

Robust Estimation of Load Performance of DC Motor using Genetic Algorithm

Jong Kwang Lee, Byung Suk Park, Jonghui Han and Il-Je Cho

Nuclear Fuel Cycle Process Technology Development Division, Korea Atomic Energy Research Institute, Daejeon, Korea

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Abstract: This paper presents a novel approach to estimate the load performance curves of DC motors whose equations are represented as a function of the torque based on a steady-state model with constraints. Since a simultaneous optimization of the curves forms a multi-objective optimization problem (MOP), we apply an optimal curve fitting method based on a real-coded genetic algorithm (RGA). In the method, we introduce a normalized ratio of errors to solve the MOP without the use of weighting factors and the nominal parameters to automatically determine the searching bounds of the curve parameters. Compared to the conventional least square fitting methods, the proposed scheme provides robust and accurate estimation characteristics even when fewer measurements with a small range of torque loading are taken and used for a data fitting.

1 INTRODUCTION

DC motors are widely used in applications ranging from toys to automobiles. To fulfil the increasing demands for quality and safety assurances, various electrical tests including a performance test, a durability test, a vibration or noise test, etc. are being performed (Soukup, 1989). Among these tests, the load performance test plays an important role in verifying an agreement on the design properties and evaluating the load performance of the motors. Accordingly, the results of the test should have a high reliability and repeatability.

To obtain the load performance curves or characteristic values of DC motors, two methods have been performed. Firstly, the steady-state test was recommended by IEEE Std 113-1985 (1985) and it is widely used in the industrial fields. Readings of the current, voltage, speed, torque, and temperatures should be obtained for six load points equally spaced from 0.25 to 1.5 times the rated load. During the load increases and decreases, two sets of readings are taken at the specified load points, and then their average value is usually chosen as a final result. The main advantage of this method is that the results are more accurate when compared with other methods based on estimation. However, this method has a disadvantage in that the time to accomplish all

the test procedures is excessive, causing the motors to heat up severely owing to the necessity of a wide range of torque loadings. Therefore, it is recommended that temperatures such as the ambient temperature, armature temperature, and field coil temperature, should be measured to compensate for the heating effects in the data.

Secondly, Nakamura, Kurosawa, Kurebayashi and Ueha (1991) proposed the transient response method to estimate the torque-speed characteristics and the torque-efficiency characteristics of an ultrasonic motor. It was assumed that the step response of the speed has first order characteristics. Its main advantage is that the torque-based characteristic curves can be obtained from the transient response of the speed without any measurements of the torque. Therefore, all the procedures can be accomplished within the transient time of the motor, which can avoid the effects arising from the temperature changes. In their experiments, however, the maximum estimation error was about 10%, which may prevent an accurate estimation of the characteristic values.

Our research was motivated by the desire to find a scheme that is faster than the steady-state test and more accurate than the transient response test. To accomplish the objectives, a new curve fitting method based on a real-coded genetic algorithm

(RGA) was proposed and implemented by using the steady-state measurements.

2 LOAD PERFORMANCE OF DC MOTOR

The loop equation for the electrical circuit of a DC motor is

$$E_a = L \frac{dI_a}{dt} + R_a I_a + E_b \quad (1)$$

where E_a , E_b , I_a , R_a , and L are the armature voltage, back-electromagnetic force (EMF), current, resistance, and inductance, respectively. The electromagnetic torque T produced by the motor is represented as

$$T = k\phi I_a = K_t I_a \quad (2)$$

where ϕ and K_t are the magnetic flux and the torque constant, respectively. The back-EMF is proportional to the rotor speed ω by the relation

$$E_b = K_b \omega \quad (3)$$

where K_b is the back-EMF constant.

Under a steady-state operation condition, substituting (2) into (1) for solving T gives

$$T = \frac{K_t}{R_a} E_a - \frac{K_t K_b}{R_a} \omega \quad (4)$$

It can be rewritten as

$$\omega = \alpha_1 T + \alpha_2 \quad (5)$$

Assuming that the speed is zero at the stall torque, T_s , then the steady-state torque-speed equation can be rewritten as

$$\omega = \alpha(T - T_s) \quad (6)$$

where α is obtained by the constrained linear least square regression of number of measurements as

$$\alpha = \frac{\sum_{i=1}^n (T_i - T_s) \omega_i}{\sum_{i=1}^n (T_i - T_s)^2} \quad (7)$$

The mechanical output power is defined by the product of the torque and the speed. Since the speed, as given in (5), is a linear function of the torque, the power equation should be a quadratic form:

$$P_o = T\omega = \beta_1 T^2 + \beta_2 T + \beta_3 \quad (8)$$

Assuming that the power should be zero when the torque or the speed is zero, then we could obtain the following torque-power equation as

$$P_o = \beta T(T - T_s) \quad (9)$$

where β is determined by a constrained 2nd order polynomial fitting based on a least square regression of n number of measurements as

$$\beta = \frac{\sum_{i=1}^n (T_i^2 - T_s T_i) P_i}{\sum_{i=1}^n (T_i^2 - T_s T_i)^2} \quad (10)$$

The efficiency of a DC motor is the ratio between the mechanical output power P_o and the electrical input power P_i determined by the product of the voltage and the current. Assuming that the electrical input power is a linear function of the torque since the input voltage to the motor is constant and the current is proportional to the torque, then we obtain the following torque-efficiency equation:

$$\eta = \frac{P_o}{P_i} = \frac{T\omega}{VI} = \frac{c_1 T^2 + c_2 T + c_3}{c_4 T + c_5} \quad (11)$$

It can be rewritten as

$$\eta = \gamma_1 T + \gamma_2 + \frac{\gamma_3}{T + \gamma_4} \quad (12)$$

Assuming that the efficiency should be zero when the torque is zero:

$$\gamma_4 = -\frac{\gamma_3}{\gamma_2} \quad (13)$$

and the efficiency should be zero at the stall torque:

$$\gamma_3 = \frac{\gamma_1 \gamma_2 T_s + \gamma_2^2}{\gamma_1} \quad (14)$$

then (12) is reduced to the following simplified form:

$$\eta = \gamma_1 T + \gamma_2 + \frac{\gamma_2 (\gamma_1 T_s + \gamma_2)}{\gamma_1 (T - T_s) - \gamma_2} \quad (15)$$

Since the efficiency function in (15) is continuous and differentiable, a nonlinear least square regression could be applied to obtain a best-fit curve.

Figure 1 shows the load performance curves of a DC motor, where the speed, current, power, and efficiency equations are drawn as a function of the torque. As previously mentioned, each curve in the load performance curves can be obtained separately based on the least square fitting methods (LSFMs).

Note that the separately fitted load performance curves are obtained by solving three single-objective optimization problems. However, owing to the existence of signal noises and the electromagnetic changes of the motor in the test, the curves based on the LSFMs do not exactly meet the physical constraint where the speed, power, and the efficiency should all be zero at the stall state. If the stall torque is able to estimated first, then the optimized curves can be easily determined by using (7), (10), and (15). As preliminary experiments to do this, we estimated the stall torque by using a linear fitting of the torque-speed curve. However, this method has a drawback in that the overall goodness-of-fit is significantly affected by the speed measurements. Therefore inaccurate results may be obtained if noisy signals exist in the speed readings and these may be increased if an insufficient range of the measurements are taken.

In the load performance curves, three curves should be optimized simultaneously while satisfying the constraint. However, since no improvement on the goodness-of-fit in any curve is possible without sacrificing at least one of the other curves, the estimation problem forms a multi-objective optimization problem. In this work, we proposed and implemented a GA based fitting method (GAFM) to solve the MOP.

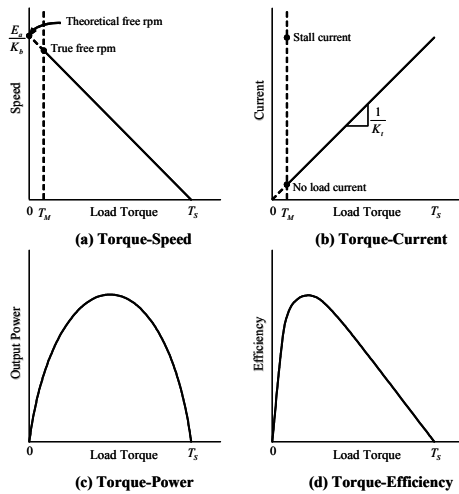


Figure 1: Typical load performance curves of a DC motor.

3 GA-BASED LOAD PERFORMANCE ESTIMATION

A real-coded genetic algorithm (RGA) has been applied to obtain the optimized load performance curves of the DC motors. The RGA is known to

provide accurate solutions even when the system model has a lack of information about the solution and when it has complex constraints. These features have enabled the genetic algorithm to be successfully applied for a parameter identification of induction motors (Nangsue, Pillay and Conry, 1999 and Huang, Wu and Turner, 2002) and a noise-free curve fitting problem (Dului-Barton and Worden, 2003).

3.1 Representation

Let \mathbf{q} be a vector consisting of the characteristic curve parameters, that is,

$$\mathbf{q} = [\alpha, \beta, \gamma_1, \gamma_2, T_s]^T \quad (16)$$

For a notational convenience, we rewrite \mathbf{q} as,

$$\mathbf{q} = [q_1, q_2, \dots, q_5]^T \quad (17)$$

where q_1, q_2 , and q_5 correspond to α, β , and T_s respectively. Searching bounds of the parameters are set as

$$\forall q_i \in [q_i^L, q_i^U]; i = 1, 2, \dots, n \quad (18)$$

where q_i^L and q_i^U denote the lower and upper bounds of q_i , respectively.

In this work we select the bounds of the curve parameters based on the nominal parameters which were obtained by the results of three curve fittings which were performed separately. Since the stall torques estimated by each curve fitting may be different, an average value of the three estimates is used for a nominal parameter of the stall torque. The searching bounds can be determined by setting the k_i in the range [0,1] as

$$\begin{aligned} q_i^L &= (1 - k_i)q_{i0} \\ q_i^U &= (1 + k_i)q_{i0} \end{aligned} \quad (19)$$

where q_{i0} is a i th nominal parameter and k_i is a scaling constant of the parameter bounds. In the following, we represent the curve parameters as real genes of RGA, \mathbf{q}_{ij}^k , where i, j , and k mean the chromosome, population and generation, respectively.

3.2 Fitness Function

The simultaneous optimization of multiple objectives is a challenging subject. In a single

objective case, we can obtain the best solution which is absolutely superior to all the other alternatives. However, in a multiple objectives case, there usually exists a set of solutions, so-called Pareto optimal solutions, which cannot be simply compared with each other because of an incommensurability and conflict among the objectives (Mitsuo and Runwei, 1990). A solution may be best for one objective but the worst for other objectives and no improvement in any objective function is possible without sacrificing at least one of the other objective functions. As a basic and the easiest approach, the weighted-sum approach was used to simplify the multi-objective optimization problem, which assigns weights to each objective function and combines them into a single objective function as

$$F = \sum_k \xi_k f_k(\mathbf{q}) \quad (20)$$

where f_k is one of the objective functions; ξ_k is a weighting factor used to ensure that one objective does not dominate the total fitness, F . However, it is usually very difficult to determine a set of appropriate weights for a given problem.

To apply a fitness function to the multi-objective optimization problems without any use of weighting factors, we introduced a normalized ratio of the errors (NRE) derived from a summed square error. A summed square error of the point-by-point difference between the measured value y_i and its estimated value \tilde{y}_i was defined as

$$S_r = \sum_{i=1}^m e_i^2 = \sum_{i=1}^m (y_i - \tilde{y}_i)^2 = \sum_{i=1}^m y_i^2 - 2 \sum_{i=1}^m y_i \tilde{y}_i + \sum_{i=1}^m \tilde{y}_i^2 \quad (21)$$

Let a NRE, R , be a performance index of the goodness-of-fit as

$$R = \frac{2 \sum_{i=1}^m y_i \tilde{y}_i}{\sum_{i=1}^m y_i^2 + \sum_{i=1}^m \tilde{y}_i^2} \quad (22)$$

then the perfect fitting with $S_r = 0$ occurs at $R=1$. Since the NRE is always less than or equal to 1, its sum can be used to solve the multi-objective optimization problem without the use of weighting factors. We can obtain an optimal solution of the curve parameters \mathbf{q} through maximizing the fitness function of the following form:

$$F(\mathbf{q}) = \frac{R_s + R_p + R_E}{N} \quad (23)$$

where R_s , R_p , and R_E are NREs related to the

speed, the power, and the efficiency curve, respectively and N , the number of single objective functions, is used to normalize the overall fitness function $F(\mathbf{q})$.

3.3 Genetic Operators

In general, GAs include operations such as a reproduction, crossover, and mutation. Selection for a reproduction is a process to choose some individuals of high fitness for breeding. The commonly used roulette wheel selection is adopted in this work. Let $F(\mathbf{q}_i)$ be a fitness function of an individual, \mathbf{q}_i , then the selection probability p_i of p_i is determined as

$$p_i = \frac{F(\mathbf{q}_i)}{\sum_{j=1}^N F(\mathbf{q}_j)} \quad (24)$$

Because the selection method given in (24) is based on a probabilistic selection, then the high fitness chromosomes may not be selected in the next generation. To solve this problem, the best individual in the old population replaces the worst one in the new population.

Crossover provides a mechanism for an individual to exchange genetic information via a probabilistic process. Let the parent, (q_{il}^k, q_{jl}^k) , denote two chromosomes selected randomly for a crossover, then the children, $(q_{il}^{*k}, q_{jl}^{*k})$, are determined by an arithmetic crossover (Mitsuo and Runwei, 1990) based on a vector convex combination as

$$\begin{aligned} q_{il}^{*k} &= \rho q_{il}^k + (1-\rho)q_{jl}^k \\ q_{jl}^{*k} &= \rho q_{jl}^k + (1-\rho)q_{il}^k \end{aligned} \quad (25)$$

where ρ is a random number uniformly distributed in the range $[0,1]$.

Even though a selection and crossover operation effectively search and recombine a possible solution, occasionally they may lose potentially useful genetic information. The role of a mutation operation is to mutate to certain genes of the individuals and recover the lost useful information. In this work, we used the dynamic mutation operator [10] designed for a fine-tuning with a high precision. If a real gene q_{ij}^k in a chromosome vector, $\mathbf{q}_i^k = [q_{i1}^k, \dots, q_{ij}^k, \dots, q_{in}^k]$ is selected for a mutation, then the result of the mutation operation is obtained as

$$\mathbf{q}_i^{rk} = [q_{i1}^k, \dots, q_{ij}^{rk}, \dots, q_{im}^k] \quad (26)$$

where q_{ij}^{rk} is given by

$$q_{ij}^{rk} = \begin{cases} q_{ij}^k + \delta(k, q_{ij}^U - q_{ij}^k), & \text{if } \tau = 0 \\ q_{ij}^k - \delta(k, q_{ij}^k - q_{ij}^L), & \text{if } \tau = 1 \end{cases} \quad (27)$$

where k is the number of generations performed, and τ is the random number with 0 or 1. $\delta(k, y)$ is determined by

$$\delta(k, y) = y[1 - r^{(1-t/T)^b}] \quad (28)$$

where r is the random number with the range of $[0,1]$, T is the number of preset maximum generations, b is a preset constant for determining the degree of a non-uniformity. The dynamic mutation operator given in (27) makes it possible to search the space uniformly with a small k and very locally with $k = T$.

4 EXPERIMENT AND DISCUSSION

4.1 Measurement System

The proposed curves fitting method based on GA has been verified by experimental studies. A measurement system, shown in Figure 2, was developed by using off-the-shelf components such as hysteresis brakes, a torque transducer with an inductive proximity sensor, a power analyzer, and power supplies. Two serially connected hysteresis brakes could produce a precise load torque up to 243kgf-cm, which is independent of the shaft speed. The inductive proximity sensor produces an open-collector speed output which is transformed into an analog signal by a frequency to voltage converter (FVC). A torque transducer TM210 manufactured by Magtrol is able to measure a torque up to

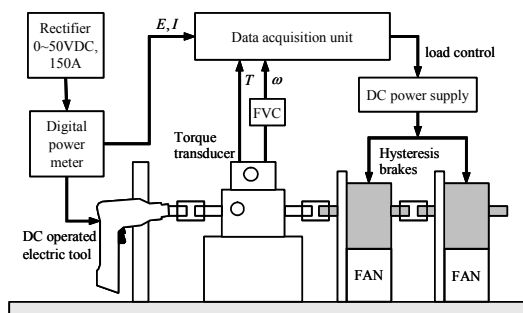


Figure 2: Schematic of the measurement system.

490kgf-cm. A power analyzer WT200 manufactured by Yokokawa was used to measure the voltage, the current, and the electric power. A data acquisition board was installed in a PC to measure the analog signals and it was also used to control the load torque. The developed algorithm was implemented by means of C++ language.

4.2 Results and Discussion

To evaluate the effectiveness of the proposed GA-based fitting method (GAFM), we conducted load performance tests on several DC operated electric tools consisting of a permanent magnet DC motor, a reduction gear, etc. Throughout the following experiments, we used a population size of 100, a maximum generation of 200, a crossover probability of 0.9, and a mutation probability of 0.1 as control parameters of the GA. Since the GA sets the initial real genes in a random manner, it may seek out different maxima depending on the initial conditions. Therefore, we selected the final parameters as those with the highest fitness value from 10 different runs. It takes less than 1 second to execute all 200 generations.

We first investigated how various searching bounds of the curve parameters affect the performance of the proposed scheme. As previously mentioned, the searching bounds were determined by setting a k_i of (19) in the range of 0 to 1, which indicates that the searching spaces cover the $\pm k_i \times 100\%$ range of the nominal parameter q_{i0} . Table 1 provides the nominal parameters and their estimated values corresponding to 5 different k_i s. Even though the searching bounds are considerably enlarged by up to $\pm 50\%$ of the nominal parameters, the estimates are converged to almost the same values. This shows the fact that the GAFM can estimate curve parameters with small errors without the need for good initial estimates and that the enlarging searching spaces have only a marginal impact on the estimation accuracy.

Next, we investigated whether the number of data sets used for a curve fitting affects the goodness-of-fit. Figure 3 shows the load performance curves obtained based on the conventional least square fitting methods (LSFMs) and GAFM. In the LSFMs, the curves are obtained by solving three single curve (or objective) optimization problems separately, while the GAFM optimizes the multiple curves simultaneously. Therefore, it is natural that the goodness-of-fit of LSFMs outperforms that of the GAFM if we are

Table 1: Parameter estimation results corresponding to 5 different searching spaces.

Parameters	Nominal values	Estimated values						
		$k_i = 0.1$	$k_i = 0.2$	$k_i = 0.3$	$k_i = 0.4$	$k_i = 0.5$	mean	Stdev.
α	-2.0	-1.989	-1.987	-1.985	-1.988	-1.990	-1.988	1.924E-3
β	-0.0198	-0.02039	-0.02036	-0.02034	-0.02038	-0.02040	-0.02037	2.408E-5
γ_1	-0.325	-0.3226	-0.3220	-0.3219	-0.3227	-0.3228	-0.3224	4.307E-4
γ_2	92.35	92.81	92.69	92.70	92.87	92.86	92.79	8.648E-2
T_s	272.3	272.3	272.5	272.7	272.4	272.3	272.5	0.1658

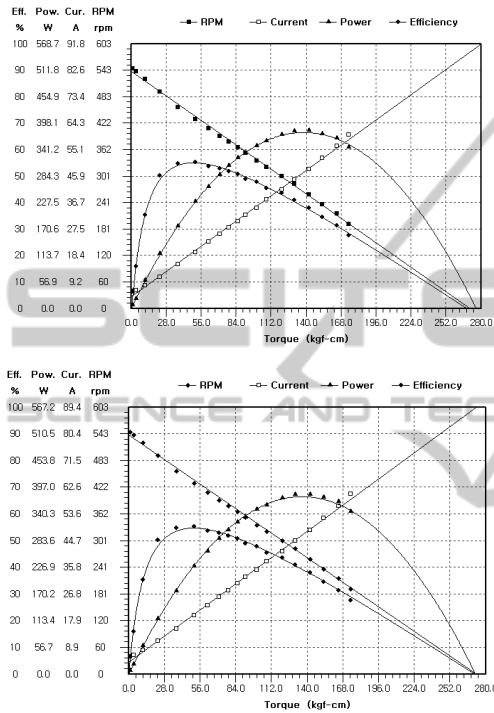


Figure 3: Load performance curves obtained with 19 data sets based on LSFMs (top) and GAFM (bottom).

concerned about each curve separately. However, owing to the existence of the signal noises and electromagnetic changes of the motor in the test, the curves based on LSFMs do not meet the physical constraints in that the three curves meet in the stall torque. Therefore a sufficient range of the measurements is required to apply LSFMs to fit the load performance curves. On the contrary, in the case with 5 data sets, GAFM is superior to the LSFMs from the aspect of the physical characteristics and an accuracy. This is due to the fact that the constraints of the curves in the GAFM can help to improve the goodness-of-fit. Furthermore, the curves shown in Figure 4 were obtained by using the data sets whose maximum load torque is about 13.4% of the stall torque, which are almost the same results as shown in Figure 3 whose maximum load torque is about 62% of the full load. This indicates that a sufficient accuracy

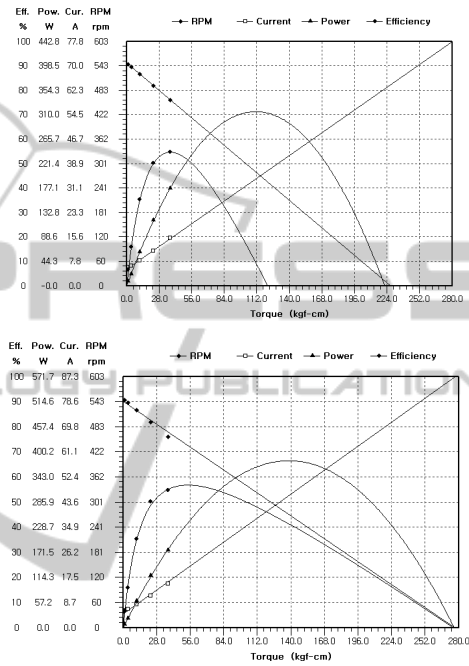


Figure 4: Load performance curves obtained with 5 data sets based on LSFMs (top) and GAFM (bottom).

can be achieved with a small number of measurements and/or a small range of torque loadings in the GAFM.

5 CONCLUSIONS

In this paper, we solved the load performance estimation problem of DC motors through an optimal fitting of multiple curves with constraints. Although the load performance curves could be fitted separately by using the conventional least square curve fitting methods, a wide range of measurements is required to improve the goodness-of-fit. As an alternative, we proposed a new curve fitting method based on a genetic algorithm. In the method, a normalized ratio of the errors was used to optimize the multi-objective functions without the use of weighting factors and the searching bounds of

the curve parameters could be automatically determined by using the nominal parameters. From the experimental studies on several DC operated electric tools, we concluded that the proposed and implemented GAFM could be applied to obtain robust and reliable load performance curves of DC motors even when fewer measurements with a small range of the torque loading are taken and used for a data fitting.

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