Development of a Model based on Evaluation Considering Explicit and Implicit Element in Multiple Criteria Decision Making

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Abstract: The Analytic Hierarchy Process (AHP) is a decision-making method for smoothly managing problems, criteria, and alternatives. AHP can be used to respond to multiple criteria, and allows for the quantification of subjective human judgments, as well as objective evaluations. In a classical AHP, a decision-maker derives a list of priorities by consciously comparing criteria and alternatives in order to deriving a comprehensive evaluation. However, when the number of criteria increases, the problem also becomes complicated and the subjective judgment of the decision-maker tends to be clouded by ambiguity and inconsistency. As the solution, this study proposes a method whereby latent elements are extracted from the data given by the decision-maker, and an evaluation is made from a different aspect based on the extracted elements. This allows for the construction of a model in which a decision is made from both explicit and implicit elements by making a final synthesis of the results obtained using the conventional method as well as the evaluation obtained using the method proposed in this study. As a result, we can conclude that it is possible to make a decision that is not affected by the ambiguity or inconsistency of the decision-maker.

1 INTRODUCTION

The analytic hierarchy process (AHP) (Saaty, 1980) is well known as the procedure to solve multiple criteria decision-making problems. AHP is the method which quantifies human's subjective judgments, and makes a decision by combining them and system approach in the analysis of problem. AHP is used in a variety of multiple-choice situations such as economic problems, management problems, medical issues, energy problems, educational problems and city planning.

When making a decision, having a large number of various criteria and alternatives tends to complicate the problem and make it impossible to arrive at the most appropriate decision. One problem is that the hierarchal structure becomes complicated. When creating a hierarchal structure, it is necessary to set independent items in the criteria. If each criterion is not independent, it is necessary to define a multi-level hierarchy, such as AHP inner-dependence method (Saaty and Takizawa, 1986) or dominant AHP (Kinoshita and Nakanishi, 1997)(Kinoshita and Nakanishi, 1998). However, even in them it is impossible to account for all of the implicit dependencies between the criteria at a level beneath the decision-maker's awareness.

Another problem is that inconsistencies may occur in choices when criteria or alternatives must be evaluated using subjective human judgment. Ambiguities and inconsistencies tend to occur more often in human judgment when the number of criteria and alternatives increase. The work involved in making a pairwise comparison therefore becomes unmanageable and consistency consequently suffers. As a result, the reliability of the final evaluation decreases, and it is difficult to make the best decision. In order to resolve this problem, the absolute measure method on AHP (Saaty, 1986) has been proposed. The method is effective in case containing too many alternatives and can avoid the rank reversal problem. However using the the method, the results often lose reliability because the comparison matrix does not always have sufficient consistency.

This study proposes a method whereby implicit elements are extracted from the data of a decision-maker's judgments using principle component analysis (PCA) (Jolliffe, 2002), and new evaluation is derived based on them. The conventional method involves the decision-maker coming to a decision based on explicit elements. The implicit elements ex-

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tracted by PCA are as new criteria. Then, to utilize new criteria enables to derive new evaluation from a different aspect. Furthermore, a final evaluation can be made by synthesizing the explicit evaluation and the implicit evaluation by the proposed method, thereby constructing a decision-making model that is not affected by ambiguity or inconsistencies of the decision-maker.

There are researches to examine about best method by comparing the evaluation by AHP with the evaluation by PCA (Kim, 2006)(Wu et al., 2011). On the other hand, there is research by which PCA is applied to the decision-making method (Lee et al., 2010). However, our approach is to integrate the evaluation by latent elements extracted by applying PCA into AHP values which the decision-maker scored subjectively. It is applied only to the process in which the absolute measure method because PCA is an effective technique to normally-distributed data. Then, our approach enables the decision-makers to achieve clearer result.

2 BASIC CONCEPT

2.1 Analytic Hierarchy Process using Absolute Measurement

The AHP is a technique used for dealing with problems which involve the consideration of multiple criteria simultaneously. It is based on the principles of decomposition structures, comparative judgments, and synthesis of priorities. Comparative judgments are necessary to perform the pairwise comparisons of criteria and alternatives. However, in case containing too many alternatives, it is burdensome for decision-maker to draw pairwise comparison in alternatives. It produces sometimes bad consistency. Saaty proposed an absolute measure method on AHP to solve the problem (Saaty, 1986). The difference between the method and the conventional relative measurement is in the procedure of scoring the alternatives corresponding to criteria. The method is adopted that indirect comparison. A decision-maker evaluates alternatives using absolute measurement by linguistic scales as ”very good”, ”good” and etc. The evaluation value of linguistic scale is acquired by pairwise comparison of criteria as in Table 1. Table 2 is a example of evaluation values for linguistic scales. They are derived from eigenvector calculated by the ratio of a linguistic scale. In our proposal model, the absolute measurement method is adopted.

Table 1: A example of pairwise comparison of linguistic scale.

	very good	good	common	bad
very good	1	2	5	7
good	1/2	1	3	5
common	1/5	1/3	1	2
bad	1/7	1/5	1/2	1

Table 2: Evaluation value about Table 1.

linguistic scale	weight
bad	0.120
common	0.209
good	0.569
very good	1.000

2.2 Principal Component Analysis (PCA)

PCA is a data representation method and is a kind of multivariate analysis. It can extract new indexes without correlation from each data and analyze weight of data in each element. Moreover, new indexes can be extracted from a few of data set. This is achieved by transforming to a new set of variables, the principal components, which are uncorrelated, and which are ordered so that the first few retain most of the variation present in all of the original data.

Suppose that matrix A is made from the result of a questionnaire filled out by decision-maker in Figure 1, and that S_a is a covariance matrix of A, and that $\lambda_k (k = 1,2,\ldots,n)$ are a eigenvalue of S_a. n is number of questionnaire items. Suppose that λ_1 is the largest eigenvalue, and v_1 is the corresponding eigenvector. It can be shown that for the second,third, . . . , nth principal component,the vectors of coefficients $\boldsymbol{v}_2$,$\boldsymbol{v}_3$, . . . ,$\boldsymbol{v}_n$ are the eigenvectors corresponding to λ_2,λ_3, . . . , λ_n. The vectors $\boldsymbol{v}_k$ are principal components and each of them is new indicator uncorrelated.

$$\boldsymbol{v}_k^T = \lfloor v_{k1} v_{k2} \ldots v_{kn} \rfloor \tag{1}$$

where $\boldsymbol{v}_k$ corresponds to kth column of new indicators in Figure 1.

The principal component score z_{ki} of ith alternative corresponding new indicator $\boldsymbol{v}_k$ is given as

$$z_{ki} = v_{k1}a_{i1} + v_{k2}a_{i2} + \cdots + v_{kn}a_{in} \tag{2}$$

where a_{in} is score of alternative A_i corresponding criterion C_n.

Criterion \ Alternative	Items C1	C2	C3	C4
A1	5	3	7	3
A2	5	7	7	3
A3	3	3	1	5
A4	3	5	1	1

$$A = \begin{bmatrix} 5 & 3 & 7 & 3 \\ 5 & 7 & 7 & 3 \\ 3 & 3 & 1 & 5 \\ 3 & 5 & 1 & 1 \end{bmatrix}$$

Application of PCA

	New indicators (Principal Component) 1st PC	2nd PC	3rd PC	4th PC
C1	-0.9540	-0.2803	-0.1064	0.0001
C2	-0.5622	0.6378	0.5265	0.0000
C3	-0.9540	-0.2803	-0.1064	-0.0001
C4	0.2035	-0.8661	0.4566	0.0000
contribution rate	54.4%	32.8%	12.7%	0.1%

Figure 1: Example of a matrix which is made from questionnaire data and new indicators which is made from the matrix by PCA.

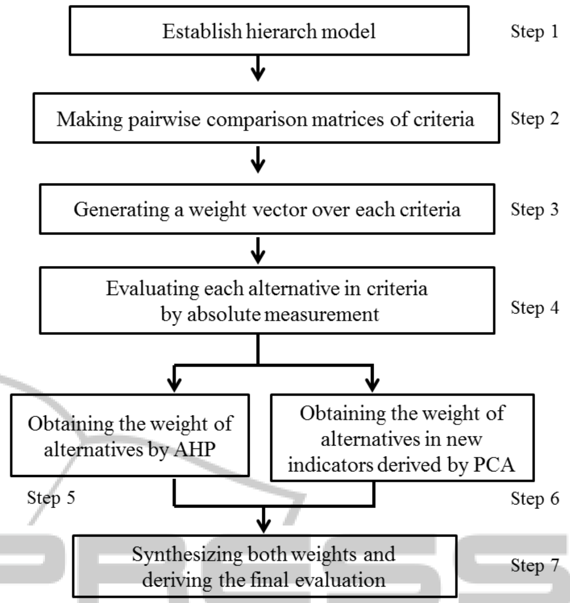

Figure 2: The flowchart of proposed decision-making model.

3 PROPOSED MODEL

We propose a method by adopting PCA. The proposed method can extract new criteria from the absolute evaluation values given by a decision-maker and evaluate based on them. We call a result of it the evaluation based on implicit elements. In contrast, we call the conventional absolute measure method on AHP the evaluation based on explicit elements. Moreover, we develop a new approach to derive clearer priority by synthesizing implicit and explicit evaluation.

Figure 2 shows the flowchart of proposal model for considering both evaluations. In Figure 2, through Step 1 to Step 5 is the same process as the conventional AHP. The evaluation X_{AHP} in Step 5 is given as

$$X_{AHP} = S\boldsymbol{w} \tag{3}$$

$$X_{AHP_i} = \sum_{k=1}^{n} s_{ki} w_k$$

where matrix S consists of scores s_{ki} of each alternative about criteria in step 4, and vector $\boldsymbol{w}$ is the weights of criteria in Step 3. X_{AHP_i} is regarded as explicit evaluation of ith alternative..

We derive the implicit evaluation in Step 6. At first, PCA is applied to the matrix S in order to acquire new indicators $v_1, v_2, \ldots, v_m$. Number of new indicators becomes less than half that of original criteria by adopting until 90% of contribution rate. Moreover, in the process of implicit evaluation, the decision-maker does not need to be conscious of dependency among criteria because $\boldsymbol{v}_k$ is independent component.

Next, the weight w'_k of each indicator v_k as new criteria is acquired by pairwise comparison. Here, vector $\boldsymbol{w}' = (w'_1, w'_2, \ldots, w'_m)$ is a weight vector of new criteria. Based on Eq.(2), a vector $\boldsymbol{z}_k = (z_{k1}, z_{k2}, \ldots, z_{kn})$ made up of kth principal scores is attained as

$$\boldsymbol{z}_k = S \cdot \boldsymbol{v}_k (k = 1, 2, \ldots, m). \tag{4}$$

Finally, the priority of alternatives based on implicit elements is acquired on the following Eq.(6).

$$X_{PCA} = Z\boldsymbol{w}' \tag{5}$$

where matrix $Z = (\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_m)$. The implicit evaluation value of ith alternative is given as

$$X_{PCA_i} = \sum_{k=1}^{m} z_{ki} w'_k \tag{6}$$

Final evaluation is obtained in Step 7. We define final evaluation of ith alternative to synthesize Eq.(3) and Eq.(6) as

$$X_i = X_{AHP_i} \cdot X_{PCA_i} \tag{7}$$

4 APPLICATION

Suppose that a family is looking for the new house. After visiting much real estate, eight houses remained as possible houses for new life. We shall call them A, B, C, D, E, F and G. A decision-maker has to decide which house is the best. The decision-maker has identified the following decision criteria. Access, Price, Safety, Comfort, Location, Width, Equipment and Appearance (hereinafter referred to as ”Ac”, ”Pr”, ”Sa”, ”Co”, ”Lo”, ”Wi”, ”Eq” and ”Ap”). We apply our procedure to the above mentioned example, following the steps in flowchart in Figure 2.

4.1 Evaluation based on Explicit Elements

The decision-maker constructs a evaluation matrix with respect to decision criteria in Step 2. It is performed through a pairwise comparison shown in Table 3. The values for pairwise comparison in Table 3 is scored on 9-point measurement at the same as the conventional AHP.

In Step 3, a weight vector $\boldsymbol{w}$ of criteria is acquired as an eigenvector for a maximum eigenvalue of the matrix composed of Table 3, given as

$$\boldsymbol{w} = (0.156, 0.233, 0.269, 0.138, 0.081, 0.067, 0.031, 0.025)^T. \tag{8}$$

In Step 4, the decision-maker evaluates each alternative about criteria. The alternatives are scored by evaluation values acquired according to the linguistic scales provided for each criterion as Table 4 to 7, and not by pairwise comparison. The result is described in Table 8.

Table 8 is regarded as matrix S and the evaluation X_{AHP} based on explicit elements is derived from Eq.(3) in Step 5.

$$\begin{aligned} X_{AHP} &= S\boldsymbol{w} \\ &= (0.370, 0.458, 0.451, 0.458, \\ &\quad 0.459, 0.327, 0.472)^T \end{aligned} \tag{9}$$

There is little difference among priorities of B, C, D and E in the result of explicit evaluation. Then, in addition to the conventional method, it is necessary to evaluate alternatives from another perspective.

4.2 Evaluation based on Implicit Elements

According to the Step 6, we obtain an evaluation based on implicit element by adopting PCA. The result of Table 9 is obtained by applying PCA to the data in Table 8. The result gives us new indicators consisting of principal component score. The first PC (principal component) can be interpreted as being highly positively related to the abundances of Ap (0.531) and Lo (0.432), and negatively related to the abundance of Sa (-0.455), that is, it expresses the beautiful urbane

Table 3: Pairwise comparison between each criterion.

	Ac	Pr	Sa	Co	Lo	Wi	Eq	Ap
Ac	1	1	1/3	1	3	3	5	5
Pr	1	1	1	3	5	3	5	7
Sa	3	1	1	3	5	3	5	7
Co	1	1/3	1/3	1	3	3	5	5
Lo	1/3	1/5	1/5	1/3	1	3	3	5
Wi	1/5	1/5	1/5	1/5	1/3	1/3	1	1
Ap	1/5	1/7	1/7	1/5	1/5	1/5	1	1

$\lambda_{max} = 8.606$ $C.I. = 0.087$

Table 4: Evaluation values of criterion ”Ac”.

linguistic scale	value
inconvenience	0.188
moderate	0.354
convenience	1.000

Table 5: Evaluation values of criterion ”Pr”.

linguistic scale	value
low	1.000
moderate	0.464
expensive	0.208
very expensive	0.098

Table 6: Evaluation values of criteria ”Sa”,”Co”, ”Lo”,”Eq” and ”Ap”.

linguistic scale	value
very good	1.000
good	0.464
moderate	0.208
bad	0.098

Table 7: Evaluation values of criterion ”Wi”.

linguistic scale	value
very large	1.000
large	0.538
moderate	0.274
narrow	0.129
very narrow	0.068

Table 8: Absolute evaluation of alternatives about each criterion.

	Ac	Pr	Sa	Co	Lo	Wi	Eq	Ap
A	0.354	0.208	0.208	1.000	0.208	0.538	0.464	0.208
B	1.000	0.098	0.208	0.464	0.464	0.274	0.464	0.464
C	0.354	1.000	0.098	0.208	0.208	0.538	0.098	0.464
D	0.188	0.464	0.464	1.000	0.464	0.274	0.098	0.098
E	1.000	0.208	0.464	0.464	0.464	1.000	1.000	0.208
F	1.000	0.208	0.098	0.464	1.000	0.129	0.208	1.000
G	0.188	1.000	0.464	0.208	0.464	0.538	0.098	0.098

Table 9: Principal component which obtained from absolute evaluation.

	Principal Component : PC			
	1st PC	2nd PC	3rd PC	4th PC
Ac	0.349	0.466	0.202	-0.015
Pr	-0.168	-0.541	0.343	0.156
Sa	-0.455	0.167	-0.220	0.543
Co	-0.162	0.182	-0.705	-0.213
Lo	0.432	0.086	-0.118	0.776
Wi	-0.351	0.274	0.476	0.077
Eq	-0.159	0.589	0.231	-0.080
Ap	0.531	0.010	0.052	-0.144
cumulative contribution ratio	41.3%	71.2%	90.7%	97.7%

Table 10: Pairwise comparison between new criterion.

	C_1	C_2	C_3	C_4
C_1	1	1/5	1/3	1/5
C_2	5	1	3	1/3
C_3	3	1/3	1	1/5
C_4	5	3	5	1

$\lambda_{max} = 4.198 \quad C.I. = 0.066$

house which is not located in safety area. The second PC, on the other hand, is positively related to the abundance of Eq (0.589) and Ac (0.466), and negatively related to the abundance of Pr (-0.541). Therefore, the second index means that a house is prized convenience more than price. Similarly, the third indicator is interpreted in terms of Pr, Wi and Co, as a house which is affordable and large but inconveniently located. The forth indicator is characterized by Lo (0.776) and Sa (0.543), as a house that is at safe place and good environment. Four indicators are identified as the following decision criteria.

- the beautiful urbane house (C'_1)
- the house at more convenient place (C'_2)
- the affordable house (C'_3)
- the safety house in a good environment (C'_4)

The decision-maker constructs a evaluation matrix with respect to four criteria by using 9-point measurement. It is shown in Table 10. The weight vector $\boldsymbol{w}'$ of new criteria C' is acquired as an eigenvector for a maximum eigenvalue of the matrix composed of Table 10, given as

$$\boldsymbol{w}' = (0.064, 0.271, 0.122, 0.544)^T \tag{10}$$

By applying Eq.(4), the vector $\boldsymbol{z}_k$ of principal component scores is acquired regarding the new criterion C'_k. Then, we obtain the implicit evaluation by normalization of Eq.(6).

$$\begin{aligned} X_{PCA} &= (\boldsymbol{z}_1\boldsymbol{z}_2\boldsymbol{z}_3\boldsymbol{z}_4)\boldsymbol{w}' \\ &= (0.126, 0.231, 0.052, 0.006, \\ &\quad 0.366, 0.196, 0.022)^T \end{aligned} \tag{11}$$

As in Step 7, the final evaluation X is obtained to synthesize X_{AHP} and X_{PCA}, given as

$$\begin{aligned} X &= (0.110, 0.251, 0.056, 0.007, \\ &\quad 0.399, 0.152, 0.025)^T \end{aligned} \tag{12}$$

The priority of each alternative is $E > B > F > A > C > G > D$.

5 CONCLUSIONS

In this paper, a decision-making model for considering both explicit and implicit element was presented. We utilized the absolute measure method on AHP for determining the evaluation based on explicit element. On the other hand, we proposed the method adopting PCA for determining the evaluation based on implicit element.

The increase in criteria or alternatives becomes frequently the cause of making vagueness in the decision-maker's judgment. In the case, we think that our proposed procedure is effective. In the conventional method, the priority of alternatives is obtained using directly the score given by the decision-maker. However, utilizing the score accompanied by vague judgments directly makes the reliability of evaluation lower. Therefore, we proposed the method which enables decision-makers to evaluate based on implicit elements extracted from the score accompanied by vague judgments. Further, we tried to develop the model for decision making by synthesizing explicit evaluation and implicit evaluation. Our approach enables decision-makers to achieve clearer result in decision making.

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