A Variable Structure Controller for a Class of Hyper-redundant Arms

Decebal Popescu[1], Nirvana Popescu[1], Mircea Ivanescu[2] and Dorin Popescu[2]
[1]*Department of Computing Science, POLITEHNICA University of Bucharest, Splaiul Independentei, Bucharest, Romania*
[2]*Department of Mechatronics, University of Craiova, A.I. Cuza, Craiova, Romania*

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Abstract: The paper treats the control problem of a class of hyper-redundant robot constituted by a chain of continuum segments. The technological model basis is a central, long and thin, highly flexible and elastic backbone. The driving system is a decoupled one. The main parameters of the arm control are determined by the curvature and curvature gradient. The dynamic model is inferred. A sliding mode control system is used in order to achieve a desired shape of the arm. The stability of the closed loop control system is proven. Numerical simulations are also provided to verify the effectiveness of the presented approach.

1 INTRODUCTION

The goal of this paper is to implement a control system for a class of hyper-redundant robots with continuum components. The tentacle robots represent one of the most attractive domains of robotics during the last decades. The control of these systems is very complex and several researchers have tried to offer solutions as it will be further discussed. In (Chirikjian, 1990), (Robinson, 1999), (Gravagne, 2000), (Gravagne and Walker, 2000), (Jones and Walker, 2006) the kinematic models were analysed, based on a "backbone curve" that captures the robot's macroscopic geometric features. In (Mochiyama, 1999), (Hirose and Umetani, 1976) the problem of controlling the shape of a robot with two-degree-of-freedom joints was also investigated using spatial curves. A controller for continuum robots was developed by using neural network feed-forward components in (Braganza, 2007),. Other researchers derived a new kinematic model by using the differential geometry (Walker, 1999), (Kapadia, Walker and Dawson, 2009), (Hannan, 2005) or introduced a real-time controller for continuum robots (Jones, 2006). In (Kapadia, 2009) it was proposed a sliding controller for extensible robots. Other papers (La Spina, 2007), (KeJun, 2009), (Webster and Jones, 2010) have developed several biomimetic prototypes with undulatory action.

Our paper treats the control problem of a class of light weight hyper-redundant robots. The technological model basis is a central, long and thin , highly flexible and elastic backbone. The driving system is a decoupled one. The main parameters of the arm control are determined by the curvature and curvature gradient. The dynamic model is inferred. A sliding mode control system is used in order to achieve a desired shape of the arm.

The paper is structured as follows: section 2 presents technological structure, section 3 analyses the dynamic model, section 4 treats the control algorithm, section 5 verifies the control laws by computer simulation and section 6 contains the conclusions.

2 TECHNOLOGICAL ARM

The technological model basis is an arm with a distributed mass. The 3D model basis from Fig 1 consists of a central, long and thin, highly flexible and elastic backbone. It is made from homogeneous materials, the bending represents the main motion and we neglect the deformations of axial tension/compression and shear. The arm is divided in several segments, each segment having its own driving system. The motion of the arm, the bending, is determined by antagonistic cables (tendons) attached to the terminal point of each segment and a DC motor driving system. These cables develop the driven torques τ_i, i=1, 2…

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The cables ensure an independent bending for each segment so that the segment driving control is a decoupled one.

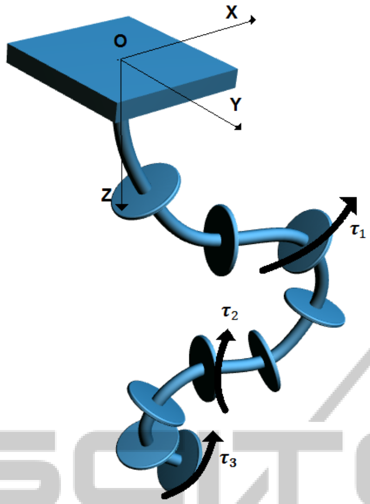

Figure 1: The technological arm.

3 DYNAMIC MODEL

We consider a hyper-redundant arm constituted by a serial connection of a number of continuum arm segments, with equal lengths L (Fig 2). For the segment i, the curvature is defined by

$$\kappa_{0i}(0)=\frac{d\omega^i(s=0)}{ds}=\begin{bmatrix}\frac{d\theta^i(s=0)}{ds}\\ \frac{dq^i(s=0)}{ds}\end{bmatrix}$$
$$\kappa_{1i}(L)=\frac{d\omega^i(s=l)}{ds}=\begin{bmatrix}\frac{d\theta^i(s=L)}{ds}\\ \frac{dq^i(s=L)}{ds}\end{bmatrix} \quad (1)$$

and we assume that the continuity of the arm curvature requires the following segment boundary conditions

$$\kappa_{0\,i}(0)=\kappa_{1\,i\text{-}1}(L) \quad (2)$$

with

$$\kappa_{0\,0}(0)=0 \quad (3)$$

Using the same procedure as in (Gravagne, 2000), we obtain the partial differential equations (PDE) of the arm segment,

$$I_\rho\ddot{\omega}^i = EI\,\omega_{ss}{}^i - B\,\dot{\omega}^i + h^i(\omega^i) \quad (4)$$

where $\omega^i=\omega^i(t,s)$, $\omega^i=(\theta^i,q^i)^T$, $s\in\Omega$, $\dot{\omega}^i=\partial\omega^i/\partial t$, $\omega_s=\partial\omega^i/\partial s$, $\mathbf{I_\rho}$ is the rotational inertial density matrix, $\mathbf{I_\rho}=\mathrm{diag}\left(I_{\rho_\theta}, I_{\rho_q}\right)$, $I_{\rho_\theta}=I_{\rho_q}=I_\rho$, B is the equivalent damping matrix of the arm, $\mathbf{B}=\mathrm{diag}(b_\theta,b_q)$, $b_\theta=b_q=b$ and h represents the nonlinear component vector determined by gravitational components, $h^i=(h_1{}^i,h^i{}_2)^T$. The initial and boundary conditions are

$$\omega^i(0,s)=\omega^i{}_0(s) \quad (5)$$
$$EI\,\omega_s^i(t,l)=\tau_i(t),\qquad \omega_s^i(t,0)=0 \quad (6)$$
$$\omega_s^0(t,0)=0 \quad (7)$$
$$\omega^0(t,0)=0 \quad (8)$$

where τ_i is the equivalent moment generated by the forces F_i at the end of the arm segment i, $\tau_i(t)=F_i.r$,r is the radix of the moment,

$$\tau_i=\begin{bmatrix}\tau_{\theta i}\\ \tau_{qi}\end{bmatrix},\ F_i=\begin{bmatrix}F_{\theta i}\\ F_{qi}\end{bmatrix} \quad (9)$$

The dynamic model of the arm can be expressed in terms of the curvature $\kappa_{1\,i}(t,L)$ as

$$I_\rho\frac{L}{2}A\,\ddot{\kappa}(t)=-b\frac{L}{2}A\dot{\kappa}(t)-\frac{EI}{L}C\kappa(t)+\frac{1}{L}\tau(t)+h(\kappa) \quad (10)$$
$$\kappa(0)=\kappa_0 \quad (11)$$

where

$$\kappa=(\kappa_{11},\kappa_{12},\ldots\kappa_{1n})^T \quad (12)$$

with

$$\kappa_i(t)=\kappa_{1\,i}(t,L) \quad (13)$$

represents the new state vector, $\kappa\in R^n$, τ denotes the general input of the arm,

$$\tau=(\tau_1,\tau_2,\ldots\ldots,\tau_n)^T \quad (14)$$

the linear components are defined by the (n x n) matrices

$$A=\begin{bmatrix}1\ 0\ 0\ 0\ .\ .\ .\ .0\\ 2\ 1\ 0\ 0\ .\ .\ .\ 0\\ 2\ 2\ 1\ 0\ .\ .\ 0\\ \ldots\ldots\ldots\ldots\\ 2\ 2\ 2\ 2\ 2\ \ 1\end{bmatrix},\quad C=\begin{bmatrix}0\ 0\ 0\ .\ .\ .\ 0\\ 1\ 0\ 0\ .\ .\ \ 0\\ 0\ 1\ 0\ .\ .\ \ 0\\ .\ .\ .\ .\ .\ .\ .\\ 0\ 0\ 0\ .\ .1\ 0\end{bmatrix} \quad (15)$$

and the nonlinear component is determined by

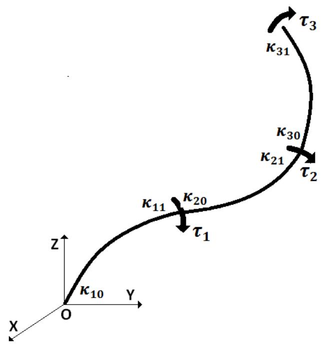

Figure 2: The backbone model.

$$h=(h^1,h^2,\ldots,h^n)^T \tag{16}$$

where $h^i = h^i(\kappa_i)$ and satisfices the inequality (Popesu,2014)

$$\|h(\omega)\| \leq M\ \|\kappa\| \tag{17}$$

4 CONTROL ALGORITHM

We consider a desired state $\kappa^d, \kappa^d \in R^n$, that satisfies (10) with initial condition (11) and we define by

$$e(t)=\kappa^d-\kappa(t) \tag{18}$$

the error variable, $e \in R^n$.

The control problem consists of finding the control law $\tau(t)$, on the boundary s=L of each segment such that the error converges to zero.

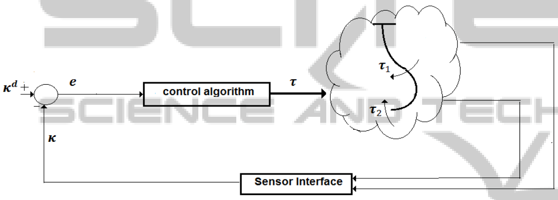

Figure 3: The control system.

The main idea of the control system is based on the variable structure control associated with the special properties of the physical system. We propose a controller with a boundary torque variable structure control. Let us define by S (t) the sliding surface, associated with the model (10) and the error (18)

$$S(t)=e(t)+\sigma\,\dot{e}(t) \tag{19}$$

where

$S=(S_1, S_2 \ldots S_n)^T$, $\sigma = \mathrm{diag}\,(\sigma_1, \sigma_2 \ldots \sigma_n)$, σ_i are positive constants.

Theorem 1. For the system described by (10) with the initial conditions (11), if the variable structure controller is given by

$$\begin{aligned}\Delta\tau &= -K_1\mathrm{sgn}(S) - K_2Ce\ \mathrm{sgn}\left(S^TK_2Ce\right)\\ &-K_3A\left(I_\rho\sigma^{-1} - bI\right)\dot{e}\ \mathrm{sgn}\left(S^TA\left(I_\rho\sigma^{-1} - bI\right)\dot{e}\right)\end{aligned} \tag{20}$$

$$K_1 > 0 \tag{21}$$

$$K_2C - M\mathbf{I} - \frac{EI}{L}C > 0 \tag{22}$$

$$\left(\frac{K_3}{L} - \frac{L}{2}I\right)\left(I_\rho A\sigma^{-1} - b\ \mathbf{I}\right) > 0 \tag{23}$$

where $K_j = \mathrm{diag}\left(k_{j1}, k_{j2},..,k_{jn}\right)$, j=1, 2, 3 represent the control coefficients, then the motion of the system will reach the sliding line S=0 and then keep it there , where $\Delta\tau$ is defined by

$$\tau = \tau^d - \Delta\tau \tag{24}$$

and τ^d represents the torque vector on the desired position κ^d that satisfies the equations,

$$-\frac{EI}{L}C\,\kappa^d + \frac{1}{L}\tau^d + h\left(\kappa^d\right)=0 \tag{25}$$

Proof. See Appendix

5 NUMERICAL SIMULATIONS

Consider a dynamic model of a vertical hyper-redundant arm, with two arm segments, with the length of the segment L=1 m, the rotational inertial density I_ρ= 0.001 kg m^2, the bending stiffness EI = 0.1Nm^3, the viscous coefficient b_θ = 0.06 Nms/rad. These constants are scaled to realistic ratios for a long thin arm. The initial and boundary conditions are: $\theta_0(s) = 0$, $\theta_s(t, 0)=0$, $EI\theta_s(t, L) = \tau$, where τ is the torque applied at the top of the arm segment. We consider that the uncertain term $h(\theta)$ is defined by the gravitational components. For the characteristic values of the arm parameters, $\rho_b = 0.8\ kg/m, g = 10\ m/s^2, A = 4.10^{-4}m^2$, associated with this thin long arm, the inequality (17) is satisfied for M = 10. The arm is simulated by a chain of vertebrae.

We simulated two motions. The first motion is a XOZ plan motion, the arm (the both segments) is moving toward the desired position defined in term of curvature, $\kappa_{\theta d} = -\frac{\pi}{14}$. A control law (20) with the controller gains, $k^i_{1\theta}$= 20, $k^i_{2\theta}$ = 8, $k^i_{3\theta}$ = 3 is used, where $k^i_{1\theta}$, $k^i_{2\theta}$, $k^i_{3\theta}$, i=1, 2…., verifies the conditions (21) - (23). The motion of the arm, several intermediary positions and final positions are illustrated in Figure 4.

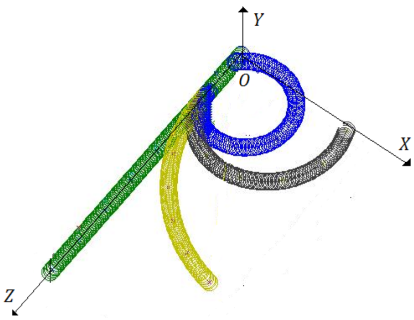

Figure 4: A XOZ plan motion.

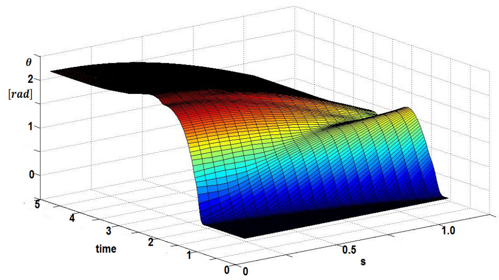

Figure 5: The state evolution for the plan motion.

The time evolution can be analysed if we use the distributed parameter dynamic model described by PDE (4) with boundary conditions defined by the control law of torque (20).The state variable evolution is presented in Fig 5.

A 3D motion of a 2-segment arm is presented in Figure 6.The desired position is defined by $\kappa_{1\theta d}=\kappa_{2\theta d}=-\pi/14$, $\kappa_{1qd}=\kappa_{2qd}=-\pi/24$. The control parameters were selected as $k_{1\theta}{}^{i}=k_{1q}{}^{i}=20$, $k_{2\theta}{}^{i}=k_{2q}{}^{i}=8$, $k_{3\theta}{}^{i}=k_{3q}{}^{i}=3$, $i=1,2$ … The both segments bend with the same curvature. The error evolution is presented in Figure 7. In Figure 8 is presented a new motion of this arm, with a different desired position of each arm segment:

$\kappa_{1\theta d}=-\pi/14$, $\kappa_{1qd}=-\pi/24$, $\kappa_{2\theta d}=\pi/14$, $\kappa_{2qd}=-\pi/24$

The good performances of the proposed control algorithm are concluded from the graphics.

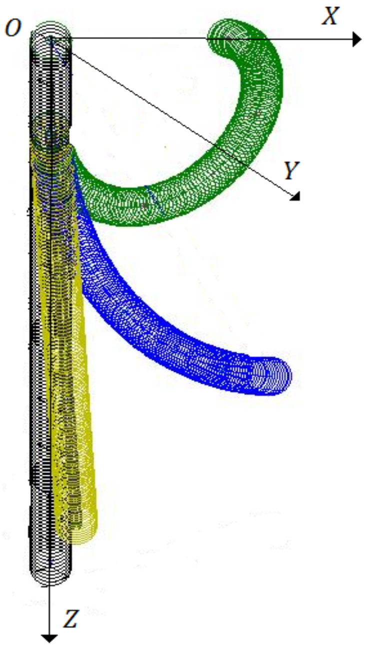

Figure 6: A 3D motion: $\kappa_{1\theta d}=\kappa_{2\theta d}=-\frac{\pi}{14}$, $\kappa_{1qd}=\kappa_{2qd}=-\frac{\pi}{64}$.

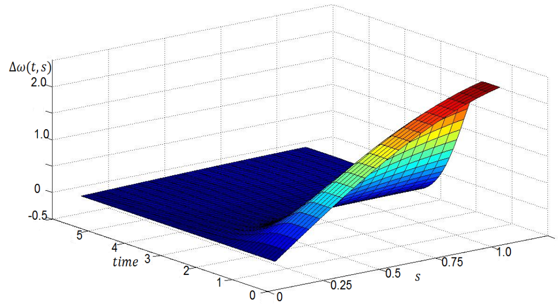

Figure 7: The error evolution for the 3D motion.

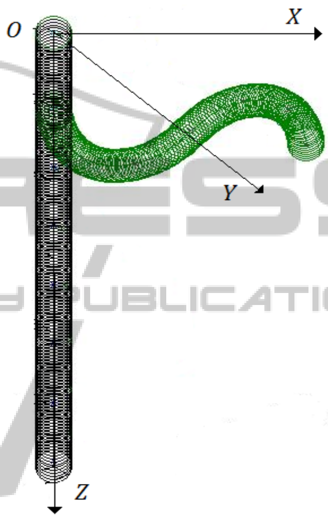

Figure 8: A 3D motion $\kappa_{1\theta d}=-\frac{\pi}{14}$, $\kappa_{1qd}=-\frac{\pi}{64}$, $\kappa_{2\theta d}=\frac{\pi}{14}$, $\kappa_{2qd}=-\frac{\pi}{64}$.

6 CONCLUSIONS

The paper presents the control problem of a class of hyper-redundant robot constituted by a chain of continuum segments. The technological model basis is a central, long and thin, highly flexible and elastic backbone. The driving system is a decoupled one. The main parameters of the arm control are determined by the curvature and curvature gradient. The dynamic model is inferred in terms of the curvature. A sliding mode control system is used in order to achieve a desired shape of the arm. The stability of the closed loop control system is proven. Numerical simulations are also provided to verify the effectiveness of the presented approach.

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APPENDIX

From (10) - (11), the error dynamics will be described by

$$I_\rho \frac{L}{2} A\, \ddot{e}(t) = -b\frac{L}{2} A\dot{\mathbf{e}}(t) - \frac{EI}{L} Ce(t) + \frac{1}{L}\boldsymbol{\Delta\tau}(t) + \boldsymbol{\Delta h}(\boldsymbol{\kappa}^d) \tag{A.1}$$

$$\mathbf{e(0) = e_0} \tag{A.2}$$

$$\Delta\mathbf{h} = \mathbf{h} - \mathbf{h}(\boldsymbol{\kappa}^d) \tag{A.3}$$

Let us consider the Liapunov function

$$V(t) = \frac{1}{4} l\, S^T(t) I_\rho A \sigma^{-1} S(t) \tag{A.4}$$

The time derivative of $V(t)$ is given by

$$\dot{V} = \frac{1}{2} S^T I_\rho A \sigma^{-1} \dot{S} \tag{A.5}$$

where in order to simplify the notation, the variable t is omitted.

$$\dot{V} = \frac{1}{2} S^T I_\rho A \sigma^{-1} (\dot{\mathbf{e}} + \boldsymbol{\sigma} \ddot{\mathbf{e}}) \tag{A.6}$$

By evaluating (A.6) along with the solutions of (A.1), it turns out that

$$\begin{aligned} \dot{V} = \frac{L}{2} S^T \left(I_\rho A \sigma^{-1} - Ab \right) \dot{\mathbf{e}} - \frac{EI}{L} S^T C \mathbf{e} + S^T \boldsymbol{\Delta} \mathbf{h} \\ + \frac{1}{L} S^T \boldsymbol{\Delta \tau} \end{aligned} \tag{A.7}$$

Now, substituting the control $\Delta\tau$ from (20), after simple additional manipulations, we obtain

$$\begin{aligned} \dot{V} \leq -\frac{1}{L} K_1 \|S\| - \|S^T\| \left(K_2 C - M\mathbf{I} - \frac{EI}{L} C \right) \|\mathbf{e} \\ - \|S^T\| \left(\frac{K_3}{L} - \frac{L}{2} \mathbf{I} \right) (I_\rho A \sigma^{-1} - b\, \mathbf{I}) \|\dot{\mathbf{e}}\| \end{aligned} \tag{A.8}$$

Using the conditions (21) - (23), this inequality can be rewritten as

$$\dot{V} \leq -\frac{1}{L} K_1 \|S\| \tag{A.8}$$