIR}; i = 1, \dots, N\} \quad (12)$$

The transitions of the CPN represent the executions of the basic actions. Set  $T$  is defined as follows.

$$T = \{t_{i-j}, t_{i-i}, t_{i-a}^P, t_{i-b}^D; \quad (i, j) \in \mathcal{E}, (i, a) \in \mathcal{L}, (i, b) \in \mathcal{D}\} \quad (13)$$

being  $\mathcal{E}, \mathcal{L}$ , and  $\mathcal{D}$  respectively the set of pairs  $(i, j)$  such that it exists an elementary movement (move straight or turn) which carries an AGV from the cell  $i$  to the cell  $j$ , the set of pairs  $(i, a)$  such that the cell  $i$  is next to the storage lane  $a$ , and the set of pairs  $(i, b)$  such that the cell  $i$  corresponds to the drop-off area  $b$ . Then, transitions of type  $t_{i-j}$  are relevant to a move straight or a turn action, those of type  $t_{i-i}$  are relevant to a rotate action, whereas transitions of types  $t_{i-a}^P$  and  $t_{i-b}^D$  model respectively a pick-up activity and a drop-off one.

### 3 THE CPN MODEL

The CPN representing the whole system can be obtained by suitably merging the nets representing each cell. Since the set of actions that an AGV can take depends on the position of the AGV in the system layout, such nets can be different one from another. In order to generalize the behaviour of an AGV within a distribution warehouse of the considered class, it is possible to refer to the generic portion of system layout illustrated in Figure 3, which consists of 121 cells ( $11 \times 11$  grid). Such a portion is generic in the sense that all basic actions (move straight, rotate  $90^\circ$ , and turn) are allowed for an AGV with the extremities of the forks in the central cell, namely  $c_{61}$ , and having any direction (the AGV in Figure 3 has north direction but it is evident that it can have any direction).

c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>	c <sub>5</sub>	c <sub>6</sub>	c <sub>7</sub>	c <sub>8</sub>	c <sub>9</sub>	c <sub>10</sub>	c <sub>11</sub>
c <sub>12</sub>	c <sub>13</sub>	c <sub>14</sub>	c <sub>15</sub>	c <sub>16</sub>	c <sub>17</sub>	c <sub>18</sub>	c <sub>19</sub>	c <sub>20</sub>	c <sub>21</sub>	c <sub>22</sub>
c <sub>23</sub>	c <sub>24</sub>	c <sub>25</sub>	c <sub>26</sub>	c <sub>27</sub>	c <sub>28</sub>	c <sub>29</sub>	c <sub>30</sub>	c <sub>31</sub>	c <sub>32</sub>	c <sub>33</sub>
c <sub>34</sub>	c <sub>35</sub>	c <sub>36</sub>	c <sub>37</sub>	c <sub>38</sub>	c <sub>39</sub>	c <sub>40</sub>	c <sub>41</sub>	c <sub>42</sub>	c <sub>43</sub>	c <sub>44</sub>
c <sub>45</sub>	c <sub>46</sub>	c <sub>47</sub>	c <sub>48</sub>	c <sub>49</sub>	c <sub>50</sub>	c <sub>51</sub>	c <sub>52</sub>	c <sub>53</sub>	c <sub>54</sub>	c <sub>55</sub>
c <sub>56</sub>	c <sub>57</sub>	c <sub>58</sub>	c <sub>59</sub>	c <sub>60</sub>	c <sub>61</sub>	c <sub>62</sub>	c <sub>63</sub>	c <sub>64</sub>	c <sub>65</sub>	c <sub>66</sub>
c <sub>67</sub>	c <sub>68</sub>	c <sub>69</sub>	c <sub>70</sub>	c <sub>71</sub>	c <sub>72</sub>	c <sub>73</sub>	c <sub>74</sub>	c <sub>75</sub>	c <sub>76</sub>	c <sub>77</sub>
c <sub>78</sub>	c <sub>79</sub>	c <sub>80</sub>	c <sub>81</sub>	c <sub>82</sub>	c <sub>83</sub>	c <sub>84</sub>	c <sub>85</sub>	c <sub>86</sub>	c <sub>87</sub>	c <sub>88</sub>
c <sub>89</sub>	c <sub>90</sub>	c <sub>91</sub>	c <sub>92</sub>	c <sub>93</sub>	c <sub>94</sub>	c <sub>95</sub>	c <sub>96</sub>	c <sub>97</sub>	c <sub>98</sub>	c <sub>99</sub>
c <sub>100</sub>	c <sub>101</sub>	c <sub>102</sub>	c <sub>103</sub>	c <sub>104</sub>	c <sub>105</sub>	c <sub>106</sub>	c <sub>107</sub>	c <sub>108</sub>	c <sub>109</sub>	c <sub>110</sub>
c <sub>111</sub>	c <sub>112</sub>	c <sub>113</sub>	c <sub>114</sub>	c <sub>115</sub>	c <sub>116</sub>	c <sub>117</sub>	c <sub>118</sub>	c <sub>119</sub>	c <sub>120</sub>	c <sub>121</sub>

Figure 3: An AGV in a generic portion of a system layout.

Since every part of the system layout (vertical and horizontal lanes, X-crossroads, T-crossroads, etc.) can be “mapped” to such generic portion of the system layout, the net representing the behaviour of an AGV in a specific cell can be obtained from the net representing the “meta-cell”  $c_{61}$  by removing all places, transitions, and arcs which are “not involved”, that is, which refer to cells that are outside of the “mapped” part. An example relevant to a cell in the middle-right side of a vertical aisle will be provided in subsection 3.3.

#### 3.1 Basic Actions

The 5 basic actions (move straight, rotate  $90^\circ$  clockwise, rotate  $90^\circ$  counterclockwise, turn left, turn right) that can be taken by the AGV illustrated in Figure 3 are now described in detail. The states of the

AGV and of the layout at the start of the movement are characterized by:

- *Start position (cell) of the AGV:*  $c_{61}$   
*Start direction of the AGV:* north
- *O-cells in the layout:*  $c_{39}, c_{50}, c_{61}$   
*V-cells in the layout:*  $c_{27}, c_{28}, c_{29}, c_{38}, c_{40}, c_{49}, c_{51}, c_{60}, c_{62}, c_{71}, c_{72}, c_{73}$

It is worth noting that the 15 further actions that can be taken by the AGV in  $c_{61}$  when its direction is east or south or west can be easily derived from those here considered.

##### 3.1.1 Move Straight

This action can be taken only when the cells  $c_{16}, c_{17}$ , and  $c_{18}$  are not O.

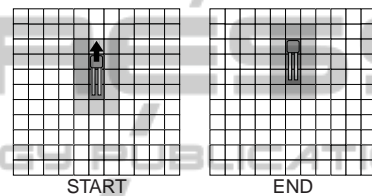


Figure 4: AGV's behavior in move straight action.

The states of the AGV and of the layout at the end of the movement are characterized by:

- *End position (cell) of the AGV:*  $c_{50}$   
*End direction of the AGV:* north
- *O-cells in the layout:*  $c_{28}, c_{39}, c_{50}$   
*V-cells in the layout:*  $c_{16}, c_{17}, c_{18}, c_{27}, c_{29}, c_{38}, c_{40}, c_{49}, c_{51}, c_{60}, c_{61}, c_{62}$

The states of the cells that have been “released” in consequence of the movement, namely  $c_{71}, c_{72}$ , and  $c_{73}$ , become either V or A depending on the states of the cells  $c_{59}, c_{63}, c_{70}, c_{74}, c_{81}, c_{82}, c_{83}, c_{84}$ , and  $c_{85}$  (for example,  $c_{71}$  is V if one of  $c_{59}, c_{70}, c_{81}, c_{82}, c_{83}$  is O, and is A otherwise).

##### 3.1.2 Rotate $90^\circ$ Clockwise

This action can be taken only when cells the  $c_{41}, c_{52}$ , and  $c_{63}$  are A.

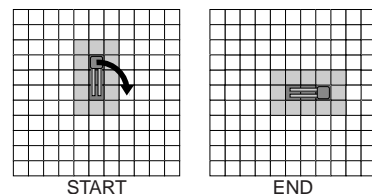


Figure 5: AGV's behavior in rotate  $90^\circ$  clockwise action.

The states of the AGV and of the layout at the end of the movement are characterized by:

- *End position (cell) of the AGV:*  $c_{61}$   
*End direction of the AGV:* east
- *O-cells in the layout:*  $c_{61}, c_{62}, c_{63}$   
*V-cells in the layout:*  $c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{60}, c_{64}, c_{71}, c_{72}, c_{73}, c_{74}, c_{75}$

The states of the cells that have been released in consequence of the movement, namely  $c_{27}, c_{28}, c_{29}, c_{38}, c_{39},$  and  $c_{40}$ , become either A or V depending on the states of the cells  $c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{26}, c_{37},$  and  $c_{48}$  (cells  $c_{39}$  and  $c_{40}$  become A for certain).

### 3.1.3 Rotate 90° Counterclockwise

This action is very similar to the rotate 90° clockwise action; the details are not given as they can be easily derived on the basis of what reported in subsection 3.1.2.

### 3.1.4 Turn Left

This action can be taken only when the cells  $c_{14}, c_{15},$  and  $c_{16}$  are A.

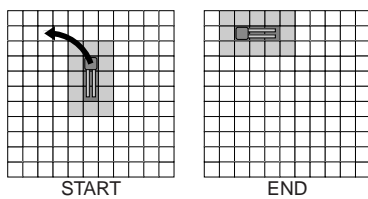


Figure 6: AGV's behavior in turn left action.

The states of the AGV and of the layout at the end of the movement are characterized by:

- *End position (cell) of the AGV:*  $c_{16}$   
*End direction of the AGV:* west
- *O-cells in the layout:*  $c_{14}, c_{15}, c_{16}$   
*V-cells in the layout:*  $c_2, c_3, c_4, c_5, c_6, c_{13}, c_{17}, c_{24}, c_{25}, c_{26}, c_{27}, c_{28}$

The states of the cells that have been released in consequence of the movement, namely  $c_{29}, c_{38}, c_{39}, c_{40}, c_{49}, c_{50}, c_{51}, c_{60}, c_{61}, c_{62}, c_{71}, c_{72},$  and  $c_{73}$ , become either A or V depending on the states of the cells  $c_{18}, c_{19}, c_{30}, c_{37}, c_{41}, c_{48}, c_{52}, c_{59}, c_{63}, c_{70}, c_{74}, c_{81}, c_{82}, c_{83}, c_{84},$  and  $c_{85}$  (cells  $c_{39}, c_{50},$  and  $c_{61}$  become A for certain).

### 3.1.5 Turn Right

This action is very similar to the turn left action; the details are not given as they can be easily derived on the basis of what reported in subsection 3.1.4.

### 3.1.6 Other Actions (Pick-up and Drop-off)

When an AGV is in a vertical aisle, has east or west direction, and is in a cell next to a storage lane, it can pick-up a load in the lane (if any) if it is either empty (in this case it can pick-up either 1 pallet or 1 roll or 2 rolls) or carrying 1 roll (in this case it can pick-up 1 roll). For what concerns the availability (physical or virtual) of cells, the pick-up activity does not require specific conditions to be fulfilled as it is assumed that the cells that are O and V when the activity starts, remain in their state throughout the execution of the activity.

Similarly, when an AGV has east or west direction and is in a cell which corresponds to a drop-off area, it can drop-off its load(s). As before, the drop-off activity does not require specific conditions on the state of the near cells (also in this case the cells that are O and V at the beginning remain in their state throughout the execution of the activity).

## 3.2 The CPN Model of the Meta-cell

The coloured Petri net which models all the considered behaviours of an automatic guided vehicle which is in cell  $c_{61}$  of the  $11 \times 11$  layout portion represented in Figure 3, is illustrated in Figure 7. Such a net is undoubtedly complex due to the presence of several arcs. However, in order to evaluate the complexity of the nets that are generated, it is necessary to take into account the fact that only a subset of basic actions are allowed in each of the cells which are present in the layout of the class of warehouses considered in this paper (such as the one illustrated in Figures 1 and 2); this significantly reduces the number of transitions. Moreover, since the aisles are 3 cells wide, the transitions representing the basic actions will be actually connected with a reduced number of  $s_i$  places; this greatly reduces the number of arcs.

In this connection, in the following subsection the CPN representing a specific part in the layout of Figure 2 (a cell in the middle / right side of the aisle V2A) is described. It can be derived from the generic one illustrated in Figure 7 but it has a limited number of transitions and especially arcs.

Before describing such a simpler net, it is worth noting that the places representing the state of a cell, namely  $s_i, i = 1, \dots, N$ , always contains 1 and only 1 token (or, equivalently, they are safe and always marked); however, the colour of the token of such places changes on the basis of the evolution of tokens through places  $p_i, i = 1, \dots, N$ , in accordance with the arc expression function (places  $p_i$  are safe but not always marked).

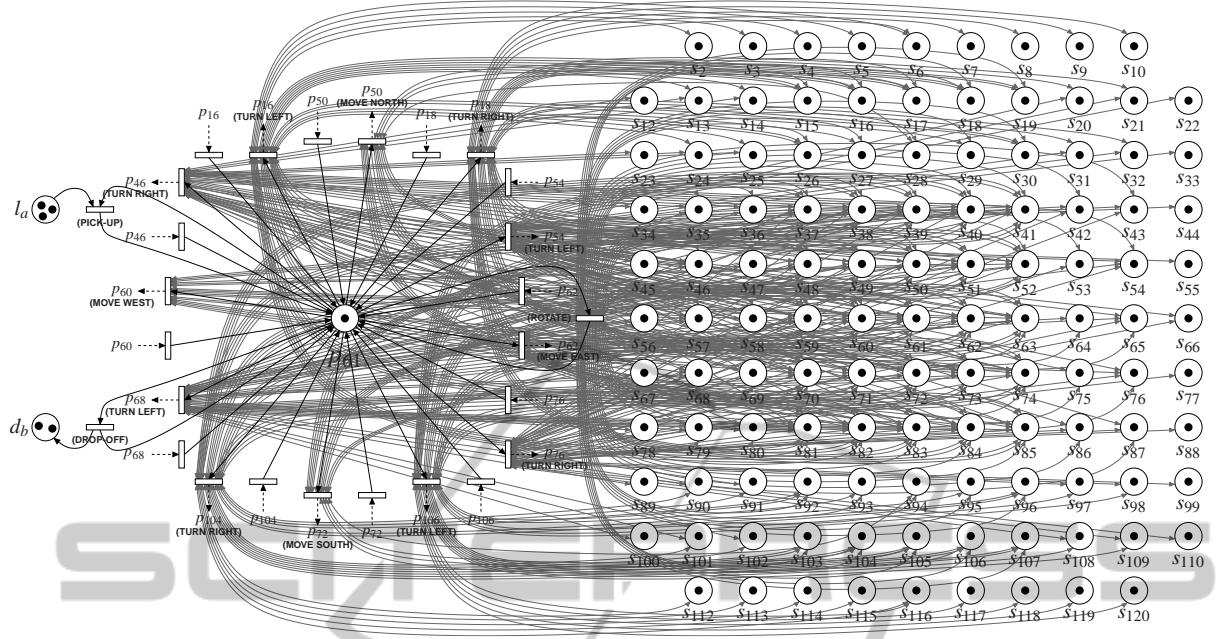


Figure 7: The CPN model of the meta-cell of the system layout.

### 3.3 The CPN Model for a Cell in a Vertical Aisle

The specific CPN representing a cell in the middle / right side of a vertical aisle is illustrated in Figure 8. In such a cell, an AGV can pick-up loads (by firing transition  $t_{132-71}^P$ ), rotate (transition  $t_{132-132}$ ), leave the cell by moving either to north (transition  $t_{132-84}$ ) or to south (transition  $t_{132-135}$ ). The arc expressions<sup>1</sup> and the guard function which characterize such actions are reported in the following subsections (the arc expressions, which update the state of the cells, are not discussed for the sake of brevity).

#### 3.3.1 Action: Move North

##### Arc Expressions

$$\begin{aligned} E(p_{132}, t_{132-84}) &= 1'(l, n, d) \\ E(t_{132-84}, p_{84}) &= 1'(l, n, d) \\ E(s_{74}, t_{132-84}) &= 1'c_{74} \quad E(t_{132-84}, s_{74}) = 1'V \\ E(s_{75}, t_{132-84}) &= 1'c_{75} \quad E(t_{132-84}, s_{75}) = 1'V \\ E(s_{78}, t_{132-84}) &= 1'V \quad E(t_{132-84}, s_{78}) = 1'O \\ E(s_{130}, t_{132-84}) &= 1'c_{130} \quad E(t_{132-84}, s_{130}) = 1'c_{130} \end{aligned}$$

<sup>1</sup>In accordance with (Jensen and Kristensen, 2009), arc expressions are specified by using the infix operator  $'$  which takes a positive integer as its left argument, specifying the number of tokens of the colour provided as the right argument that are removed from (resp., added to) the place which is the origin (resp., the destination) of the arc.

$$\begin{aligned} E(s_{132}, t_{132-84}) &= 1'O \quad E(t_{132-84}, s_{132}) = 1'V \\ E(s_{133}, t_{132-84}) &= 1'c_{133} \quad E(t_{132-84}, s_{133}) = 1'c_{133} \\ E(s_{134}, t_{132-84}) &= 1'V \end{aligned}$$

$$\begin{aligned} E(t_{132-84}, s_{134}) &= \text{if } (c_{130} = O) \vee (c_{133} = O) \\ &\vee (c_{193} = O) \vee (c_{194} = O) \vee (c_{195} = O) \text{ then } 1'V \\ &\text{else } 1'A \end{aligned}$$

$$\begin{aligned} E(s_{135}, t_{132-84}) &= 1'V \\ E(t_{132-84}, s_{135}) &= \text{if } (c_{194} = O) \vee (c_{195} = O) \\ &\text{then } 1'V \text{ else } 1'A \end{aligned}$$

$$\begin{aligned} E(s_{193}, t_{132-84}) &= 1'c_{193} \quad E(t_{132-84}, s_{193}) = 1'c_{193} \\ E(s_{194}, t_{132-84}) &= 1'c_{194} \quad E(t_{132-84}, s_{194}) = 1'c_{194} \\ E(s_{195}, t_{132-84}) &= 1'c_{195} \quad E(t_{132-84}, s_{195}) = 1'c_{195} \end{aligned}$$

##### Guard Function

$$G(t_{132-84}) = (d = N) \wedge [(c_{74} \neq O) \wedge (c_{75} \neq O)]$$

The AGV can move towards north if its direction is N and if the two cells  $c_{74}$  and  $c_{75}$  are not physically occupied.

#### 3.3.2 Action: Rotate

##### Arc Expressions

$$\begin{aligned} E(p_{132}, t_{132-132}) &= 1'(l, n, d) \\ E(t_{132-132}, p_{132}) &= 1'(l, n, e) \\ E(s_{73}, t_{132-132}) &= 1'c_{73} \quad E(t_{132-132}, s_{73}) = 1'c_{73} \end{aligned}$$



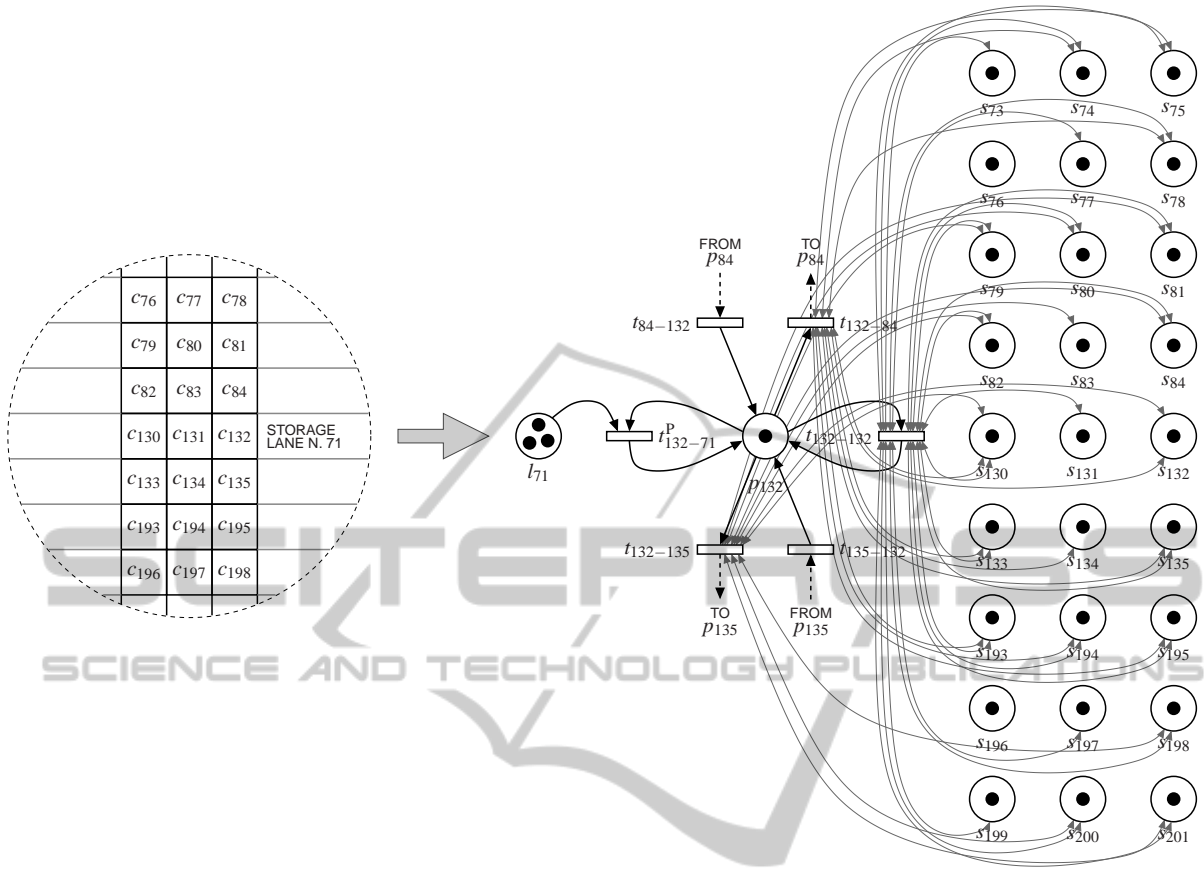


Figure 8: Specific CPN model for a cell in the middle/right side of a vertical aisle.

$$\begin{aligned}
 E(s_{74}, t_{132-132}) &= 1'c_{74} & E(t_{132-132}, s_{74}) &= 1'c_{74} \\
 E(s_{75}, t_{132-132}) &= 1'c_{75} & E(t_{132-132}, s_{75}) &= 1'c_{75} \\
 E(s_{77}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'V; \text{ else } \rightarrow 1'c_{77} \\
 E(t_{132-132}, s_{77}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &\text{if } (c_{73} = O) \vee (c_{74} = O) \vee (c_{75} = O) \text{ then } 1'V \\
 &\text{else } 1'A; \text{ case } (W, N) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{77} \\
 E(s_{78}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'V; \text{ else } \rightarrow 1'c_{78} \\
 E(t_{132-132}, s_{78}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &\text{if } (c_{74} = O) \vee (c_{75} = O) \text{ then } 1'V \text{ else } 1'A; \\
 &\text{case } (W, N) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{78} \\
 E(s_{79}, t_{132-132}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\vee (W, N)] \rightarrow 1'A; \text{ else } \rightarrow 1'c_{79} \\
 E(t_{132-132}, s_{79}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\vee (W, N)] \rightarrow 1'A; \text{ else } \rightarrow 1'c_{79} \\
 E(s_{80}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'V; \text{ case } (W, N) \rightarrow 1'A; \text{ else } \rightarrow 1'c_{80}
 \end{aligned}$$

$$\begin{aligned}
 E(t_{132-132}, s_{80}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'A; \text{ case } (W, N) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{80} \\
 E(s_{81}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'O; \text{ case } (W, N) \rightarrow 1'A; \text{ else } \rightarrow 1'c_{81} \\
 E(t_{132-132}, s_{81}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'A; \text{ case } (W, N) \rightarrow 1'O; \text{ else } \rightarrow 1'c_{81} \\
 E(s_{82}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'A; \text{ case } (S, W) \rightarrow 1'c_{82}; \text{ else } \rightarrow 1'V \\
 E(t_{132-132}, s_{82}) &= \text{select case } (d, e): \text{ case } (W, N) \rightarrow \\
 &1'A; \text{ case } (W, S) \rightarrow \text{if } (c_{79} = O) \vee (c_{80} = O) \\
 &\text{then } 1'V \text{ else } 1'A; \text{ else } \rightarrow 1'V \\
 E(s_{84}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &1'O; \text{ else } \rightarrow 1'V \\
 E(t_{132-132}, s_{84}) &= \text{select case } (d, e): \text{ case } (W, N) \rightarrow \\
 &1'O; \text{ else } \rightarrow 1'V \\
 E(s_{130}, t_{132-132}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\vee (S, W)] \rightarrow 1'A; \text{ else } \rightarrow 1'O
 \end{aligned}$$

$$\begin{aligned}
 E(t_{132-132}, s_{130}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\quad \vee (S, W)] \rightarrow 1'O; \text{ else } \rightarrow 1'A \\
 E(s_{131}, t_{132-132}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\quad \vee (S, W)] \rightarrow 1'V; \text{ else } \rightarrow 1'O \\
 E(t_{132-132}, s_{131}) &= \text{select case } (d, e): \text{ case } [(N, W) \\
 &\quad \vee (S, W)] \rightarrow 1'O; \text{ else } \rightarrow 1'V \\
 E(s_{133}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (N, W) \rightarrow \\
 &\quad 1'c_{133}; \text{ case } (S, W) \rightarrow 1'A; \text{ else } \rightarrow 1'V \\
 E(t_{132-132}, s_{133}) &= \text{select case } (d, e): \text{ case } (W, N) \rightarrow \\
 &\quad \text{if } (c_{193} = O) \vee (c_{194} = O) \text{ then } 1'V \text{ else } 1'A; \\
 &\quad \text{case } (W, S) \rightarrow 1'A; \text{ else } \rightarrow 1'V \\
 E(s_{135}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'O; \text{ else } \rightarrow 1'V \\
 E(t_{132-132}, s_{135}) &= \text{select case } (d, e): \text{ case } (W, S) \rightarrow \\
 &\quad 1'O; \text{ else } \rightarrow 1'V \\
 E(s_{193}, t_{132-132}) &= \text{select case } (d, e): \text{ case } [(S, W) \\
 &\quad \vee (W, S)] \rightarrow 1'A; \text{ else } \rightarrow 1'c_{193} \\
 E(t_{132-132}, s_{193}) &= \text{select case } (d, e): \text{ case } [(S, W) \\
 &\quad \vee (W, S)] \rightarrow 1'A; \text{ else } \rightarrow 1'c_{193} \\
 E(s_{194}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'V; \text{ case } (W, S) \rightarrow 1'A; \text{ else } \rightarrow 1'c_{194} \\
 E(t_{132-132}, s_{194}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'A; \text{ case } (W, S) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{194} \\
 E(s_{195}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'O; \text{ case } (W, S) \rightarrow 1'A; \text{ else } \rightarrow 1'c_{195} \\
 E(t_{132-132}, s_{195}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'A; \text{ case } (W, S) \rightarrow 1'O; \text{ else } \rightarrow 1'c_{195} \\
 E(s_{197}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'V; \text{ else } \rightarrow 1'c_{197} \\
 E(t_{132-132}, s_{197}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad \text{if } (c_{199} = O) \vee (c_{200} = O) \vee (c_{201} = O) \text{ then } 1'V \\
 &\quad \text{else } 1'A; \text{ case } (W, S) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{197} \\
 E(s_{198}, t_{132-132}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad 1'V; \text{ else } \rightarrow 1'c_{198} \\
 E(t_{132-132}, s_{198}) &= \text{select case } (d, e): \text{ case } (S, W) \rightarrow \\
 &\quad \text{if } (c_{200} = O) \vee (c_{201} = O) \text{ then } 1'V \text{ else } 1'A; \\
 &\quad \text{case } (W, S) \rightarrow 1'V; \text{ else } \rightarrow 1'c_{198} \\
 E(s_{199}, t_{132-132}) &= 1'c_{199} \quad E(t_{132-132}, s_{199}) = 1'c_{199} \\
 E(s_{200}, t_{132-132}) &= 1'c_{200} \quad E(t_{132-132}, s_{200}) = 1'c_{200} \\
 E(s_{201}, t_{132-132}) &= 1'c_{201} \quad E(t_{132-132}, s_{201}) = 1'c_{201}
 \end{aligned}$$

### Guard Function

$$\begin{aligned}
 G(t_{132-132}) &= \{ (d = N) \wedge (e = W) \wedge [(c_{79} = A) \\
 &\quad \wedge (c_{82} = A) \wedge (c_{130} = A)] \} \vee \{ (d = W) \\
 &\quad \wedge [(e = N) \wedge [(c_{79} = A) \wedge (c_{80} = A) \\
 &\quad \wedge (c_{81} = A)]] \vee [(e = S) \wedge [(c_{193} = A) \\
 &\quad \wedge (c_{194} = A) \wedge (c_{195} = A)]]] \} \vee \{ (d = S) \\
 &\quad \wedge (e = W) \wedge [(c_{130} = A) \wedge (c_{133} = A) \\
 &\quad \wedge (c_{193} = A)] \}
 \end{aligned}$$

The AGV can rotate:

- counterclockwise, from direction N to W, if its direction is N and if the three cells  $c_{79}$ ,  $c_{82}$ , and  $c_{130}$  are available;
- clockwise, from direction W to N, if its direction is W and if the three cells  $c_{79}$ ,  $c_{80}$ , and  $c_{81}$  are available;
- counterclockwise, from direction W to S, if its direction is W and if the three cells  $c_{193}$ ,  $c_{194}$ , and  $c_{195}$  are available;
- clockwise, from direction S to W, if its direction is S and if the three cells  $c_{130}$ ,  $c_{133}$ , and  $c_{193}$  are available.

### 3.3.3 Action: Move South

#### Arc Expressions

$$\begin{aligned}
 E(p_{132}, t_{132-135}) &= 1'(l, n, d) \\
 E(t_{132-135}, p_{135}) &= 1'(l, n, d) \\
 E(s_{79}, t_{132-135}) &= 1'c_{79} \quad E(t_{132-135}, s_{79}) = 1'c_{79} \\
 E(s_{80}, t_{132-135}) &= 1'c_{80} \quad E(t_{132-135}, s_{80}) = 1'c_{80} \\
 E(s_{81}, t_{132-135}) &= 1'c_{81} \quad E(t_{132-135}, s_{81}) = 1'c_{81} \\
 E(s_{82}, t_{132-135}) &= 1'c_{82} \quad E(t_{132-135}, s_{82}) = 1'c_{82} \\
 E(s_{83}, t_{132-135}) &= 1'V \\
 E(t_{132-135}, s_{83}) &= \text{if } (c_{79} = O) \vee (c_{80} = O) \\
 &\quad \vee (c_{81} = O) \vee (c_{82} = O) \vee (c_{130} = O) \text{ then } 1'V \\
 &\quad \text{else } 1'A \\
 E(s_{84}, t_{132-135}) &= 1'V \\
 E(t_{132-135}, s_{84}) &= \text{if } (c_{80} = O) \vee (c_{81} = O) \\
 &\quad \text{then } 1'V \text{ else } 1'A \\
 E(s_{130}, t_{132-135}) &= 1'c_{130} \quad E(t_{132-135}, s_{130}) = 1'c_{130} \\
 E(s_{132}, t_{132-135}) &= 1'O \quad E(t_{132-135}, s_{132}) = 1'V \\
 E(s_{198}, t_{132-135}) &= 1'V \quad E(t_{132-135}, s_{198}) = 1'O \\
 E(s_{200}, t_{132-135}) &= 1'c_{200} \quad E(t_{132-135}, s_{200}) = 1'V \\
 E(s_{201}, t_{132-135}) &= 1'c_{201} \quad E(t_{132-135}, s_{201}) = 1'V
 \end{aligned}$$

### Guard Function

$$G(t_{132-135}) = (d = S) \wedge [(c_{200} \neq O) \wedge (c_{201} \neq O)]$$

The AGV can move towards south if its direction is S and if the two cells  $c_{200}$  and  $c_{201}$  are not physically occupied.

#### 3.3.4 Action: Pick-up Pallet/Roll(s)

##### Arc Expressions

$$E(l_{71}, t_{132-71}^P) = m'q$$

$$E(p_{132}, t_{132-71}^P) = 1'(l, n, d)$$

$$E(t_{132-71}^P, p_{132}) = 1'(q, n + m, d)$$

##### Guard Function

$$G(t_{132-71}^P) = (d = W) \wedge (m \geq 1) \wedge \{ [(n = 0) \wedge (l = \text{none}) \wedge [ [(m = 1) \wedge (q = P)] \vee [(m \leq 2) \wedge (q = R)]] ] \vee [(n = 1) \wedge (l = R) \wedge (m = 1) \wedge (q = R)] \}$$

First of all, the AGV can pick-up loads if its direction is W, and some loads are present in the storage lane; moreover, an unloaded AGV can pick-up either 1 pallet or 1 roll or 2 rolls (if available), whereas a loaded AGV can pick-up 1 roll if its actual load is 1 roll.

## 4 DEADLOCK ISSUES

Deadlocks may block a multi-AGV system, and such an issue is indeed critical in open path multi-AGV systems. In the considered class of systems, some strategies are taken into account to prevent deadlocks and to recover from them. However, it is worth noting that this paper is focused on the modelling aspects of a distribution warehouse; therefore, such strategies are here only briefly discussed, with no claim of being exhaustive.

### 4.1 Deadlock Prevention

The deadlock prevention strategies which are proposed consist, as usually done in this field, in applying some constraints to the behaviour of the forklift AGVs, so that the probability that two or more AGVs incur in a deadlock is reduced. The constraints will be formalized in the CPN as Generalized Mutual Exclusion Constraints (GMEC) which are implemented by adding some monitor places whose marking either

prevents or allows the firing of transitions, on the basis of the actual system state.

In the considered class of distribution warehouses, in which lanes are wide 3 cells, a first set of constraints for deadlock prevention is the following:

- two AGVs at most can travel in the same aisle (both horizontal and vertical);
- if two AGVs travel in the same aisle:
  - they cannot use the central lane;
  - they can use the same lane only if they have identical direction;
- only one AGV can approach the nine cells of a crossroad at a time.

Such constraints work well when the number of forklift AGVs is limited (for example, 6-8 AGVs in the distribution warehouse illustrated in Figures 1 and 2). However, these deadlock prevention strategies are not sufficient to guarantee the absence of deadlocks, since, for example, it can happen that an AGV in an aisle may require, to complete its turn action, the room of another aisle which is actually occupied by an AGV that, in turn, needs to go to the room occupied by the first AGV. For this reason, deadlock recovery strategies are also considered in the model.

### 4.2 Deadlock Recovery

The recovery strategy consists in forcing the overcome of the deadlock situation after a certain amount of time. This is accomplished by the AGVs which are assumed to be autonomously able to recover from a deadlock situation by performing some specific actions; when, for example, an AGV encounters another AGV and a deadlock occurs (because each of the two AGVs has to go to the room occupied by the other one), the two AGVs stop their nominal movements and perform some specific actions which allow them to swap their position, thus overcoming the deadlock situation. Such actions are carried out safely, being the AGVs equipped with lasers to detect obstacles or other AGVs.

This capability of AGVs is modelled into the CPN by adding some net structures (places, timed transitions, and arcs) which recover deadlocks after the estimated amount of time. In particular, the added transitions are enabled by the possible deadlock markings; when a deadlock occurs, one of these transitions is enabled and thus can fire; the firing drives the CPN to a new marking in which the deadlock has been solved.

## 5 CONCLUSIONS

A coloured Petri net model of a distribution warehouse, in which AGVs move pallets and rolls from the storage area to the gates of the warehouse, has been described in this paper. The AGV system is open path and the proposed CPN model is able to represent the behaviour of a variable number of AGVs which freely travel on the system layout. To ensure safety, the proposed CPN model includes colours, guards and arc expressions which constrain each AGV to be at least 0.8-1 meter distant from any other AGV.

This CPN model has been also used to build a discrete-event simulator of the distribution warehouse. Such a simulator, implemented in Extend-Sim, is currently used to test a scheduling procedure (based on the solution of some mathematical programming problems and a heuristic algorithm) aimed at assigning the AGVs to the trucks which arrive at the warehouse and sequencing transportation tasks on the AGVs. Moreover, the use of the proposed CPN model for determining the actual paths that the AGVs have to follow to minimize travel times and to reduce interactions with other AGVs is currently investigated.

## ACKNOWLEDGEMENTS

This work has been supported by A.I.R.O.N.E. project funded by European Union and Regione Liguria (Italy) under POR-FESR 2007/2013. Special thanks to the partners of the project: Softeco Sismat SpA, Sogegross SpA, and Genova Robot Srl.

## REFERENCES

- Aized, T. (2009). Modelling and performance maximization of an integrated automated guided vehicle system using coloured Petri net and response surface methods. *Computers and Industrial Engineering*, 57(3):822–831.
- Ajmone Marsan, M., Conte, G., and Balbo, G. (1984). A class of Generalized Stochastic Petri Nets for the performance evaluation of multiprocessor systems. *ACM Transactions on Computer Systems*, 2(2):93–122.
- Castillo, I., Reyes, S. A., and Peters, B. A. (2001). Modeling and analysis of tandem AGV systems using generalized stochastic Petri nets. *Journal of Manufacturing Systems*, 20(4):236–249.
- Dotoli, M. and Fantì, M. (2004). Coloured timed Petri net model for real-time control of automated guided vehicle systems. *International Journal of Production Research*, 42(9):1787–1814.
- Diunkerken, M. B., ter Hoeven, T., and Lodewijks, G. (2006). Simulating the operational control of free ranging AGVs. In Perrone, L. F., Wieland, F. P., Liu, J., Lawson, B. G., Nicol, D. M., and Fujimoto, R. M., editors, *Proceedings of the 2006 Winter Simulation Conference*, pages 1515–1522.
- Holloway, L. E. and Krogh, B. H. (1990). Synthesis of feedback control logic for a class of controlled Petri nets. *IEEE Transactions on Automatic Control*, 35(5):514–523.
- Hsieh, S. (1998). Synthesis of AGVS by coloured-timed Petri nets. *International Journal of Computer Integrated Manufacturing*, 11(4):334–346.
- Hsieh, S. and Chen, Y.-F. (1999). AgvSimNet: A Petri-net-based AGVS simulation system. *International Journal of Advanced Manufacturing Technology*, 15(11):851–861.
- Jensen, K. and Kristensen, L. M. (2009). *Coloured Petri Nets*. Springer.
- Le-Ahn, T. and De Koster, M. B. M. (2006). A review of design and control of automated guided vehicle systems. *European Journal of Operational Research*, 171:1–23.
- Lee, D. Y. and DiCesare, F. (1994). Integrated scheduling of flexible manufacturing systems employing automated guided vehicles. *IEEE Transactions on Industrial Electronics*, 41(6):602–610.
- Martínez-Barberá, H. and Herrero-Pérez, D. (2010). Autonomous navigation of an automated guided vehicle in industrial environments. *Robotics and Computer-Integrated Manufacturing*, 26:296–311.
- Murata, T. (1989). Petri Nets: Properties, analysis and applications. *Proceedings of the IEEE*, 77(4):541–580.
- Nishi, T. and Maeno, R. (2010). Petri net decomposition approach to optimization of route planning problems for AGV systems. *IEEE Transactions on Automation Science and Engineering*, 7(3):523–537.
- Nishi, T. and Tanaka, Y. (2012). Petri net decomposition approach for dispatching and conflict-free routing of bidirectional automated guided vehicle systems. *IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans*, 42(5):1230–1243.
- Petri, C. A. (1962). *Kommunikation mit Automaten*. Bonn: Institut für Instrumentelle Mathematik, Schriften des IIM Nr. 2.
- Seelinger, M. and Yoder, J.-D. (2006). Automatic visual guidance of a forklift engaging a pallet. *Robotics and Autonomous Systems*, 54:1026–1038.
- Sen, A., Wang, C., Ristic, M., and Besant, C. (1991). The supervisory system of the imperial college free ranging automated guided vehicle project. In *Proceedings of the 1991 IEEE International Conference on Systems, Man, and Cybernetics*, pages 1017–1022.
- Vis, I. F. A. (2006). Survey of research in the design and control of automated guided vehicle systems. *European Journal of Operational Research*, 170:677–709.
- Wu, N. and Zhou, M. (2005). Modeling and deadlock avoidance of automated manufacturing systems with multiple automated guided vehicles. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 35(6):1193–1202.