Evolutionary Tuning of Optimal Controllers for Complex Systems

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Abstract: The Proportional Integral Derivative controller is the most widely used industrial device for monitoring and controlling processes. Although there are alternatives to the traditional rules of tuning, there is not yet a study showing that the use of heuristic algorithms it is indeed better than using the classic methods of optimal tuning. Current trends in controller parameter estimation minimize an integral performance criterion. In this paper, an evolutionary algorithm (MAGO - *Multidynamics Algorithm for Global Optimization*) is used as a tool to optimize the controller parameters minimizing the ITAE (*Integral of Time multiplied by Absolut Error*) performance index. The procedure is applied to a set of standard plants modelled as a *Second Order System Plus Time Delay (SOSPD)*. Operating on servo and regulator modes and regardless the plant used, the evolutionary approach gets a better overall performance comparing to traditional methods (Bohl and McAvoy, Minimum ITAE-Hassan, Minimum ITAE-Sung). The solutions obtained cover all restrictions and extends the maximum and minimum boundaries between them.

1 INTRODUCTION

A comparative study of performance of different tuning classical methods for PID (proportional-integral-derivative) controllers is achieved in (Desanti, 2004). This study concludes that tuning methods that require a *Second Order System Plus Time Delay* model *(SOSPD)* perform better than those that require a *First Order Lag Plus time Delay* model *(FOLPD)*. O'Dwyer (2009) reports that 90% of the tuning rules developed are based on a model of first and second order plus time delay. The most frequently tuning rules used are not based on an integral performance criterion. The optimal tuning rules based on second-order models are just 14 of the 84 reported until 2009. In general, those rules are based on several relationships and/or conditions of the parameters defining the process model. The SOSPD model was selected in this paper as representing the plants in order to compare the performance of a heuristic algorithm with the "best" techniques developed for PID controllers optimal tuning. For SOSPD general models 147 tuning rules have been defined based on the ideal PID structure (O'Dwyer, 2009).

In (Mora, 2004; Solera, 2006) the performance and robustness of some tuning rules are evaluated, and a complete analysis of the methods of tuning controllers based on SOSPD is made. Each of the developed tuning rules for PID controllers has only been applied to a certain group of processes. Usual tuning methodologies, such as design based on the root locus, pole-zero cancellation, location of the closed-loop poles, among others, require cumbersome procedures and specialized knowledge. Additionally, most methods for optimal tuning of SOSPD require extra system information from experiments carried out directly on the plant; activities that are not always possible to perform because the presence of extreme stresses and oscillations which may create instability and damage to the system.

The studies mentioned suggest the lack of a general rule for tuning PID controllers. Due to the large number of existing tuning rules it is necessary to find a tuning method that best satisfies the requirements of each problem and also ensures optimal values for the controller parameters according to the selected performance criterion. The tuning of controllers that minimize an integral performance criterion can be established as an optimization problem consisting of minimizing an objective function.

There is a trend to develop new methods for tuning PI and PID controllers (Liu, 2001; Solera, 2005; Tavakoli, 2007), posed as a nonlinear

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optimization problem. In reviewing the literature is found that evolutionary algorithms (EA) are applied to the tuning of controllers on particular cases and not in the general case as in this paper. Nor are compared with traditional methods that minimize some tuning performance index (Chang and Yan, 2004; Junli et al, 2011; Saad et al, 2012a; Saad et al, 2012b). This implies that although there are alternatives to the traditional rules of tuning, there is not yet a study showing that the use of heuristic algorithms it is indeed better than using the traditional rules of optimal tuning. Hence, this matter is addressed. Other applications of the EA in control systems, among them, are system identification (Hernández-Riveros and Arboleda-Gómez, 2013) and optimal configuration of sensors (Michail et al, 2012). The use of an EA for tuning PID controllers in processes represented by SOSPD models is proposed in this paper.

This paper is concerned with PID controllers for processes modeled as SOSPD, optimizing the ITAE (Integral of Time multiplied by Absolute Error) and not requiring additional system information.

EA are a proven tool for solving nonlinear systems and optimization problems. The weaknesses of these algorithms are in the large number of control parameters of the EA to be determined by the analyst and the lack of a solid mathematical foundation (Whitley, 2001). Looking address these weaknesses arise recently the Estimation of Distribution Algorithms, EDA (Lozano, 2006). These algorithms do not use genetic operators, but are based on statistics calculated on samples of the population, which is constantly evolving. This variant when introduce statistics operators provides a strong way to demonstrate the evolution. Nevertheless, they are difficult to manage and do not eliminate the large number of control parameters of classical EA. Set a classic EA is itself a difficult optimization problem; the analyst must try with probabilities of crossover, mutation, replication, operator forms, legal individuals, loss of diversity, etc. Whereas, the EDA require expert skills as the formulation of simultaneous complex distributions or the Bayesian networks structure.

For its part, Multi dynamics Algorithm for Global Optimization (MAGO) also works with statistics from the evolution of the population (Hernandez and Ospina, 2010). MAGO is a heuristic algorithm resulting from the combination of Lagrangian Evolution, Statistical Control and Estimation of Distribution. MAGO has shown to be an efficient and effective tool to solve problems whose search space is complex (Hernandez and Villada, 2012) and works with a real-valued representation. MAGO only requires two parameters provided by the analyst: the number of generations and the population size. The traditional EA, additionally to the number of generations and the population size requires from the user the definition of the selection strategy, the individuals' representation, probabilities of mutation, crossover, replication, as well as, the crossover type, the locus of crossing, among others. Depending of its design, some EA also have extra parameters of tuning as control variables, number of branch and nodes, global step size, time constant, etc. (Xinjie and Mitsuo, 2010). Because of that, MAGO becomes a good choice as a tool for solving controller tuning as an optimization problem.

The results obtained by MAGO are compared with traditional tuning methods not requiring additional system information. An integral performance criterion (Integral of Absolute Error – IAE; Integral of Time multiplied by Absolute Error –ITAE) is optimized to penalize the error. As it is further shown, the system model used makes no difference for the MAGO, because to calculate the controller parameters only input and output signals from the closed loop system are required. Regardless of the relationship between the parameters of the system (time delay, constant time, etc.) the results obtained by MAGO overcome those from the traditional methods of optimal tuning.

This paper begins with an introduction of controller parameters estimation and performance index calculation. The tuning of PID controllers on SOSPD using both the traditional methods and the evolutionary algorithm MAGO follow. A results analysis and some conclusions come after.

2 PID CONTROLLER TUNING

The control policy of an ideal PID controller is shown in equation (1), where *E(s) = (R(s) – Y(s))*. The current value Y(s) of the controlled variable is compared to its desired value R(s), to obtain an error signal *E(s)* (feedback). This error is processed to calculate the necessary change in the manipulated variable U(s) (control action). Some rules of tuning controllers are based on critical system information, on reaction curves and on closed loop tests (Åström and Hägglund, 1995).

$$U(s) = Kc\left[1 + \frac{1}{T_i s} + T_d s\right]E(s) \tag{1}$$

This paper is concerned to PID controllers for processes modeled as SOSPD, optimizing the ITAE and not requiring additional system information.

In (O'Dwyer, 2009), it is indicated that 20.7% of the rules of tuning PID controllers have been developed from SOSPD models (with or without a zero in the numerator). This implies 84 rules, 66 of them do not include the zero in the numerator. Of these, only 14 optimize an integral performance criterion, from which 4 rules propose selecting controller parameters by means of tables and other 6 require additional system information (ultimate gain, *Ku*; ultimate frequency, *Tu*). There are only 4 tuning rules that optimize an integral performance criterion and are only function of the SOSPD parameters. For regulators these rules are: Bohl and McAvoy, Minimum ITAE - Hassan, Minimum ITAE - Sung; for servomechanisms: Minimum ITAE - Sung. Table 1 shows the summary of the study, the chosen rules are shadowed. The equations for the calculation of proportional gain, Kc; integral time, Ti and derivative time, Td can be consulted in (Bohl and McAvoy, 1976; Hassan, 1993; Sung, 1996; Lagunas, 2004). These tuning rules define restrictions on the behavior of the plant, expressed in the range of validity.

2.1 Performance Criteria of PID Controllers

The criterion used for tuning a controller is directly related to the expected performance of the control loop. It can be based on desired characteristics of the response, in time or frequency. Searching for a way to quantify the behavior of control loops led to the establishment of performance indexes based on the error signal, *e(t)* (feedback). The objective is to determine the controller setting that minimizes the chosen cost function. The parameters are optimal under fixed performance criteria. Of these, the best known are the so-called integral criteria (Åström and Hägglund, 1995), defined in equations (2) and (3).

Integral of Absolute Error

$$IAE = \int_0^\infty |e(t)|dt \tag{2}$$

Integral of Time multiplied by Absolut Error

$$ITAE = \int_0^\infty t|e(t)|dt \tag{3}$$

Where the error is given by:

$$e(t) = r(t) - y(t) \tag{4}$$

r(t) is the reference value, and y(t) is the current value of the controlled variable, both expressed in time.

Table 1: PID Controller methods requiring only system parameters and minimizing an integral performance criterion.

Tuning Methods for PID Controllers Optimizing an Integral Criterion on SOSPD System					
Method	**Type of Operation**	**Performance Criterion**	**Type of plant**	**Range of Pertinence**	**Observation**
Minimum IAE - Wills	Regulator	IAE	2	$Tm2 = \tau = 0.1Tm1$	Requires critical system information (Ku, Tu).
Minimum IAE - López	Regulator	IAE	1	$0.5 < \xi < 4$, $0.1 < \tau/Tm1 < 10$	Tuning rule base on tables.
Minimum IAE - Shinskey	Regulator	IAE	2	$Tm2/(Tm2+\tau) = 0.25, 0.5, 0.75$	Requires critical system information (Ku, Tu).
Minimum IAE - Kang	Regulator	IAE	2	$\tau/Tm1$, $Tm2/Tm1$	Tuning rule base on tables.
Minimum ITAE - López	Regulator	ITAE	1	$0.5 < \xi < 4$, $0.1 < \tau/Tm1 < 1$	Tuning rule base on tables.
Bohl and McAvoy	Regulator	ITAE	2	$0.12 < Tm1/Tm2 < 0.9$, $0.1 < \tau/Tm1 < 0.5$	Tuning rule requiring only SOSPD model parameters.
Minimum ITAE - Hassan	Regulator	ITAE	1	$0.5 < \xi < 2$, $0.1 < \tau/Tm1 < 4$	Tuning rule requiring only SOSPD model parameters.
Minimum ITAE - Sung	Regulator	ITAE	1	$0.05 < \tau/Tm1 < 2$	Tuning rule requiring only SOSPD model parameters.
Nearly minimum IAE, ISE, ITAE - Hwang	Regulator	IAE, ISE, ITAE	1	$0.6 < \xi < 4.2$, $0.2 < \tau/Tm1 < 2$	Requires critical system information (Ku, Tu).
Minimun IAE - Wills	Servomechanism	IAE	2	$Tm2 = \tau = 0.1Tm1$	Requires critical system information (Ku, Tu).
Minimum IAE - Gallier and Otto	Servomechanism	IAE	1 & 2	$0.05 < \tau/2Tm1 < 4$	Tuning rule base on tables.
Minimum ITAE - Wills	Servomechanism	ITAE	2	$Tm1 = Tm2$; $\tau = 0.1Tm1$	Requires critical system information (Ku, Tu).
Minimum ITAE - Sung	Servomechanism	ITAE	1	$0.05 < \tau/Tm1 < 2$	Tuning rule requiring only SOSPD model parameters.
Nearly minimum IAE, ISE, ITAE - Hwang	Servomechanism	IAE, ISE, ITAE	1	$0.6 < \xi < 4.2$, $0.2 < \tau/Tm1 < 2$	Requires critical system information (Ku, Tu).

2.2 Plant Parameters and Performance Indexes

To compare the performance of the studied controllers it is necessary to tune them with the same plants. The plant models used are given in equations (5) and (6) (Åström and Hägglund, 2000).

$$G(s) = \frac{K_p e^{-\tau_m s}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1} \tag{5}$$

$$G(s) = \frac{K_p e^{-\tau_m s}}{(1 + T_{m1} s)(1 + T_{m2} s)} \tag{6}$$

The following considerations are taken for equation (5): Kp = 1, τm = 1, ξ = 1 and Tm1 ranging from 1, 10 and 20. For equation (6), the following considerations are taken: Kp = 1, τm = 1, Tm1 = 1 and Tm2 = a*Tm1, where a ≤ 1. Table 2 and Table 3 presents a set of transfer functions according to the parameter values of each plant given by equations (5) and (6).

Table 2: Transfer Functions of Plants 1, for the tuning.

Plants given by Equation (5)	
$G_{p1\_servo1}(s) = \frac{e^{-s}}{s^2 + 2s + 1}$	$G_{p1\_servo1}(s) = G_{p1\_reg1}(s)$
$G_{p1\_servo2}(s) = \frac{e^{-s}}{100s^2 + 20s + 1}$	$G_{p1\_servo2}(s) = G_{p1\_reg3}(s)$
$G_{p1\_servo3}(s) = \frac{e^{-s}}{400s^2 + 40s + 1}$	$G_{p1\_servo3}(s) = G_{p1\_reg5}(s)$

Table 3: Transfer Functions of Plants 2, for the tuning.

Plants given by Equation (6)	
$G_{p2\_servo1}(s) = \frac{e^{-s}}{(1+s)(1+0.1s)}$	$G_{p2\_servo1}(s) = G_{p2\_reg1}(s)$
$G_{p2\_servo2}(s) = \frac{e^{-s}}{(1+s)(1+0.5s)}$	$G_{p2\_servo2}(s) = G_{p2\_reg2}(s)$
$G_{p2\_servo3}(s) = \frac{e^{-s}}{(1+s)(1+s)}$	$G_{p2\_servo3}(s) = G_{p2\_reg3}(s)$

The values of the PID controller parameters for each selected tuning rules are presented, further on, on Table 4. The parameters are calculated according to the formulas proposed for each kind of plant. The selected methods for tuning controllers minimize the integral performance criterion, ITAE. Therefore, in Table 4, besides the values of controller parameters, the ITAE is also reported. The ITAE is calculated in all cases using the commercial software MATLAB function "trapz". For the Hassan method, the controller parameter values are not reported because there was no convergence in the closed loop system response for the selected plants given by equation (5), operating as regulator.

3 TUNING PID CONTROLLERS USING AN EA

Different solutions there may exist in optimization problems, therefore a criterion for discriminating between them, and finding the best, is required. The tuning of controllers that minimize an integral performance criterion can be seen as an optimization problem, inasmuch as the ultimate goal is to find the combination of parameters *Kc*, *Ti* and *Td*, such that the value of the integration of a variable of interest is minimal (error between the actual output of the plant and the desired value).

EA are widely studied as a heuristic tool for solving optimization problems. They have shown to be effective in problems that exhibit noise, random variation and multimodality. Genetic algorithms, for example, have proven to be valuable in both obtaining the optimal values of the PID controller parameters, and in computational cost (Lagunas, 2004). One of the recent trends in EA is Estimation of Distribution Algorithms (Lozano et al, 2006). These do not use genetic operators but are based on statistics from the same evolving population. The Multidynamics Algorithm for Global Optimization (MAGO) (Hernández and Ospina, 2010) also works with statistics from the evolving population. MAGO is autonomous in the sense that it regulates its own behavior and does not need human intervention.

3.1 Optimization and Evolutionary Algorithms

There are techniques used to obtain better results (general or specific) for a problem. The results can greatly improve the performance of a process, which is why this kind of tools is known as optimization. When speaking of an optimization problem is to minimize or maximize depending of the design requirements.

These mean representative criteria of the system efficiency. The chosen criterion is called objective function. The design of an optimization problem is subject to specific restraints of the system, decision variables and design objectives, which leads to an expression such that the optimizer can interpret. Given its nature of global optimizer, an evolutionary algorithm (EA) is used in this work. EA have been used in engineering problems (Fleming and Purshouse, 2002) and the tuning of PID controllers (Chang and Yan, 2004, Li, 2006). The late is the case tries in this work, where successful results have been obtained. The tuning of controllers that minimize an objective function can be formulated as an optimization problem; it is a case of optimal control (Vinter, 2000). The optimal control consists in selecting a control structure (including a PID controller) and adjusts its parameters such that a criterion of overall performance is minimized. In the case of a PID controller (equation 1), the ultimate goal is to find the combination of the Kc, Ti and Td parameters, given some restrictions, such that the value of the integral of a variable of interest (error between the plant's actual output and the desired value or control effort) is minimal. The problem consists of minimizing an objective function, where its minimum is the result of obtaining a suitable combination of the three parameters of PID controller.

3.2 Multidynamics Algorithm for Global Optimization

MAGO inspires by statistical quality control for a self-adapting management of the population. In control charts it is assumed that if the mean of the process is out of some limits, the process is

Table 4: PID Controller parameters. (NC* = Not converged; B&M*= Bohl and McAvoy).

Plant (2)	PID Operating as Regulator						ITAE	
	Kc		Ti		Td			
	B&M	MAGO	B&M	MAGO	B&M	MAGO	B&M	MAGO
$G_{P2\text{-}reg1}(s)$	1.7183	1.4296	1.8978	1.5433	1.8988	0.3341	7.7760	3.1052
$G_{P2\text{-}reg2}(s)$	1.0300	1.4656	1.4164	1.5552	1.6702	0.5597	6.8722	3.6071
$G_{P2\text{-}reg3}(s)$	0.3092	1.8527	0.5854	1.7791	0.7286	0.7575	3.8073	3.6738
Plant (2)	**PID Operating as Servomechanism**						**ITAE**	
	Kc		**Ti**		**Td**			
	Hassan	**MAGO**	**Hassan**	**MAGO**	**Hassan**	**MAGO**	**Hassan**	**MAGO**
$G_{P2\text{-}servo1}(s)$	N C*	0.5658	N C*	1.6705	N C*	1.0318	NC*	72.6860
$G_{P2\text{-}servo2}(s)$	N C*	0.2731	N C*	1.0966	N C*	0.4871	NC*	69.4943
$G_{P2\text{-}servo3}(s)$	N C*	0.9074	N C*	2.0666	N C*	0.5258	NC*	63.2413
Plant (1)	**PID Operating as Servomechanism**						**ITAE**	
	Kc		**Ti**		**Td**			
	SUNG	**MAGO**	**SUNG**	**MAGO**	**SUNG**	**MAGO**	**SUNG**	**MAGO**
$G_{P1\text{-}servo1}(s)$	1.2420	1.2318	2.0550	2.1167	0.6555	0.6050	2.0986	2.0486
$G_{P1\text{-}servo2}(s)$	9.0500	10.3237	18.009	16.8942	4.9386	5.5162	3.7911	2.8532
$G_{P1\text{-}servo3}(s)$	16.4953	19.7929	35.689	29.7905	9.5595	10.7718	3.7937	2.7827
Plant (1)	**PID Operating as Regulator**						**ITAE**	
	Kc		**Ti**		**Td**			
	SUNG	**MAGO**	**SUNG**	**MAGO**	**SUNG**	**MAGO**	**SUNG**	**MAGO**
$G_{P1\text{-}reg1}(s)$	1.8160	1.8557	1.9120	1.7563	0.7073	0.7518	3.8100	3.6623
$G_{P1\text{-}reg2}(s)$	12.8460	17.3252	16.7995	7.4691	-1.99e-6	2.3730	894.5522	3.6427
$G_{P1\text{-}reg3}(s)$	21.8276	31.8262	37.7393	11.0993	-1.17e-4	3.7005	314.5554	4.4240

suspicious of being out of control. Then, some actions should be taken to drive the process inside the control limits (Montgomery, 2008). MAGO takes advantage of the concept of control limits to produce individuals on each generation simultaneously from three distinct subgroups, each one with different dynamics. MAGO starts with a population of possible solutions randomly distributed throughout the search space. The size of the whole population is fixed, but the cardinality of each sub-group changes in each generation according to the first, second and third deviation of the actual population. The exploration is performed by creating new individuals from these three sub-populations. For the exploitation MAGO uses a greedy criterion in one subset looking for the goal.

In every generation, the average location and the first, second and third deviations of the whole population are calculated to form the groups. The first subgroup of the population is composed of improved elite which seeks solutions in a neighbourhood near the best of all the current individuals. *N1* individuals within one standard deviation of the average location of the current population of individuals are displaced in a straight line toward the best of all, suffering a mutation that incorporates information from the best one. The mutation is a simplex search as the Nelder–Mead method (Xinjie and Mitsuo, 2010) but only two individuals are used, the best one and the trial one. A movement in a straight line of a fit individual toward the best one occurs. If this movement generates a better individual, the new one passes to the next generation; otherwise its predecessor passes on with no changes. This method does not require gradient information. For each trial individual $X_i^{(j)}$ at generation *j* a shifted one is created according to the rule in equation (7).

$$X_T^{(j)} = X_i^{(j)} + F^{(j)}(X_B^{(j)} - X_m^{(j)})$$
$$F^{(j)} = S^{(j)} / \|S^{(j)}\| \tag{7}$$

Where $X_B^{(j)}$ is the best individual, $X_m^{(j)}$ is an individual randomly selected. To incorporate information of the current relations among the variables, the factor $F^{(j)}$ depending on the covariance matrix is chosen in each generation. $S^{(j)}$ is the population covariance matrix at generation j. This procedure compiles the differences among the best individuals and the very best one. The covariance matrix of the current population takes into account the effect of the evolution. This information is propagated on new individuals. Each mutant is compared to his father and the one with better performance is maintained for the next generation. This subgroup, called Emergent Dynamics, has the function of making faster convergence of the algorithm.

The second group, called Crowd Dynamics, is formed by creating *N2* individuals from a uniform distribution determined by the upper and lower limits of the second deviation of the current population of individuals. This subgroup seeks possible solutions in a neighborhood close to the population mean. At first, the neighborhood around the mean can be large, but as evolution proceeds it reduced, so that across the search space the population mean is getting closer to the optimal. The third group, or Accidental Dynamics, is the smaller one in relation to its operation on the population. *N3* individuals are created from a uniform distribution throughout the search space, as in the initial population. This dynamic has two functions: maintaining the diversity of the population, and ensuring numerical stability of the algorithm.

The Island Model Genetic Algorithm also works with subpopulations (Skolicki, 2005). But in the Island model, more parameters are added to the genetic algorithm: number of islands, migration size, migration interval, which island migrate, how migrants are selected and how to replace individuals. Instead, in MAGO only two parameters are needed: number of generations and population size. On another hand, the use of a covariance matrix to set an exploring distribution can also be found in (Hansen, 2006), where, in only one dynamics to explore the promising region, new individuals are created sampling from a Gaussian distribution with an intricate adapted covariance matrix. In MAGO a simpler distribution is used.

To get the cardinality of each dynamics, consider the covariance matrix of the population, $S^{(j)}$, at generation *j*, and its diagonal, *diag*($S^{(j)}$). If $Pob^{(j)}$ is the set of potential solutions being considered at generation *j*, the three groups can be defined as in equation (8), where: XM(j) = mean of the actual population. If *N1*, *N2* and *N3* are the cardinalities of the sets *G1*, *G2* and *G3*, the cardinalities of the

$$G_1 = \left\{ x \in Pob(j) \middle/ \begin{array}{l} XM(j) - \sqrt{diag(S(j))} <= x \\ <= XM(j) + \sqrt{diag(S(j))} \end{array} \right\}$$
$$G_2 = \left\{ x \in Pob(j) \middle/ \begin{array}{l} XM(j) - 2\sqrt{diag(S(j))} < x \\ <= XM(j) + \sqrt{diag(S(j))}, or, \\ XM(j) + \sqrt{diag(S(j))} <= x \\ < XM(j) + 2\sqrt{diag(S(j))} \end{array} \right\} \tag{8}$$
$$G_3 = \left\{ x \in Pob(j) \middle/ \begin{array}{l} x <= XM(j) - 2\sqrt{diag(S(j))}, or, \\ x >= XM(j) + 2\sqrt{diag(S(j))} \end{array} \right\}$$

Emergent Dynamics, the Crowd Dynamics and the Accidental Dynamics are set, respectively, and Pob(j) = G1 U G2 U G3.

This way of defining the elements of each group is dynamical by nature. The cardinalities depend on the whole population dispersion in the generation *j*. The Emergent Dynamics tends to concentrate *N1* individuals around the best one. The Crowd Dynamics concentrates *N2* individuals around the mean of the actual population. These actions are reflected in lower values of the standard deviation in each of the problem variables. The Accidental Dynamics, with *N3* individuals, keeps the population dispersion at an adequate level. The locus of the best individual is different from the population's mean. As the evolution advances, the location of the best individual and of the population's mean could be closer between themselves. This is used to self-control the population diversity. Following is MAGO's pseudo code.

MAGO's pseudo code.

1: j = 0, Random initial population generation uniformly distributed over the search space.
2: Repeat
3: Evaluate each individual with the objective function.
4: Calculate the population covariance matrix and the first, second and third dispersion.
5: Calculate the cardinalities N1, N2 and N3 of the groups G1, G2 and G3.
6: Select N1 best individuals, modify them according to equation (7), make them compete and translate the winners towards the best one. Pass the fittest to the generation j + 1.
7: Sample from a uniform distribution in hyper rectangle [LB(j), UB(j)] N2 individuals, pass to generation j+ 1.
8: Sample N3 individuals from a uniform distribution over the whole search space and pass to generation j+1
9: j = j + 1
10: Until an ending criterion is satisfied.

3.3 Statement of the Problem

An EA represents a reliable approach when adjusting controllers is proposed as an optimization problem (Fleming and Purshouse, 2002). Given their nature of global optimizers, EA could face non-convex, nonlinear and highly restrictive optimization problems (Herreros et al, 2002; Tavakoli et al 2007; Iruthayarajan and Baskar, 2009). The MAGO has been shown as a very efficient instrument to solve problems in a continuous domain (Hernandez and Villada, 2012). Thus, the MAGO is applied as a tool for estimating the parameters of a PID controller that minimizes an integral performance index.

In the case where the system is operating as servomechanism, the control problem consists of minimizing the integral of the error multiplied by the time (ITAE). This involves finding the values for the parameters *Kc*, *Ti* y *Td*, such that the system gets the desired *r(t)* value as fast as possible and with few oscillations. In the case where the system operates as a regulator, the reference is a constant *R*, but the control problem is also to minimize the ITAE index. This implies, again, finding the values of the parameters *Kc*, *Ti* and *Td*, but the goal in this mode is that at the appearance of a disturbance the system returns as quickly as possible to the point of operation. The optimization problem is defined in equation (9).

$$J(\underbrace{K_c, \quad T_i, \quad T_d}_{x}) = \min_x \left\{ J_{ITAE} = \int_0^\infty t\left|e(t)\right|dt \right\} \qquad (9)$$

3.4 Evolutionary Design of PID Controller

The controller design is made for the modes servo and regulator. For the servo, a change in a unit step reference is applied. For the regulator, the same change is applied but as a unit step disturbance to the second-order plant. The controllers are tuned for the six plants defined in Table 2 and Table 3. The two parameters of MAGO: number of generations (ng) and number of individuals (n), are very low and fixed for all cases (ng = 150, n = 100). MAGO is a real-valued evolutionary algorithm, so that the representation of the individual is a vector containing the controller parameters. The parameters are positive values in a continuous domain. See Table 5.

Table 5: Structure of the EA.

$Kc \in R^+$	$Ti \in R^+$	$Td \in R^+$

The fitness function is in equation (9). The error is calculated as the difference between the system output and the reference signal. The error is calculated for each point of time throughout the measurement horizon. MAGO does not use genetic operators as crossover or mutation. The adaptation of the population is based on moving N1 individuals to the best one with a Simplex Search, creating N2 individuals over the average location of the actual population and creating N3 individuals through a uniform distribution over the whole search space, as previously discussed.

3.5 Controller Parameters and Performance Indexes

The comparison between the PID controller parameters obtained with the traditional tuning rules and the MAGO algorithm are shown in Table 4. These values minimize the ITAE. Figure 1 illustrates the time response, in closed loop, for the plants given in Table 2 and Table 3. Figure 2 illustrates the time response of the plants defined by equation (6), given in Table 3. For this mode of operation, in the literature review, no tuning rule has been found that could compute the PID controller parameters requiring only the parameters of the plant. However, with MAGO is possible to find controller parameters that minimize the ITAE, without additional information and regardless of the operating mode. The closed-loop system simulations from which the controller was tuned using the MAGO are presented.

4 ANALYSIS OF RESULTS

The study of traditional tuning methods shows that despite the large amount of available tuning rules, there is no one that is effective for the solution of all control problems based on SISO systems. It is evident that a single tuning rule applies only to a small number of problems. A tendency to develop new methods for tuning PID controllers (Tavakoli et al, 2007; Iruthayarajan and Baskar, 2009; Solera, 2005; Liu and Daley, 2001) has been noticed. The most recent are focused on controller's parameter calculation achieving a desired performance, where this index is one of those mentioned before (IAE, ITAE). Table 4 shows the results when tuning PID controllers for different plant models based on equations (5) and (6). The parameters obtained minimize the ITAE criterion. In the case of plants based on the model of equation (5), when the system operates as servomechanism, the tuning rules used are those proposed by Sung. Obtaining an ITAE close to 3, the response behavior of the system is a smooth one, free of oscillations (Figure 1).

For the system operating as a regulator the rules by Sung are employed. In this case the ITAE value is considerably higher for plants Gp1_servo3 and Gp1_servo2, and the system presents oscillations. From this result, it has to be concluded that the rules proposed by Sung are a good choice for the system operating as a servomechanism; while for the case where the system operates as a regulator the use of these rules should be reconsidered.

On another hand, in the case of plants operating as regulators, whose model is given by the equation (6), the rules proposed by Bohl and McAvoy were used to calculate the controller parameters. The results for this experiment are reported in Table 4.

The response of the closed loop system is smooth using the parameters found by this method. The value for the ITAE performance index, in all cases, is below 10. Due to the features that the control problem has, where the objective is to minimize a function by a suitable combination of controller parameters which can be expressed as a function of cost, the solution is presented as an optimization problem. The algorithm MAGO is used to calculate the controller parameters seeking to minimize the ITAE. The results, reported in Table 4, are compared with those obtained by the traditional tuning rules.

The results obtained by MAGO were very satisfactory for all cases. The ITAE performance index is low when the controller parameters are calculated by the MAGO, whatever the plant is represented by equation (5) or equation (6), and for the two modes of operation, servo and regulator. Additional to the above, the responses of closed loop systems where the controller parameters are obtained using the MAGO could be observed in Figure 1. These responses are softer and exhibit less oscillation with respect to the response where controllers are calculated with traditional methods. It can be appreciate in the Sung case as regulator, that the addressed problem has a big variability.

Table 4 also reports the results obtained for the plant based on equation (6). For this case no comparative data are available, because the only traditional tuning rule found that minimizes the performance index ITAE and requires no additional system information is proposed by Hassan (See Table 1). However, in the experiments with this tuning rule it was not possible to obtain convergence to a real value of the parameters of the controller and thus it was not possible to calculate the ITAE. Whereas with MAGO, requiring only the minimum information of the model, it was possible to find the controller parameters reaching an acceptable answer, because in a finite time less than the open-loop system settling time the reference value is achieved, see Figure 2.

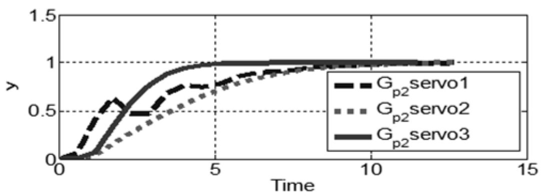

Figure 1: Response to step change in the input of the plant (6), as servomechanism (MAGO only).

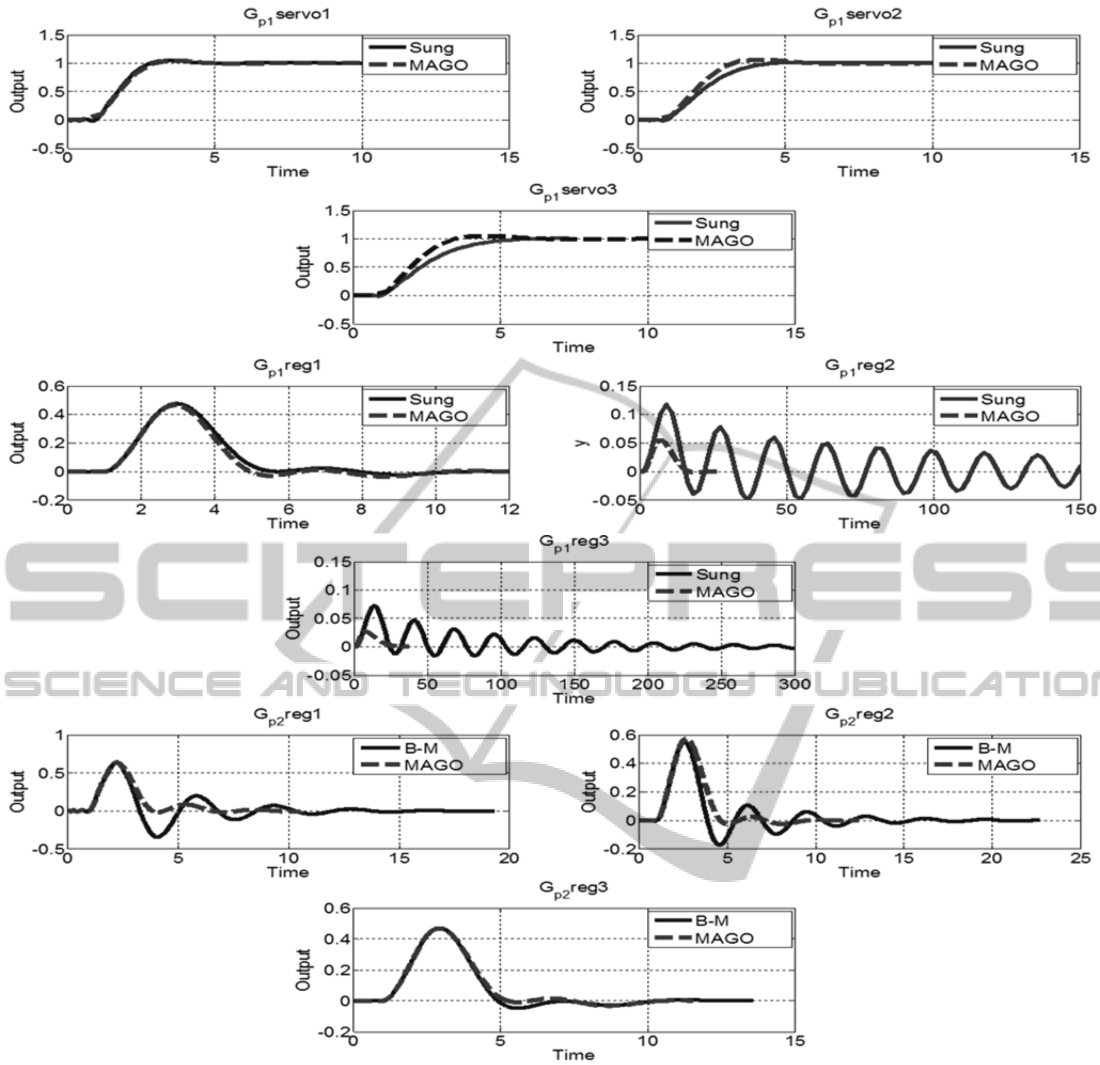

Figure 2: Time response of the plants given by equations (5) and (6), operating as servomechanism and regulator.

5 CONCLUSION

A method of optimal tuning of PID controllers through the evolutionary algorithm MAGO has been successfully developed and implemented. The process resolves the controller tuning as an optimization problem. The PID controller tuning was made for SOSPD, without additional knowledge of the plant. MAGO calculates the parameters of PID controllers minimizing the ITAE performance index, and penalizing the error between the reference value and the output of the plant.

The results showed that MAGO, operating on servo and regulator modes, gets a better overall performance comparing to traditional methods (Bohl and McAvoy (1976), Minimum ITAE - Hassan (1993), Minimum ITAE - Sung (1996)). Each of these methods is restricted to certain values on the behavior of the plant and is limited to an only one type of operation. The solution obtained with the evolutionary approach cover all these restrictions and extends the maximum and minimum between them. Finally, it should be noted that the MAGO successful results are obtained regardless of, both, the plant or controller models used.

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REFERENCES

Åström K. J., Hägglund T., 1995. PID controllers: theory, design and tuning. *Instrument Society of America*.

Åström K.J., Hägglund T., 2000. Benchmark Systems for PID Control. *IFAC Workshop on Digital Control -- Past, present, and future of PID Control*. Spain.

Bohl A. H., McAvoy T. J., 1976. Linear Feedback vs. Time Optima Control. II. The Regulator Problem. *Industrial & Engineering Chemistry Process Design and Development*, *15*(1), 30-33.

Chang W. D., Yan J. J., 2004. Optimum setting of PID controllers based on using evolutionary programming algorithm. *Journal of the Chinese Institute of Engineers*, *27*(3), 439-442.

Desanti J., 2004. Robustness of tuning methods of based on models of first-order plus dead time PI and PID controllers (in Spanish). *Escuela de Ingeniería Eléctrica, Universidad de Costa Rica*.

Fleming P. J., Purshouse R. C., 2002. Evolutionary algorithms in control systems engineering: a survey. *Control Engineering Practice*, *10*(11), 1223-1241.

Hassan G. A., 1993. Computer-aided tuning of analog and digital controllers. *Control and Computers*, *21*, 1-1.

Hansen N. (2006). The CMA Evolution Strategy. *In: Towards a new evolutionary computation. Lozano, Larrañaga, Inza, Bengoetxea,* Heidelberg, Springer.

Hernández J. A., Ospina J. D., 2010. A multi dynamics algorithm for global optimization. *Mathematical and Computer Modelling*, *52*(7), 1271-1278.

Hernández-Riveros Jesús-Antonio, Villada-Cano Daniel., 2012. Sensitivity Analysis of an Autonomous Evolutionary Algorithm. *Lecture Notes in Computer Science. Volume 763. Advances in Artificial Intelligence.* Springer.

Hernández-Riveros J., Arboleda-Gómez A., 2013. Multi-criteria decision and multi-objective optimization for constructing and selecting models for systems identification. *Trans.Modelling and Simulation*, 55.

Herreros A., Baeyens E., Perán J. R., 2002. Design of PID-type controllers using multiobjective genetic algorithms. *ISA transactions*, *41*(4), 457-472.

Iruthayarajan M. W., Baskar S., 2009. Evolutionary algorithms based design of multivariable PID controller. *Expert systems with Applications*, *36*(5).

Junli L., Jianlin M.. Guanghui Z., 2011. Evolutionary algorithms based parameters tuning of PID controller. *Control and Decision Conference,* IEEE. 416-420.

Lagunas J. 2004. Tuning of PID controllers using a multi-objective genetic algorithm, *(NSGA-II)*. (in Spanish) (Doctoral dissertation) *Departamento de Control Automático. Centro de Investigación y de Estudios Avanzados*. México.

Li, Y., Ang, K.H., Chong, G.C., 2006. PID control system analysis and design. *IEEE Control Systems Magazine 26*(1), 32-41.

Liu G. P., Daley S., 2001. Optimal-tuning PID control for industrial systems. *Control Engineering Practice*, *9*(11), 1185-1194.

Lozano J. A., Larrañaga P., Inza I., Bengoetxea E. (Eds.). 2006. Towards a new evolutionary computation: Advances on estimation of distribution algorithms. *Springer.*

Michail, K., Zolotas, A.C., Goodall, R.M., Whidborne, J.F., 2012. Optimised configuration of sensors for fault tolerant control of an electro-magnetic suspension system, *International Journal of Systems Science*, 43 (10), 1785-1804.

Montgomery D. C., 2008. Introduction to statistical quality control. *John Wiley & Sons.* Wiley, New York.

Mora J. 2004. Performance and robustness of the methods based on second order models plus dead time tuning PID controllers. (in Spanish). *Escuela de Ingeniería Eléctrica, Universidad de Costa Rica.*

Dwyer A., 2009. Handbook of PI and PID controller tuning rules (Vol. 2). London: *Imperial College Press.*

Saad M. S., Jamaluddin H., Darus I. Z. M., 2012a. PID Controller Tuning Using Evolutionary Algorithms. WSEAS Transactions on Systems and Control. Issue 4.

Saad M. S., Jamaluddin H., Darus I. Z. M., 2012b. Implementation of PID controller tuning using differential evolution and genetic algorithms. *Int. Journal of Innovative Computing Information and Control*, *8*(11), 7761-7779.

Skolicki Z., De Jong K., 2005. The influence of migration sizes and intervals on island models. *Proceedings of the conference on Genetic and evolutionary computation* ACM. 1295-1302.

Solera Saborío E. 2005. PI / PID Controller Tuning with IAE and ITAE criteria for double pole plants. (in Spanish). *Escuela de Ingeniería Eléctrica, Universidad de Costa Rica.*

Sung S. W., Lee O, J., Lee I. B., Yi S. H. 1996. Automatic Tuning of PID Controller Using Second-Order Plus Time Delay Model. *Journal of chemical engineering of Japan*, *29*(6), 990-999.

Tavakoli S., Griffin I., Fleming P. J., 2007. Multi-objective optimization approach to the PI tuning problem. *Evolutionary Computation Congress. IEEE.* 3165-3171.

Vinter, R., 2000. *Optimal Control*. Systems & Control: Foundations & Applications. Springer, London.

Whitley Darrell., 2001. An overview of evolutionary algorithms: practical issues and common pitfalls. *Information and Software Technology.* 43(14).

Xinjie Yu, Mitsuo Gen., 2010. Introduction to Evolutionary Algorithms. Springer, London.