Fuzzy Signal Processing of Sound and Electromagnetic Environment by Introducing Probability Measure of Fuzzy Events

Akira Ikuta and Hisako Orimoto
Department of Management Information Systems, Prefectural University of Hiroshima, 1-1-71 Ujina-higashi,Minaki-ku, Hiroshima, 734-8558 Japan

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Abstract: The specific signal in real sound and electromagnetic waves frequently shows some very complex fluctuation forms of non-Gaussian type owing to natural, social and human factors. Furthermore, the observed data often contain fuzziness due to the existence of confidence limitation in measuring instruments, permissible error in experimental data, and the variety of human response to phenomena, etc. In this study, by introducing the probability measure of fuzzy events, static and dynamic signal processing methods based on fuzzy observations are proposed for specific signal in the sound and electromagnetic environment with complex probability distribution forms. The effectiveness of the proposed theoretical method is experimentally confirmed by applying it to estimation problems in the real sound and electromagnetic environment.

1 INTRODUCTION

The Probability distribution of a specific signal in the real sound and electromagnetic environment can take various forms, not necessarily characterized by a standard Gaussian distribution. This is due to the diverse nature of factors affecting the properties of the signal (Ikuta et al., 1997). Therefore, it is necessary for the estimation of the evaluation quantities such as the peak value, the amplitude probability distribution, the average crossing rate, the pulse spacing distribution, and the frequent distribution of occurrence etc. of the specific signal, to consider the lower order statistical properties of the signal such as mean and variance as well as the higher order statistics associated with non-Gaussian properties.

On the other hand, the observed data often contain fuzziness due to confidence limitations in sensing devices, permissible errors in the experimental data, and quantizing errors in digital observations (Ikuta et al., 2005). For reasons of simplicity, many previously proposed estimation methods have not considered fuzziness in the observed data under the restriction of Gaussian type fluctuations (Bell and Cathey, 1993; Kalman, 1960; Kalman and Buch, 1961; Kushner, 1967; Julier, 2002). Although several state estimation methods for a stochastic environment system with non-Gaussian fluctuations and many analyses based on Gaussian Mixture Models have previously been proposed (Kitagawa, 1996; Ohta and Yamada, 1984; Ikuta et al., 2001; Orimoto and Ikuta, 2014; Guoshen, 2012), the fuzziness contained in the observed data has not been considered in these studies. Therefore, it is desirable to develop a method that is flexible and is applicable to ill-conditioned fuzzy observations.

In this study, a new estimation theory is proposed for a signal based on observations with non-Gaussian properties, from both static and dynamic viewpoints by regarding the observation data with fuzziness as fuzzy observations.

First, a static signal processing method considering not only linear correlation but also the higher order nonlinear correlation information is proposed on the basis of fuzzy observation data, in order to find the mutual relationship between sound and electromagnetic waves leaked from electronic information equipment. More specifically, a conditional probability expression for fuzzy variables is derived by applying probability measure of fuzzy events (Zadeh, 1968) to a joint probability function in a series type expression reflecting various correlation relationships between the variables. By use of the derived probability expression, a method for estimating precisely the

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correlation information based on the observed fuzzy data is theoretically proposed. On the basis of the estimated correlation information, the probability distribution for a specific variable (e.g. electromagnetic wave) based on the observed fuzzy data of the other variable (e.g. sound) can be predicted.

Next, a dynamic state estimation method for estimating a specific signal based on fuzzy observations with the existence of background noise is proposed in a recursive form suitable for use with a digital computer. More specifically, by paying attention to the power state variable for a specific signal in the sound environment, a new type of signal processing method for estimating a specific signal on a power scale is proposed. In the case of considering the power state variable, a physical mechanism of contamination by background noise can be reflected in the state estimation algorithm by using the additive property between the specific signal and the background noise. There is a restriction for power state variables fluctuating only in the non-negative region (i.e., any fluctuation width around the mean value has necessary to tend zero when the mean value tends zero), and it is obvious that the Gaussian distribution and Gaussian Mixture Models regarding the mean and variance as independent parameters are not adequate for power state variables. The proposed method positively utilizes Gamma distribution and Laguerre polynomial suitable to represent the power state variable, which fluctuates only within the positive region (Ohta and Koizumi, 1968).

The effectiveness of the theoretically proposed static and dynamic fuzzy signal processing methods for estimating the specific signal is experimentally confirmed by applying those to real data in the sound and electromagnetic environment.

2 STATIC SIGNAL PROCESSING BASED ON FUZZY OBSERVATIONS IN SOUND AND ELECTROMAGNETIC ENVIRONMENT

2.1 Prediction for Probability Distribution of Specific Signal from Fuzzy Fluctuation Factor

The observed data in the real sound and electromagnetic environment often contain fuzziness due to several factors such as limitations in the measuring instruments, permissible error tolerances in the measurement, and quantization errors in digitizing the observed data. In this study, the observation data with fuzziness are regarded as fuzzy observations.

In order to evaluate quantitatively the complicated relationship between sound and electromagnetic waves leaked from an identical electronic information equipment, let two kinds of variables (i.e. sound and electromagnetic waves) be x and y, and the observed data based on fuzzy observations be X and Y respectively. There exist the mutual relationships between x and y, and also between X and Y. Therefore, by finding the relations between x and X, and also between y and Y, based on probability measure of fuzzy events (Zadeh, 1968), it is possible to predict the true value y (or x) from the observed fuzzy data X (or Y). For example, for the prediction of the probability density function $P_s(y)$ of y from X, averaging the conditional probability density function $P(y \mid X)$ on the basis of the observed fuzzy data X, $P_s(y)$ can be obtained as: $P_s(y) = < P(y \mid X) >_X$. The conditional probability density function $P(y \mid X)$ can be expressed under the employment of the well-known Bayes' theorem:

$$P(y \mid X) = \frac{P(X, y)}{P(X)}. \tag{1}$$

The joint probability distribution $P(X, y)$ is expanded into an orthonormal polynomial series on the basis of the fundamental probability distribution $P_0(X)$ and $P_0(y)$, which can be artificially chosen as the probability function describing approximately the dominant parts of the actual fluctuation pattern, as follows:

$$P(X, y) = P_0(X) P_0(y) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \varphi_m(X) \phi_n(y),$$
$$A_{mn} \equiv \langle \varphi_m(X) \phi_n(y) \rangle, \tag{2}$$

where $< \cdot >$ denotes the averaging operation with respect to the random variables. The information on the various types of linear and nonlinear correlations between X and y is reflected in each expansion coefficient A_{mn}. When X is a fuzzy number expressing an approximated value, it can be treated as a discrete variable with a certain level difference. Therefore, as $P_0(X)$, the generalized binomial distribution with a level difference interval h_X can be chosen (Ikuta et al., 1997):

$$P_0(X)=\frac{\left(\frac{N_X-M_X}{h_X}\right)!}{\left(\frac{X-M_X}{h_X}\right)!\left(\frac{N_X-X}{h_X}\right)!} p_X^{\frac{X-M_X}{h_X}}(1-p_X)^{\frac{N_X-X}{h_X}},$$

$$p_X\equiv\frac{\mu_X-M_X}{N_X-M_X} \quad , \quad \mu_X\equiv\langle X\rangle, \tag{3}$$

where M_X and N_X are the maximum and minimum values of X. Furthermore, as the fundamental probability density function $P_0(y)$ of y, the standard Gaussian distribution is adopted:

$$P_0(y)=\frac{1}{\sqrt{2\pi\sigma_y^2}}e^{-\frac{(y-\mu_y)^2}{2\sigma_y^2}},$$

$$\mu_y\equiv\langle y\rangle \quad , \quad \sigma_y^2\equiv\left\langle (y-\mu_y)^2\right\rangle. \tag{4}$$

The orthonormal polynomials $\varphi_m(X)$ and $\phi_n(y)$ with the weighting functions $P_0(X)$ and $P_0(y)$ can be determined as (Ikuta et al., 1997)

$$\varphi_m(X)=\left\{\left(\frac{N_X-M_X}{h_X}\right)^{(m)}m!\right\}^{-\frac{1}{2}}\left(\frac{1-p_X}{p_X}\right)^{\frac{m}{2}}\frac{1}{h_X^m} \sum_{j=0}^{m}\frac{m!}{(m-j)!\,j!}(-1)^{m-j}\left(\frac{p_X}{1-p_X}\right)^{m-j} (N_X-X)^{(m-j)}(X-M_X)^{(j)},$$

$$\left(X^{(n)}\equiv X(X-h_X)\cdots(X-(n-1)h_X)\ ,\ X^{(0)}\equiv 1\right), \tag{5}$$

$$\phi_n(y)=\frac{1}{\sqrt{n!}}H_n\left(\frac{y-\mu_y}{\sigma_y}\right);\ \text{Hermite polynomial.} \tag{6}$$

Thus, the predicted probability density function $P_s(y)$ can be expressed in an expansion series form:

$$P_s(y)=P_0(y)\sum_{n=0}^{\infty}\left\langle\frac{\sum_{m=0}^{\infty}A_{mn}\varphi_m(X)}{\sum_{m=0}^{\infty}A_{m0}\varphi_m(X)}\right\rangle_X\phi_n(y). \tag{7}$$

2.2 Estimation of Correlation Information based on Fuzzy Observation Data

The expansion coefficient A_{mn} in (2) has to be estimated on the basis of the fuzzy observation data X and Y, when the true value y is unknown. Let the joint probability distribution of X and Y be $P(X,Y)$. By applying probability measure of fuzzy events (Zadeh, 1968), $P(X,Y)$ can be expressed as:

$$P(X,Y)=\frac{1}{K}\int\mu_Y(y)P(X,y)\,dy\,, \tag{8}$$

where K is a constant satisfying the normalized condition: $\sum_X\sum_Y P(X,Y)=1$. The fuzziness of Y can be characterized by the membership function $\mu_Y(y)$ $(=\exp\{-\alpha(y-Y)^2\}$, α; a parameter).

Substituting (2) in (8), the following relationship is derived.

$$P(X,Y)=\frac{1}{K}P_0(X)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}a_n\varphi_m(X),$$

$$a_n=\int e^{-\alpha(y-Y)^2}P_0(y)\phi_n(y)\,dy\,. \tag{9}$$

The conditional N-th order moment of the fuzzy variable X is given from (9) as

$$\langle X^N\mid Y\rangle=\sum_X P(X\mid Y)=\frac{\sum_X X^N P(X,Y)}{P(Y)}$$

$$=\frac{\sum_X P_0(X)X^N\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}a_n\varphi_m(X)}{\sum_{n=0}^{\infty}A_{0n}a_n}. \tag{10}$$

After expanding X^N in an orthogonal seri es expression, by considering the orthonormal relationship of $\varphi_m(X)$, (10) is expressed explicitly as

$$\langle X^N\mid Y\rangle=\frac{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}d_m^N A_{mn}a_n}{\sum_{n=0}^{\infty}A_{0n}a_n}, \tag{11}$$

($X^N\equiv\sum_{m=0}^{N}d_m^N\varphi_m(X)$, d_m^N; appropriate constant).

The right side of the above equation can be evaluated numerically from the fuzzy observation data. Accordingly, by regarding the expansion coefficients A_{mn} as unknown parameters, a set of simultaneous equations in the same form as in (11) can be obtained by selecting a set of N and/or Y values equal to the number of unknown parameters. By solving the simultaneous equations, the expansion coefficients A_{mn} can be estimated. Furthermore, using these estimates, the probability density function $P_s(y)$ can be predicted from (7).

3 DYNAMIC SIGNAL PROCESSING BASED ON FUZZY OBSERVATIONS IN SOUND ENVIRONMENT

3.1 Formulation of Fuzzy Observation under Existence of Background Noise

Consider a sound environmental system with background noise having a non-Gaussian distribution. Let the specific signal power of interest in the environment at a discrete time k be x_k, and the dynamical model of the specific signal be:

$$x_{k+1} = Fx_k + Gu_k , \tag{12}$$

where u_k denotes the random input power with known statistics, and F , G are known system parameters and can be estimated by use of the system identification method (Eykhoff, 1984) when these parameters cannot be determined on the basis of the physical mechanism of system.

The observed data in the real sound environment often contain fuzziness due to several factors, as indicated earlier. Therefore, in addition to the inevitable background noise, the effects of the fuzziness contained in the observed data have to be considered in developing a state estimation method for the specific signal of interest. From a functional viewpoint, the observation equation can be considered as involving two types of operation:

1. The additive property of power state variable with the background noise can be expressed as:

$$y_k = x_k + v_k , \tag{13}$$

where it is assume that the statistics of the background noise power v_k are known in advance.

2. The fuzzy observation z_k is obtained from y_k. The fuzziness of z_k is characterized by the membership function $\mu_{z_k}(y_k)$.

3.2 State Estimation based on Fuzzy Observation Data

To obtain an estimation algorithm for the signal power x_k based on the fuzzy observation z_k, the Bayes' theorem for the conditional probability density function can be considered (Ohta and Yamada, 1984).

$$P(x_k \mid Z_k) = \frac{P(x_k, z_k \mid Z_{k-1})}{P(z_k \mid Z_{k-1})} , \tag{14}$$

where $Z_k (\equiv (z_1, z_2, ..., z_k))$ is a set of observation data up to a time k . By applying probability measure of fuzzy events (Zadeh, 1968) to the right side of (14), the following relationship is derived.

$$P(x_k \mid Z_k) = \frac{\int_0^\infty \mu_{z_k}(y_k) P(x_k, y_k \mid Z_{k-1}) dy_k}{\int_0^\infty \mu_{z_k}(y_k) P(y_k \mid Z_{k-1}) dy_k} . \tag{15}$$

The conditional probability density function of x_k and y_k can be generally expanded in a statistical orthogonal expansion series.

$$P(x_k, y_k \mid Z_{k-1}) = P_0(x_k \mid Z_{k-1}) P_0(y_k \mid Z_{k-1}) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{mn} \theta_m^{(1)}(x_k) \theta_n^{(2)}(y_k) , \tag{16}$$

$$B_{mn} \equiv < \theta_m^{(1)}(x_k) \theta_n^{(2)}(y_k) \mid Z_{k-1} > , \tag{17}$$

where the functions $\theta_m^{(1)}(x_k)$ and $\theta_n^{(2)}(y_k)$ are the orthogonal polynomials of degrees m and n with weighting functions $P_0(x_k \mid Z_{k-1})$ and $P_0(y_k \mid Z_{k-1})$, which can be artificially chosen as the probability density functions describing the dominant parts of $P(x_k \mid Z_{k-1})$ and $P(y_k \mid Z_{k-1})$. These two functions must satisfy the following orthonormal relationships:

$$\int_0^\infty \theta_m^{(1)}(x_k) \theta_{m'}^{(1)}(x_k) P_0(x_k \mid Z_{k-1}) dx_k = \delta_{mm'} , \tag{18}$$

$$\int_0^\infty \theta_n^{(2)}(y_k) \theta_{n'}^{(2)}(y_k) P_0(y_k \mid Z_{k-1}) dy_k = \delta_{nn'} . \tag{19}$$

By substituting (16) into (15), the conditional probability density function $P(x_k \mid Z_k)$ can be expressed as:

$$P(x_k \mid Z_k) = \frac{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{mn} P_0(x_k \mid Z_{k-1}) \theta_m^{(1)}(x_k) I_n(z_k)}{\sum_{n=0}^{\infty} B_{0n} I_n(z_k)} \tag{20}$$

with

$$I_n(z_k) \equiv \int_0^\infty \mu_{z_k}(y_k) P_0(y_k \mid Z_{k-1}) \theta_n^{(2)}(y_k) dy_k . \tag{21}$$

Based on (20), and using the orthonormal relationship of (18), the recurrence algorithm for estimating an arbitrary N -th order polynomial type function $f_N(x_k)$ of the specific signal can be derived as follows:

$$\hat{f}_N(x_k) \equiv < f_N(x_k) \mid Z_k > = \frac{\sum_{m=0}^{N} \sum_{n=0}^{\infty} B_{mn} C_{Nm} I_n(z_k)}{\sum_{n=0}^{\infty} B_{0n} I_n(z_k)} , \tag{22}$$

where C_{Nm} is the expansion coefficient determined by the equality:

$$f_N(x_k) = \sum_{m=0}^{N} C_{Nm}\varphi_m^{(1)}(x_k)\ . \qquad (23)$$

In order to make the general theory for estimation algorithm more concrete, the well-known Gamma distribution is adopted as $P_0(x_k \mid Z_{k-1})$ and $P_0(y_k \mid Z_{k-1})$, because this probability density function is defined within positive region and is suitable to the power state variables.

$$P_0(x_k \mid Z_{k-1}) = P_\Gamma(x_k; m_{x_k}^*, s_{x_k}^*)\ ,$$
$$P_0(y_k \mid Z_{k-1}) = P_\Gamma(y_k; m_{y_k}^*, s_{y_k}^*) \qquad (24)$$

with

$$P_\Gamma(x; m, s) \equiv \frac{x^{m-1}}{\Gamma(m)s^m} e^{-\frac{x}{s}}\ ,$$
$$m_{x_k}^* \equiv (x_k^*)^2/\Gamma_k\ ,\quad s_{x_k}^* \equiv x_k^*/m_{x_k}^*\ ,$$
$$x_k^* = < x_k \mid Z_{k-1} >\ ,\quad \Gamma_k \equiv < (x_k - x_k^*)^2 \mid Z_{k-1} >\ ,$$
$$m_{y_k}^* \equiv (y_k^*)^2/\Omega_k\ ,\quad s_{y_k}^* \equiv y_k^*/m_{y_k}^*\ ,$$
$$y_k^* = < y_k \mid Z_{k-1} > = x_k^* + < v_k >\ ,$$
$$\Omega_k \equiv < (y_k - y_k^*)^2 \mid Z_{k-1} >$$
$$= \Gamma_k + < (v_k - < v_k >)^2 >\ . \qquad (25)$$

Then, the orthonormal functions with two weighting probability density functions in (24) can be given in the Laguerre polynomial (Ohta and Koizumi, 1968):

$$\theta_m^{(1)}(x_k) = \sqrt{\frac{\Gamma(m_{x_k}^*)m!}{\Gamma(m_{x_k}^* + m)}} L_m^{(m_{x_k}^* - 1)}\left(\frac{x_k}{s_{x_k}^*}\right)\ ,$$
$$\theta_n^{(2)}(y_k) = \sqrt{\frac{\Gamma(m_{y_k}^*)n!}{\Gamma(m_{y_k}^* + n)}} L_n^{(m_{y_k}^* - 1)}\left(\frac{y_k}{s_{y_k}^*}\right)\ . \qquad (26)$$

As the membership function $\mu_{z_k}(y_k)$, the following function suitable for the Gamma distribution is newly introduced.

$$\mu_{z_k}(y_k) = (z_k^{-\beta} e^{\beta}) y_k^{\beta} \exp\{-\frac{\beta}{z_k} y_k\}\ , \qquad (27)$$

where $\beta (> 0)$ is a parameter. Accordingly, by considering the orthonormal condition of Laguerre polynomial (Ohta and Koizumi, 1968), (21) can be given by

$$I_n(z_k) = \frac{z_k^{-\beta} e^{\beta}}{\Gamma(m_{y_k}^*)(s_{y_k}^*)^{m_{y_k}^*}} \Gamma(M_k) D_k^{M_k}$$
$$\int_0^\infty \frac{y_k^{M_k - 1}}{\Gamma(M_k) D_k^{M_k}} e^{-\frac{1}{D_k}} \sqrt{\frac{\Gamma(m_{y_k}^*)n!}{\Gamma(m_{y_k}^* + n)}}$$
$$\sum_{r=0}^{n} d_{nr} L_r^{(M_k - 1)}\left(\frac{y_k}{D_k}\right) dy_k$$
$$= \frac{z_k^{-\beta} e^{\beta}}{\Gamma(m_{y_k}^*)(s_{y_k}^*)^{m_{y_k}^*}} \Gamma(M_k) D_k^{M_k} \sqrt{\frac{\Gamma(m_{y_k}^*)n!}{\Gamma(m_{y_k}^* + n)}} d_{n0} \qquad (28)$$

with

$$M_k \equiv m_{y_k}^* + \beta\ ,\qquad D_k \equiv \frac{s_{y_k}^* z_k}{\beta s_{y_k}^* + z_k}\ , \qquad (29)$$

where d_{nr} (r =0, 1, 2, …, n) are the expansion coefficients in the equality:

$$L_n^{(m_{y_k}^* - 1)}\left(\frac{y_k}{s_{y_k}^*}\right) = \sum_{r=0}^{n} d_{nr} L_r^{(M_k - 1)}\left(\frac{y_k}{D_k}\right)\ . \qquad (30)$$

Especially, the estimates for mean and variance can be obtained as follows:

$$\hat{x}_k \equiv < x_k \mid Z_k >$$
$$= \frac{\sum_{n=0}^{\infty} \{B_{0n}C_{10} + B_{1n}C_{11}\} I_n(z_k)}{\sum_{n=0}^{\infty} B_{0n} I_n(z_k)}\ , \qquad (31)$$

$$P_k \equiv < (x_k - \hat{x}_k)^2 \mid Z_k >$$
$$= \frac{\sum_{n=0}^{\infty} \{B_{on}C_{20} + B_{1n}C_{21} + B_{2n}C_{22}\} I_n(z_k)}{\sum_{n=0}^{\infty} B_{0n} I_n(z_k)} \qquad (32)$$

with

$$C_{10} = m_{x_k}^* s_{x_k}^*\ ,\quad C_{11} = -\sqrt{m_{x_k}^*}\, s_{x_k}^*\ ,$$
$$C_{20} = \hat{x}_k^{\ 2} - 2m_{x_k}^* s_{x_k}^* \{\hat{x}_k - (m_{x_k}^* + 1)s_{x_k}^*\}$$
$$- m_{x_k}^*(m_{x_k}^* + 1) s_{x_k}^{*\ 2}\ ,$$
$$C_{21} = 2\sqrt{m_{x_k}^*}\, s_{x_k}^* \{\hat{x}_k - (m_{x_k}^* + 1)s_{x_k}^*\}\ ,$$
$$C_{22} = \sqrt{2m_{x_k}^*(m_{x_k}^* + 1)}\, s_{x_k}^*\ . \qquad (33)$$

Finally, by considering (12), the prediction step which is essential to perform the recurrence estimation can be given by

$$x_{k+1}^* \equiv < x_{k+1} \mid Z_k > = F\hat{x}_k + G < u_k >\ ,$$
$$\Gamma_{k+1} \equiv < (x_{k+1} - x_{k+1}^*)^2 \mid Z_k >$$
$$= F^2 P_k + G^2 < (u_k - < u_k >)^2 >\ . \qquad (34)$$

By replacing k with $k+1$, the recurrence estimation can be achieved.

4 APPLICATION TO SOUND AND ELECTROMAGNETIC ENVIRONMENT

4.1 Prediction of Sound and Electric Field in PC Environment

By adopting a personal computer (PC) in the real working environment as specific information equipment, the proposed static method was applied to investigate the mutual relationship between sound and electromagnetic waves leaked from the PC under the situation of playing a computer game. In order to eliminate the effects of sound from outside, the PC was located in an anechoic room (cf. Fig. 1). The RMS value (V/m) of the electric field radiated from the PC and the sound intensity level (dB) emitted from a speaker of the PC were simultaneously measured. The data of electric field strength and sound intensity level were measured by use of an electromagnetic field survey mater and a sound level meter respectively. The slowly changing non-stationary 600 data for each variable were sampled with a sampling interval of 1 (s). Two kinds of fuzzy data with the quantized level widths of 0.1 (v/m) for electric field strength and 5.0 (dB) for sound intensity level were obtained.

Based on the 400 data points, the expansion coefficients A_{mn} were first estimated by use of (11). Furthermore, the parameters of the membership functions in (8) for sound level and electric field strength with rough quantized levels were decided so as to express the distribution of data as precisely as possible, as shown in Figs. 2 and 3. Next, the 200 sampled data within the different time interval which were non-stationary different from data used for the estimation of the expansion coefficients were adopted for predicting the probability distributions of (i) the electric field based on sound and (ii) the sound based on electric field.

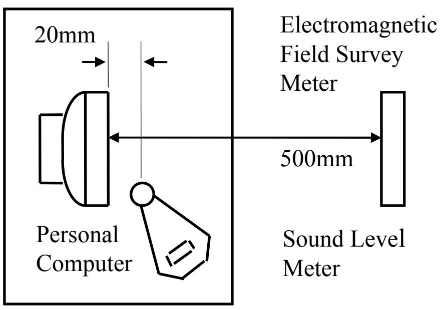

Figure 1: A schematic drawing of the experiment.

The experimental results for the prediction of electric field strength and sound level are shown in Figs. 4 and 5 respectively in a form of cumulative distribution. From these figures, it can be found that the theoretically predicted curves show good agreement with experimental sample points by considering the expansion coefficients with several higher orders.

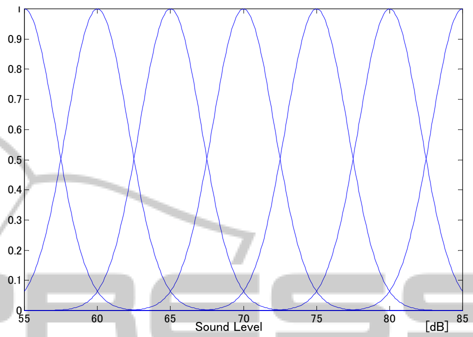

Figure 2: Membership function of sound level.

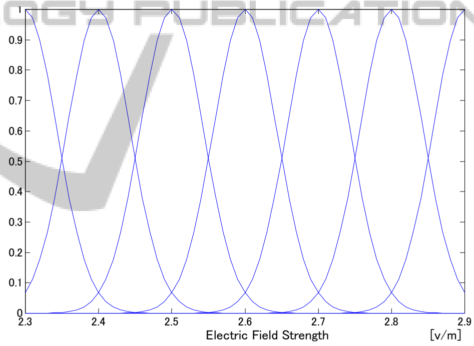

Figure 3: Membership function of electric field.

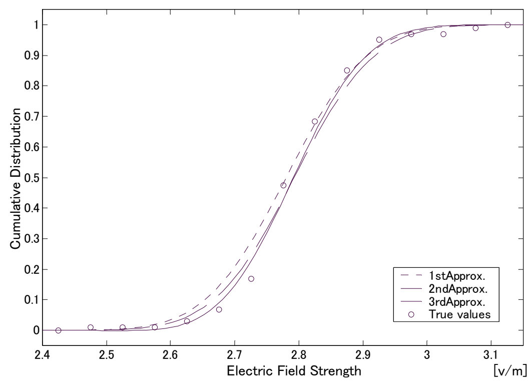

Figure 4: Prediction of the cumulative distribution for the electric field strength based on the fuzzy observation of sound.

For comparison, the generalized regression analysis method (Ikuta et al., 1997) without using fuzzy theory was applied to the fuzzy data X and Y. The prediction results are shown in Figs. 6 and 7. As compared with Figs. 4 and 5, it is obvious that the proposed method considering fuzzy theory is more effective than the previous method.

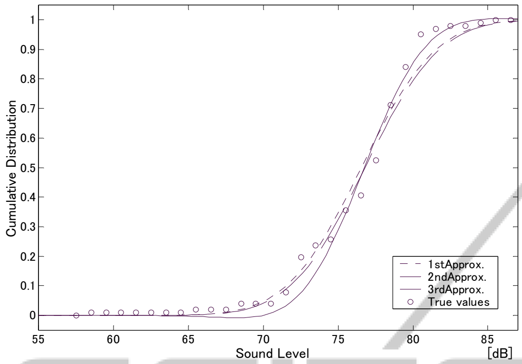

Figure 5: Prediction of the cumulative distribution for the sound level based on the fuzzy observation of electric field.

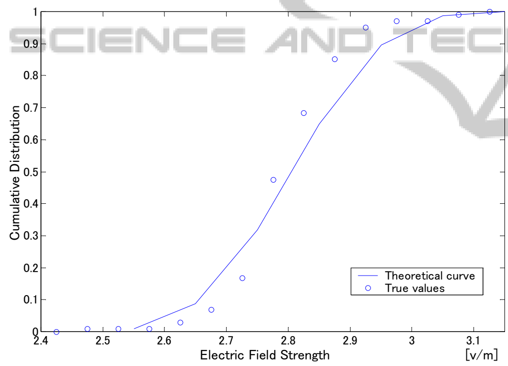

Figure 6: Prediction of the cumulative distribution for the electric field strength by use of the extended regression analysis method.

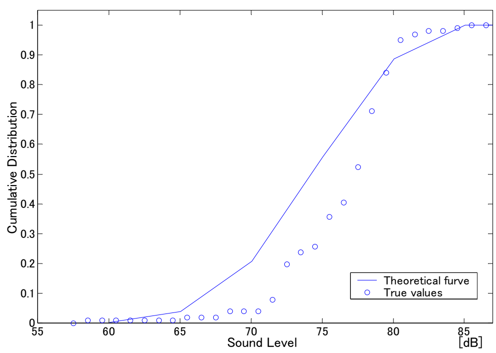

Figure 7: Prediction of the cumulative distribution for the sound level by use of the extended regression analysis method.

4.2 Estimation of Specific Signal in Sound Environment

In order to examine the practical usefulness of the proposed dynamic signal processing based on the fuzzy observation, the proposed method was applied to the real sound environmental data. The road traffic noise was adopted as an example of a specific signal with a complex fluctuation form. Applying the proposed estimation method to actually observed data contaminated by background noise and quantized with 1 dB width, the fluctuation wave form of the specific signal was estimated. The statistics of the specific signal and the background noise used in the experiment are shown in Table 1.

Figures 8-10 show the estimation results of the fluctuation wave form of the specific signal. In this estimation, the finite number of expansion coefficients $B_{mn}(m,n \leq 2)$ was used for the simplification of the estimation algorithm. In these figures, the horizontal axis shows the discrete time k, of the estimation process, and the vertical axis expresses the sound level taking a logarithmic transformation of power-scaled variables, because the real sound environment usually is evaluated on dB scale connected with human effects. For comparison, the estimation results calculated using the usual method without considering any membership function are also shown in these figures. Since Kalman's filtering theory is widely used in the field of stochastic system (Kalman, 1960; Kalman and Buch, 1961; Kushner, 1967), this method was also applied to the fuzzy observation data as a trail.

The results estimated by the proposed method considering the membership function show good agreement with the true values. On the other hand, there are great discrepancies between the estimates

Table 1: Statistics of the specific signal and the background noise.

Statistics of Specific Signal		Statistics of Background Noise	
Mean (watt/m^2)	Standard Deviation (watt/m^2)	Mean (watt/m^2)	Standard Deviation (watt/m^2)
2.9×10^{-5}	2.8×10^{-5}	2.9×10^{-5}	1.4×10^{-6}

based on the standard type dynamical estimation method (i.e., Kalman filter) without consideration of the membership function and the true values, particularly in the estimation of the lower level values of the fluctuation.

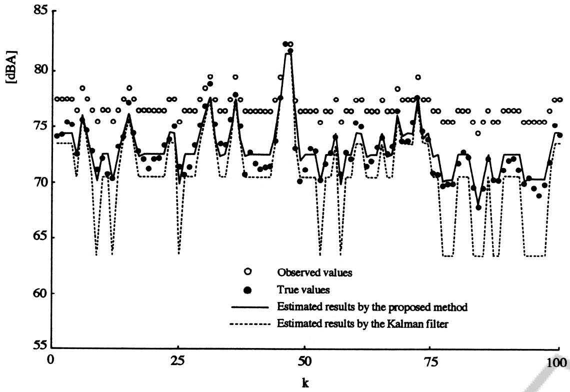

Figure 8: State estimation results for the road traffic noise during a discrete time interval of [1, 100] sec., based on the quantized data with 1 dB width.

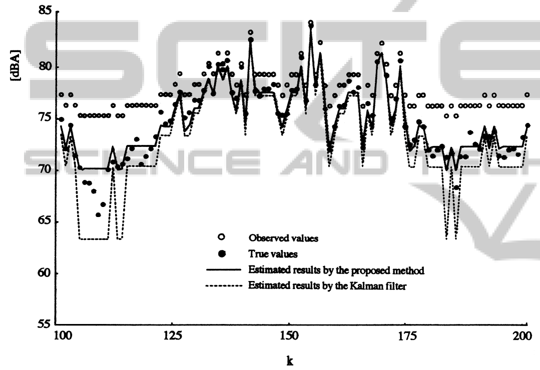

Figure 9: State estimation results for the road traffic noise during a discrete time interval of [101, 200] sec., based on the quantized data with 1 dB width.

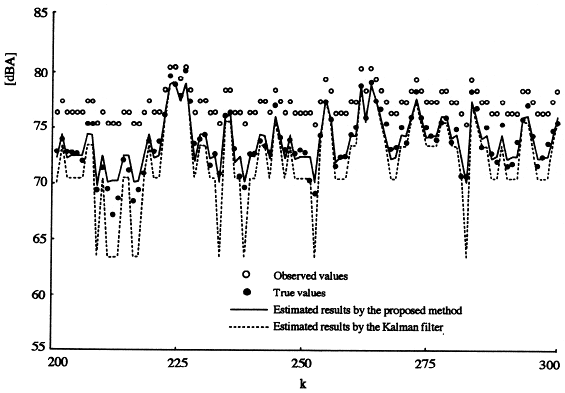

Figure 10: State estimation results for the road traffic noise during a discrete time interval of [201, 300] sec., based on the quantized data with 1 dB width.

5 CONCLUSIONS

In this study, new methods for estimating a specific signal embedded in fuzzy observations have been proposed within static and dynamic frameworks. The proposed estimation methods have been realized by introducing a fuzzy logic approach into the probabilistic description of the signal. The effectiveness of the proposed fuzzy signal processing method has been confirmed experimentally by applying it to the real sound and electromagnetic data.

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